An inverse radiation-conduction problem of estimating temperature-dependent emissivity using a combined method of genetic algorithm and conjugate gradient method

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Abstract

In this paper, the solution of the inverse, conduction-radiation problem in a two-dimensional system was analyzed to determine the temperature-dependent emissivity at the boundary for a participating media. The inverse problem was solved through the minimization of performance function, which was expressed by the sum of square residuals between estimated and exact heat fluxes, using a combined method of the genetic algorithm and the conjugate gradient method. For this, we supposed that the emissivity was represented as a function of temperature at the boundary surface. The effects of the measurement errors on the accuracy of the inverse analysis are investigated.

Keywords: Combined method of genetic algorithm and conjugate gradient method, Emissivity, Inverse radiation- conduction analysis, Measurement error

Introduction

The Inverse conduction-radiation heat transfer problems are in turn classified as identification and design problems. In identification problems, temperature is measured at some locations within the enclosure or heat flux is measured at some locations on boundary surface, and based on these measurements and using inverse algorithms, the unknown parameters such as the thermal properties of the enclosure boundaries are estimated [1]. The inverse design techniques attempts to solve directly for the conditions in the system that would provide the two specifications on the design surface. Many researchers study to estimate thermal properties [2-6]. Ki Wan Kim et al. [7] estimated the emissivities in a two-dimensional irregular geometry by inverse radiation analysis using hybrid genetic algorithm.

In the present investigation, we consider heat transfer by combined radiation with conduction through participating media capable of absorbing, emitting and isotropic scattering thermal radiation. We solved an inverse conduction-radiation heat conduction problem. The finite volume method was employed to solve energy equation. The radiative transfer equation was solved with the discrete ordinate method. A combined method of the genetic algorithm and the conjugate gradient method were used.

Direct Solution

The energy conservation equation for steady state condition in the absence of convection and heat generation, assuming constant properties in a two-dimensional Cartesian coordinate system is expressed as:

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=(1-\omega_\lambda)\beta(4\pi I_b-G) \tag{1}$$

The radiative intensity can be found from radiative transfer equation (RTE). RTE in the presence of an absorbing, emitting and scattering medium is written as:

$$\frac{dI}{ds}=s.\nabla I(s)=\sigma_{a\lambda}I_{b\lambda}-\beta_\lambda I_\lambda + \frac{\sigma_{s\lambda}}{4\pi}\int_{4\pi} I_\lambda(s_i)\phi_\lambda(s_i,s)d\Omega_i \tag{2}$$

The equation of transfer in its quasi-steady form, equation (2), is a first-order differential equation in intensity (for a fixed direction s). For a surface that emits and reflects diffusely, the exiting intensity is independent of direction. Therefore, at a point r_w on the surface, leading to:

$$I(r_w)=\varepsilon_w(r_w,T)I_b(r_w)+\frac{\rho(r_w)}{\pi}\int_{n.\hat{s}'<0} I(r_w,\hat{s}')|n.\hat{s}'|\,d\Omega' \tag{3}$$

The emissivity at the boundary was represented as the function of temperature in the boundary such as:

$$\varepsilon_w=\sum c_k\phi_k(T) \tag{4}$$

In direct solution, these coefficient are known, but in inverse analysis should be estimated.

The conjugate gradient method is combined with the genetic algorithm to solve a set of equations in inverse problem. Genetic algorithm is used to select an initial guess for the conjugate gradient method and the conjugate gradient method used to estimate coefficient. The inverse problem tries to estimation of the coefficients c_k defined by (4). The objective function for minimization is defined as follow as:

$$G(P^k)=\sum_{n=1}^{M}(q_e^*-q_m^*)^2 \tag{5}$$

Results and Discussion

In this section, to demonstrate the accuracy and efficiency of this method, we have chosen three examples. To show the effects of measurement errors on the emissivity, we consider the random errors. The simulated measured heat fluxes with random errors are obtained by adding normally distributed errors into the exact heat fluxes on the domain as:

$$q_m=(1+\lambda)q_{ex} \tag{6}$$

$$|\lambda| \leq \omega \tag{7}$$

Where λ is the random error of measurement, and ω is the bound of λ. q_{ex} in (6) is the exact heat flux and q_m is the measured heat flux at the bottom wall.

Example1. In the first example, we considered that the boundary condition for temperature at the top wall is polynomial form and the emissivity is a sinusoidal function as:

$$T^*_4 = 0.2 + 0.5X - 0.3X^2$$
$$\varepsilon_4 = c_1 + c_2 \sin(c_3 T^*_4) \tag{8}$$

Where c_1, c_2 and c_3 are supposed to be 0.5, 0.3 and 0.8 respectively in the exact solution. Fig. 1 shows the estimated emissivity in example 1.

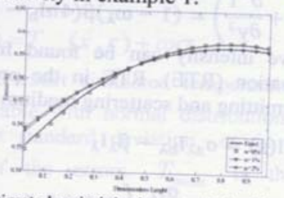

Fig. 1: The estimated emissivity in example 1 with measurement errors $\omega = 0,\ 1\%, 2\%$

Example2. The emissivity and the boundary condition for temperature at the top wall in the second example are presented in the following form,

$$T^*_4 = 0.5 + 0.5X \tag{11}$$
$$\varepsilon_4 = c_1\sqrt{T^*_4} + c_2 \exp(c_3 T^*_4)$$

Where c_1, c_2 and c_3 are supposed to be 0.3, 0.2 and 0.5 in the exact solution respectively. Fig. 2 illustrates that the estimated emissivity is acceptable even if measurement errors increases.

Example3. The emissivity is presented as a sinusoidal-exponential form. The emissivity and the boundary condition for temperature at the top wall are assumed in following form:

$$T^*_4 = 0.5 + 0.5X \tag{12}$$
$$\varepsilon_4 = c_1 + c_2 \exp(c_3 T^*_4) + c_4 \sin(c_5 T^*_4)$$

Where $c_1 = 0.3$, $c_2 = 0.2$, $c_3 = 0.4$, $c_4 = 0.1$ and $c_5 = 2$. Fig. 3 shows the comparison of the emissivity between the exact solution and the inverse solution. This indicates that that present approach provide an accurate estimation.

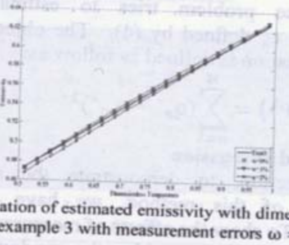

Fig. 2: Variation of estimated emissivity with dimensionless temperature in example 3 with measurement errors $\omega = 0,\ 1\%, 2\%$

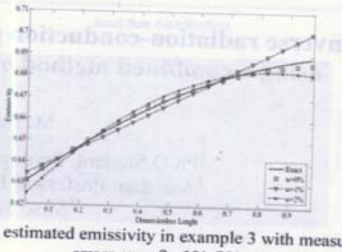

Fig. 3: The estimated emissivity in example 3 with measurement errors $\omega = 0,\ 1\%, 2\%$

Conclusions

In this paper, we solved an inverse conduction-radiation problem in a two-dimensional system to determine the temperature-dependent emissivity at the boundary for an absorbing, emitting, isotropic scattering and gray rectangular medium with opaque and diffuse bounding surfaces from the knowledge of the wall heat flux. For this, the emissivity was approached with the function of temperature. Three examples were used to show this algorithm. Results show the algorithm can estimate the unknown emissivity. Also the accuracy of algorithm decreases when measurement error increases.

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Abstract
In this paper, the solution of the inverse, conduction-radiation problem in a two-dimensional system was analyzed to determine the temperature-dependent emissivity at the boundary for a participating media. The inverse problem was solved through the minimization of performance function, which was expressed by the sum of square residuals between estimated and exact heat fluxes, using a combined method of the genetic algorithm and the conjugate gradient method. For this, we supposed that the emissivity was represented as a function of temperature at the boundary surface. The effects of the measurement errors on the accuracy of the inverse analysis are investigated.
Keywords: Combined method of genetic algorithm and conjugate gradient method, Emissivity, Inverse radiation- conduction analysis, Measurement error

Introduction
Conduction-radiation heat transfer is an important heat transfer mode in various engineering application. Two studies can be done about combined heat transfer by conduction and radiation. In direct radiation-conduction investigation, boundary conditions and the radiative properties of walls and medium are given. These types of problems are mathematically well posed and therefore, the radiation intensities, temperature distribution and heat flux distribution can be determined. In the second study, determination of the radiation properties, boundary condition, and the temperature profile or source term distribution from various types of radiation measurements are done by inverse analysis. In this problem, the mathematical formulation often leads to an ill-posed problem, and special numerical techniques must be employed to solve the corresponding set of ill-conditioned linear equations [1].
The Inverse conduction-radiation heat transfer problems are in turn classified as identification and design problems. In identification problems, temperature is measured at some locations within the enclosure or heat flux is measured at some locations on boundary surface, and based on these measurements and using inverse algorithms, the unknown parameters such as the thermal properties of the enclosure boundaries are estimated [2]. The inverse design techniques attempts to solve directly for the conditions in the system that would provide the two specifications on the design surface. Many researchers study to estimate thermal properties [3-7]. Ki Wan Kim et al. [8] estimated the emissivities in a two-dimensional irregular geometry by inverse radiation analysis using hybrid genetic algorithm.
In the present investigation, we consider heat transfer by combined radiation with conduction through participating media capable of absorbing, emitting and isotropic scattering thermal radiation. We solved an inverse conduction-radiation heat conduction problem. The finite volume method was employed to solve energy equation. The radiative transfer equation was solved with the discrete ordinate method. A combined method of the genetic algorithm and the conjugate gradient method were used.

Direct Solution
Consider steady state combined conduction and radiation heat transfer in a gray, absorbing, emitting and isotropic scattering slab. Fig. 1 shows regular quadrilateral enclosure which is filled with an absorbing, emitting, scattering and gray gas with $\sigma_a = 0.1\ \mathrm{m}^{-1}$ and $\sigma_s = 0.1\ \mathrm{m}^{-1}$.

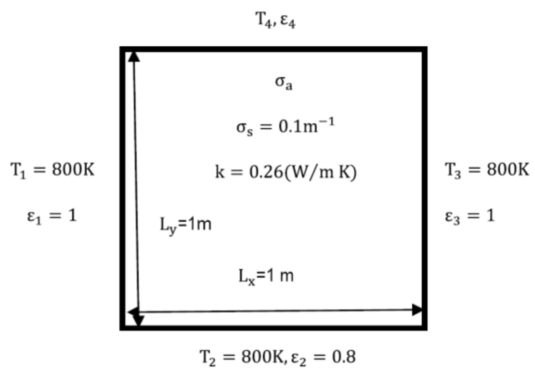

Fig. 1: A regular quadrilateral enclosure

The energy conservation equation in the absence of convection and heat generation, assuming constant properties in a two-dimensional Cartesian coordinate system is expressed as:

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = (1 - \omega_\lambda)\beta(4\pi I_b - G) \tag{1}$$

where ω_λ is the single scattering albedo and it is defined as:

$$\omega_\lambda = \frac{\sigma_{s\lambda}}{\beta_\lambda} \tag{2}$$

where $\sigma_{s\lambda}$is the (linear) scattering coefficient for scattering and β_λ is extinction coefficient. An extinction coefficient is defined as:

$$\beta_\lambda = \sigma_{a\lambda} + \sigma_{s\lambda} \tag{3}$$

where $\sigma_{a\lambda}$ is known as the (linear) absorption coefficient, $I_{b\lambda}$ is blackbody intensity:

$$I_{b\lambda} = \sigma_b T^4/\pi \tag{4}$$

σ_b is the Stefan–Boltzmann constant. G is known as the incident radiation function (since it is the total intensity impinging on a point from all sides) and can be expressed as follows:

$$G_\lambda = \int_{4\pi} I_\lambda(s)\,d\Omega \tag{5}$$

where $d\Omega$ is the solid angle and $I_\lambda$is radiative intensity. The radiative intensity can be found from radiative transfer equation (RTE). RTE in the presence of an absorbing, emitting and scattering medium is written as:

$$\frac{dI}{ds} = s.\nabla I(s) = \sigma_{a\lambda} I_{b\lambda} - \beta_\lambda I_\lambda + \frac{\sigma_{s\lambda}}{4\pi}\int_{4\pi} I_\lambda(s_i)\phi_\lambda(s_i, s)\,d\Omega_i \tag{6}$$

The equation of transfer in its quasi-steady form, equation (6), is a first-order differential equation in intensity (for a fixed direction s). For a surface that emits and reflects diffusely, the exiting intensity is independent of direction. Therefore, at a point $r_w$on the surface, boundary condition are presented in the following form:

$$I(r_w) = \varepsilon_w(r_w, T) I_b(r_w) + \frac{\rho(r_w)}{\pi}\int_{n.\hat{s}'<0} I(r_w, \hat{s}')|n.\hat{s}'|\,d\Omega' \tag{7}$$

where n is the local outward surface normal and $n.s_i = \cos\theta$ is the cosine of the angle between any incoming direction s_i and the surface normal. Therefore, the outgoing intensity is not generally known explicitly, but is related to the incoming intensity and wall temperature profile. The emissivity at the boundary was represented as the function of temperature in the boundary such as:

$$\varepsilon_w = \sum c_k \emptyset_k(T) \tag{8}$$

In direct solution, these coefficient are known, but in inverse analysis should be estimated.

DISCRETE ORDINATE METHOD

Discrete ordinate method is used to transform the equation of transfer into a set of simultaneous partial differential equations. In the discrete ordinates method (S_N), equation (6) is solved for a set of different directions:

$$s_i = \xi_i i + \eta_i j + \mu_i k_i \tag{9}$$

1,2,..., N, and the integrals over direction are replaced by numerical quadratures, that is:

$$s_i.\nabla I(s_i) = \sigma_{a\lambda} I_{b\lambda} - \beta_\lambda I_\lambda + \frac{\sigma_{s\lambda}}{4\pi}\sum_{j=1}^{N} w_j I_\lambda(s_j)\phi_\lambda(s_j, s) \tag{10}$$

Where the w_i are the quadrature weights associated with the directions s_i. The radiative heat flux, inside the medium or at a surface, may be found from its definition is given as:

$$q = \int_{4\pi} I(\hat{s})\hat{s}\,d\Omega = \sum_{i=1}^{N} w_j I_i\,\hat{s_j} \tag{11}$$

The incident radiation G is similarly determined as:

$$G = \sum_{i=1}^{N} w_i\, I_i \tag{12}$$

For 2D Cartesian coordinates, RTE can be rewritten as:

$$\zeta_i \frac{\partial I_i}{\partial x} + \zeta_i \frac{\partial I_i}{\partial x} + \beta I_i = \beta S_i \quad i = 1, 2, \ldots, N \tag{13}$$

where S_i is again shorthand for the radiative source function:

$$S_i = (1 - \omega_\lambda) I_b + \frac{\omega}{4\pi}\sum_{j=1}^{N} w_j I_\lambda(s_j)\phi_\lambda(s_j, s) \tag{14}$$

FINITE VOLUME METHOD

In solving the radiative transfer equation, the FVM is adopted for its convenience in selecting the solid angle while guaranteeing an exact global conservation of radiative energy. A general volume element is shown in Fig. 2. The volume element has four face areas $A_E$and A_W (in the x direction), and $A_N$and A_S (in the y direction).

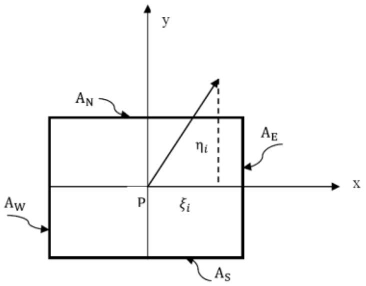

Fig. 2: A general two dimensional control volume

For each volume element, equation (6) is integrated over the volume element and over each of the solid angle elements Ω_i. The RTE with finite volume method appears as:

$$I_{pi} = \frac{\beta_\lambda S_{pi} V + |\zeta_i| A_{EW} I_{Wi}/\gamma_x + |\eta_i| A_{NS} I_{si}/\gamma_y}{\beta_\lambda V + |\zeta_i| A_{EW}/\gamma_x + |\eta_i| A_{NS}/\gamma_y} \tag{15}$$

in which γ_x and γ_y are constants $1/2 < \gamma_x$, $\gamma_y < 1$, and the scheme is known as weighted diamond differencing as proposed by Carlson and Lathrop. Where I_{pi} and S_{pi} are volume averages. A_{EW} and A_{EW} are:

$$\begin{aligned} A_{EW} &= (1-\gamma_x)A_E + \gamma_x A_W \\ A_{NS} &= \left(1-\gamma_y\right) A_N + \gamma_y A_S \end{aligned} \tag{16}$$

In the present work, as per symmetry scheme $\gamma_x = \gamma_y = 0.5$ has been adopted. Intensity on the other of faces can be calculated by relating cell-edge intensities to the volume-averaged intensity. Most often a linear relationship is chosen, i.e.:

$$I_{pi} = \gamma_y I_{Ni} + \left(1-\gamma_y\right) I_{Si} = \gamma_y I_{Ei} + (1-\gamma_y) I_{Wi} \tag{17}$$

The following parameters are used for expressing above governing equations in dimensionless form [9]:

$$\begin{aligned} &T^* = \frac{T}{T_h} \quad x^* = \frac{x}{L} \quad y^* = \frac{y}{L} \\ &RC = \frac{\sigma T_h{}^3 L}{k} \\ &I^* = \frac{I}{\sigma T_h{}^4} \quad S^* = \frac{S}{\sigma T_h{}^4} \\ &G^* = \frac{G}{\sigma T_h{}^4} \quad \tau = \beta L \end{aligned} \tag{18}$$

where RC and τ are radiation–conduction parameter and optical thickness respectively. The energy conservation equation (1) for the transport phenomena in dimensionless form is represented as:

$$\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right) = (1-\omega).\tau.RC(4T^{*4} - G^*) \tag{19}$$

where:

$$G^* = \sum_{i=1}^{N} w_i I^*{}_i \tag{20}$$

By using FVM, the energy conservation equation can be rewritten as:

$$a_P T_P{}^* + a_w T_w{}^* + a_s T_s{}^* + a_N T_N{}^* = b \tag{21}$$

where:

$$b = RC(1-\omega)\tau(12 T^*{}_{oP}{}^4 + G^*)\Delta V \tag{22}$$

By using DOM, the dimensionless form of equation (7) can be expressed as:

$$\begin{aligned} &I_w{}^*(r_w) \\ &= \varepsilon_w(r_w, T^*) I_{bw}{}^* \\ &+ \frac{\rho_w(r_w, T^*)}{\pi} \sum_{n.s_j<0} w_j I^*(r_w, s_j)|n.s_j| \quad n.s_i > 0 \end{aligned} \tag{23}$$

In this paper, the spatial domains are discretized into 50 × 50 control volumes. The discrete ordinate S_4 method is employed to solve the radiative transfer equation. By suppose emissivity at top boundary is known, equation (6)–(23) are solved in this way:

- Assume an initial temperature filed and radiative source ($T^* = 0$, $S^* = 0$).
- Calculate $I^*{}_b$.
- Solve the radiative transfer equation, equation (15) by using the S4 method.
- Obtain the temperature field by using equation (19).
- Repeat the above procedure until the converged intensity.

6 INVERSE SOLUTION

The conjugate gradient method is combined with the genetic algorithm to solve a set of equations in inverse problem, which are expressed by errors between estimated and desired heat flux. Genetic algorithm is used to select an initial guess for the conjugate gradient method and the conjugate gradient method used to estimate coefficient.

For this, we considered a known emissivity profile for top wall and calculated heat flux at the bottom wall. The inverse problem tries to estimation of the coefficients c_k defined by (8). The objective function for minimization is defined as follow as:

$$G(P^k) = \sum_{n=1}^{M} (q_e{}^* - q_d{}^*)^2 \tag{24}$$

where M is the total number of measurements $q_d{}^*$ is the vector of measured heat flux obtained from the solution of the direct problem and $q_e{}^*$ is the vector of estimated heat flux obtained from the solution of the inverse problem.

GENETIC ALGORITHM

The genetic algorithm (GA) is a search-based method that mimics the process of natural evolution using techniques influenced by natural evolution such as selection, mutation and crossover. Genetic algorithms were first used by Holland (1975). Any possible solution is an individual as shown in Fig. 3. Variables are coded in float-point or binary strings. A collection of individuals is called a population. At first generation, an initial population is randomly generated. After evaluating the fitness of each individual, fitter individuals are selected for reproducing offspring for the next generation.

There are many different types of selection such as roulette wheel selection. In roulette wheel selection, individuals are given a probability of being selected that is directly proportionate to their fitness. Two individuals are then chosen randomly based on these probabilities and produce offspring.

Individuals chosen by selection operation endure the operation of crossover and mutation for the reproduction of renewed individuals.

In crossover operation, individuals, meet their mates and swap their genes. There are various crossover schemes such as one point crossover, two points crossover, multi point crossover, matrix crossover and uniform crossover. In the one point crossover, a locus is chosen at which the remaining alleles exchanged from one parent to the other. The point at which the chromosome is broken depends on the randomly selected crossover point.

The mutation operator permits some genes to change their values within the design space in order to ensure that the individuals are not all exactly the same. In the mutation, genes changed with a small amount or replaced with a new value. The probability of mutation controls the number of genes suffering the mutation operation. The probability of mutation is usually between 1 and 2 tenths of a percent.

In this study, the population size is fixed to 20 to decrease the computational time and the float-point representation is used to reduce the length of the chromosome. A total generation number of 100 is used to product initial guess for conjugate gradient method. The nonuniform mutation [10] and the one point crossover are employed to breed new individuals. The algorithm continues since the maximum number of generation is achieved.

The computational procedure for the genetic algorithm can be presented as follows

- Select the initial population
- Calculate the fitness of each individual in that population
- Repeat on this generation until the maximum number of generation is achieved:
    1. Select the best individuals for reproduction
    2. Reproduce new individuals through crossover and mutation operations to give birth to offspring
    3. Evaluate the individual fitness of new individuals
    4. Replace old population with new individuals

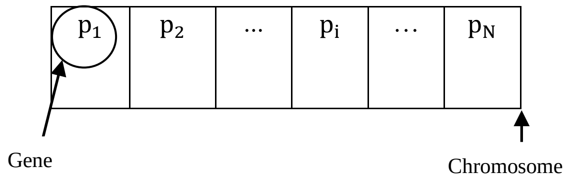

Fig. 3: Illustration of individual and gene

CONJUGATE GRADIENT METHOD

In the inverse parameter estimation problem considered in this study, we intend to recover the vector of unknown parameters $P = (c_1, c_2, \dots, c_k)$ by the minimization of the sum of squares function. Equation (28) can be rewritten as:

$$G(P^k) = [q_d(P^k) - q_e]^T [q_d(P^k) - q_e] \tag{25}$$

In this study, $q_e(P^k)$ is the vector of estimated heat fluxes achieved from the solution of the direct problem by using the current available estimate for P. In this study, the conjugate gradient method is employed to solve the inverse conduction problem; the iterative process is [11].

$$P^{k+1} = P^k - \beta^k d^k \tag{26}$$

where β^k is the step size, d^k is the direction of descent which is determined from:

$$d^k = \nabla G(P^k) + \alpha^k d^{k-1} \tag{27}$$

and the conjugate coefficient α^k is computed from:

$$\alpha^k = \begin{cases} 0 & k = 0 \\ \dfrac{\nabla G(P^k) \nabla G^T(P^k)}{\nabla G(P^{k-1}) \nabla G^T(P^{k-1})} & k = 1, 2, .. \end{cases} \tag{28}$$

Here the row vector $\nabla G(P^k)$ is the gradient of the objective function. The step size is determined from

$$\beta^k = \frac{[J^k d^k]^T (q_d(P^k) - q_e)}{[J^k d^k]^T [J^k d^k]} \tag{29}$$

where J is the sensitivity coefficient vector. The sensitivity coefficient vector is computed from (30). In order to calculate the sensitivity coefficient, finite-difference approximation is used.

$$J_{ij} = \frac{\partial q_{e,i}}{\partial p_j} = \frac{q_{e,i}(p_1, \dots, p_j + \epsilon p_j, \dots, p_{n_p}) - q_{e,i}(p_1, \dots, p_j, \dots, p_{n_p})}{\epsilon p_j} \tag{30}$$

The gradient of the objective function is determined by differentiating (25) with respect to P to obtain

$$\nabla G(P^k) = -2(J^k)^T (q_d(P^k) - q_e) \tag{31}$$

The square root of the variance is given by:

$$\delta = \frac{1}{M}\sum_{n=1}^{M}\left|\frac{({q_d}^n - {q_e}^n)}{{q_d}^n}\right| \qquad (32)$$

where M is total number of data points. The computational procedure for the solution of the inverse problem can be summarized as follows:
Step 1: Pick an initial guess P . Set k=0.
Step 2: Solve the direct problem to compute the dimensionless heat fluxes.
Step 3: Calculate the objective function. Terminate the iteration process if the specified stopping criterion is satisfied(δ). Otherwise go to Step 4.
Step 4: Compute the sensitivity coefficient vector by utilizing (30).
Step 5: Compute the gradient of the objective function $\nabla G(P^k)$ from (31).
Step 6: Knowing $\nabla G(P^k)$, compute the conjugate coefficient α^k and the direction of descent d^k.
Step 7: Compute the step size β^k.

Results and Discussion
In this section, to demonstrate the accuracy and efficiency of this method, we have chosen three examples. To show the effects of measurement errors on the emissivity, we consider the random errors. The simulated measured heat fluxes with random errors are obtained by adding normally distributed errors into the exact heat fluxes on the domain as:

$$q_m = (1 + \lambda) q_{ex} \qquad (33)$$

$$|\lambda| \leq \omega \qquad (34)$$

where λ is the random error of measurement, and ω is the bound of λ. q_{ex} in (33) is the exact heat flux and q_m is the measured heat flux at the bottom wall.

Example1. In the first example, we considered that the boundary condition for temperature at the top wall is polynomial form and the emissivity is a sinusoidal function as:

$$T^*{}_4 = 0.2 + 0.5X - 0.3X^2 \qquad (35)$$
$$\varepsilon_4 = c_1 + c_2 \sin(c_3 T^*{}_4)$$

where c_1, c_2 and c_3 are supposed to be 0.5, 0.3 and 0.8 respectively in the exact solution. Fig. 4 shows the estimated emissivity in example 1.

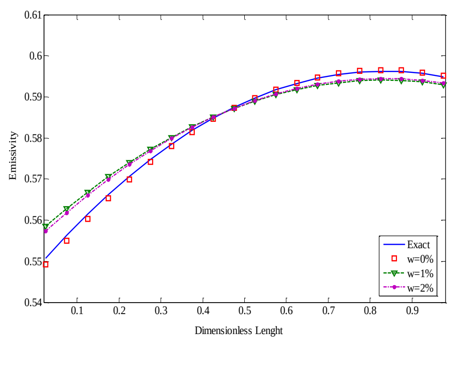

Fig. 4: The estimated emissivity in example 1 with measurement errors $\omega = 0,\ 1\%, 2\%$

Example2. The emissivity and the boundary condition for temperature at the top wall in the second example are presented in the following form:

$$T^*{}_4 = 0.5 + 0.5X \qquad (36)$$
$$\varepsilon_4 = c_1\sqrt{T^*{}_4} + c_2 \exp(c_3 T^*{}_4)$$

where c_1, c_2 and c_3 are supposed to be 0.3, 0.2 and 0.5 in the exact solution respectively. Fig. 5 illustrates that the estimated emissivity is acceptable even if measurement errors increases.

Example3. The emissivity is presented as a sinusoidal-exponential form. The emissivity and the boundary condition for temperature at the top wall are assumed in following form:

$$T^*{}_4 = 0.5 + 0.5X \qquad (36)$$
$$\varepsilon_4 = c_1 + c_2 \exp(c_3 T^*{}_4) + c_4 \sin(c_5 T^*{}_4)$$

where $c_1 = 0.3$, $c_2 = 0.2$, $c_3 = 0.4$, $c_4 = 0.1$ and $c_5 = 2$. Fig. 6 shows the comparison of the emissivity between the exact solution and the inverse solution. This indicates that that present approach provide an accurate estimation.

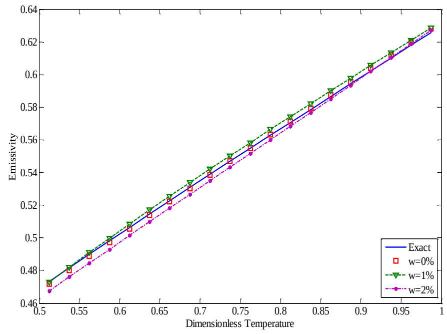

Fig. 5: Variation of estimated emissivity with dimensionless temperature in example 3 with measurement errors $\omega = 0,\ 1\%, 2\%$

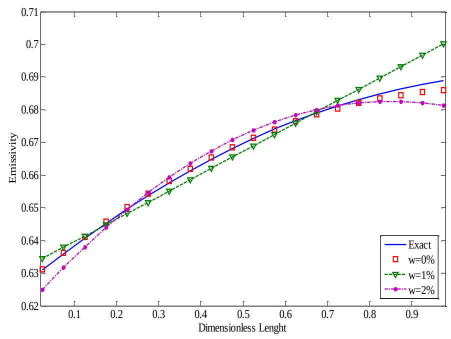

Fig. 6: The estimated emissivity in example 3 with measurement errors $\omega = 0,\ 1\%, 2\%$

Conclusions
In this paper, we solved an inverse conduction-radiation problem in a two-dimensional system to determine the

temperature-dependent emissivity at the boundary for an absorbing, emitting, isotropic scattering and gray rectangular medium with opaque and diffuse bounding surfaces from the knowledge of the wall heat flux. For this, the emissivity was approached with the function of temperature. Three examples were used to show this algorithm. Results show the algorithm can estimate the unknown emissivity. Also the accuracy of algorithm decreases when measurement error increases.

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