Market Oriented Reactive Power Expansion Planning

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***Abstract*-- This paper presents a method for market oriented reactive power expansion. In this method first ISO proposes some locations for reactive power expansion based on system requirements and operator's experiences. The investors determine the optimal locations for reactive power expansion by computing annual expansion profit of different candidates. Finally, the presented method is applied to 8-bus PJM power system to determine optimal amount and location for reactive power expansion.**

I. INTRODUCTION

Power system has been faced with variety of optimization problems such as economic dispatch and power system expansion planning [1]. In regulated power systems, the main objective function of these problems was total cost of the system. Deregulation has changed the aim of power system operation and planning and increased system uncertainties. Increasing competition among power producers, decreasing discrimination, and compromising among conflicting/supporting desires of power system stakeholders are new objectives of power system operation and planning. Approaches for taking into account the above objectives in transmission expansion planning are presented in [2] and [3].

Power system expansion planning is divided into generation, transmission, distribution, and reactive power expansion planning. Capacitor optimal placement is among the accomplished works in reactive power planning. Capacitor optimal placement was paid attention to since 1960s. The issue of capacitor optimal placement was studied by J. J. Gringer in 1980s [4]-[5]. In [6], M. E. Baran formulated capacitor optimal placement as a nonlinear mixed integer problem. Different mathematical and heuristic methods have been presented to solve this nonlinear mixed integer optimization problem. A survey on reactive power planning methods was fulfilled by W. Zhang in [7]. In [7] formulation, advantage, and disadvantage of nine different methods which has been used to solve reactive planning problem are discussed.

In this paper Market Oriented Reactive Power Expansion Planning (MORPEP) is addressed. The task of MORPEP is to determine the optimal location and capacity for reactive power expansion. Annual profit per unit investment is used as criterion for determining the optimal expansion plan. Effective factors on the profit of a specified reactive power producer can be classified as follows:

(a) Submitted bid of the producer for reactive power, and
(b) Location of the reactive power producer in the network.

MORPEP expands reactive power of candidates who have appropriate location in the network, and/or submit suitable bids for reactive power production. Expansion the capacity of these candidates will lead to decrease the operational cost of the system.

II. OVERVIEW

For reactive expansion planning first ISO proposes some locations as candidate. Investors should determine the optimal location for investment. In order to determine the optimal candidate, an unlimited reactive generator is added to each candidate location. Dispatched reactive power of each candidate is determined for peak load operating point of the system. The candidates which have acceptable level of dispatched reactive power are selected as proper candidates for expansion planning. Annual Profit-Bid curve is drawn for each candidate to determine the max annual profit of each candidate. The candidate which has the max annual expansion profit per unit investment has the optimal location for investment. The reactive power corresponding to the max annual profit specifies the optimal amount of reactive power expansion. The bid corresponding to the max annual profit specifies the optimal bid which maximizes the annual profit. In order to consider the uncertainty in competitor's bid in expansion planning, probable occurring scenarios for competitor's bid are identified. For each candidate max Annual Expansion Profit per Unit of Investment (AEPPUI) is computed under different scenarios. A probability density function is fitted to AEPPUI of each candidate using Kernel method. The candidate which has the max average and min variance in AEPPUI is selected as the final plan. This plan has max average profit and less risk concerning investment and capital return.

The rest of the paper is organized as follows. In section III simultaneous scheduling of active and reactive power is modeled. Determining proper candidate locations for reactive

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power expansion is discussed in section IV. In section V a method for computing max AEPPUI is presented. Computing probability density function of AEPPUI and selecting the final plan is discussed in section VI. The presented method is applied to an 8-bus test system in section VII. Conclusion in section VIII closes the paper.

III. ACTIVE & REACTIVE POWER SCHEDULING

Simultaneous scheduling of active and reactive power is modeled by an optimization problem. The objective function is the total cost of system operation. Power flow equations, transmission line limitations, voltage limitations, power production restrictions, and power consumption restrictions are constraints of this optimization problem. The problem is formulated as follow:

Min:
$$J(\mathbf{P}_G,\mathbf{P}_D,\mathbf{Q}_G,\mathbf{Q}_D) = S_{Base}[\mathbf{C}_{\mathbf{P}G}{}^T\,\mathbf{P}_G + \mathbf{C}_{\mathbf{Q}G}{}^T\,\mathbf{Q}_G + \mathbf{C}_{\mathbf{P}D}{}^T\,(\overset{\max}{\mathbf{P}_D} - \mathbf{P}_D) + \mathbf{C}_{\mathbf{Q}D}{}^T\,(\overset{\max}{\mathbf{Q}_D} - \mathbf{Q}_D)] \qquad (1)$$

S.t.:

$$P_{Gi} - P_{Di} = \sum_{j=1}^{n} V_i V_j Y_{ij}\cos(\delta_j - \delta_i + \gamma_{ij}) \qquad i=1,\dots,n \qquad (2)$$

$$Q_{Gi} - Q_{Di} = -\sum_{j=1}^{n} V_i V_j Y_{ij}\sin(\delta_j - \delta_i + \gamma_{ij}) \qquad i=1,\dots,n \qquad (3)$$

$$V_{i\min} \le V_i \le V_{i\max} \qquad i=1,\dots,n \qquad (4)$$

$$P_{Gi}{}^{\min} \le P_{Gi} \le P_{Gi}{}^{\max} \qquad i=1,\dots,n \qquad (5)$$

$$Q_{Gi}{}^{\min} \le Q_{Gi} \le Q_{Gi}{}^{\max} \qquad i=1,\dots,n \qquad (6)$$

$$S_{ij}^{\min} \le S_{ij} \le S_{ij}^{\max} \qquad i=1,\dots,n \qquad (7)$$

Where:

J	1×1	total operation cost in \$/h
S_{base}	1×1	base of power in MVA
C_{PG}	$N_b\times 1$	vector of generator bids in \$/MWh (this vector is submitted by producers)
C_{PD}	$N_b\times 1$	vector of load bids in \$/MWh (this vector is submitted by consumers)
C_{QG}	$N_b\times 1$	vector of generator bids in \$/MVarh (this vector is submitted by producers)
C_{QD}	$N_b\times 1$	vector of load bids in \$/MVarh (this vector is submitted by consumers)
P_G	$N_b\times 1$	vector of active power generations in pu (this vector is the output of optimal power flow)
P_D	$N_b\times 1$	vector of active power consumption in pu (this vector is the output of optimal power flow)
Q_G	$N_b\times 1$	vector of reactive power generations in pu (this vector is the output of optimal power flow)
Q_D	$N_b\times 1$	vector of reactive power consumption in pu (this vector is the output of optimal power flow)
$P_G^{\min}, P_G^{\max}$	$N_b\times 1$	vectors of min and max active power generation limits in pu (these vectors are submitted by producers)
$P_D^{\min}, P_D^{\max}$	$N_b\times 1$	vectors of min and max active loads limits in pu (these vectors are submitted by consumers)
$\overset{\min}{\mathbf{Q}_G}, \overset{\max}{\mathbf{Q}_G}$	$N_b\times 1$	vectors of min and max reactive power generation limits in pu (these vectors are submitted by producers)
$\overset{\min}{\mathbf{Q}_D}, \overset{\max}{\mathbf{Q}_D}$	$N_b\times 1$	vectors of min and max reactive loads limits in pu (these vectors are submitted by consumers)
$\overset{\min}{\mathbf{S}_{ij}}, \overset{\max}{\mathbf{S}_{ij}}$	$N_b\times 1$	vectors of min and max of line thermal limits in pu(these vectors are submitted by transmission owners)
V_i	1×1	voltage magnitude of bus i
δ_i	1×1	voltage angle of bus i
Y_{ij}	1×1	magnitude of member ij of admittance matrix
γ_{ij}	1×1	angle of member ij of admittance matrix

First and second terms of (1) refer to the active and reactive power generation cost respectively. Third and forth terms of (1) refer to active and reactive load curtailment cost respectively. Equations (2)-(7) show active and reactive power flow equations, bus voltages limits, active and reactive power generation limits, and transmission lines limits respectively. Note since ac power flow equations are considered in constraints generation is equal to consumption plus losses. Hence, transmission losses are considered in this model.

IV. DETERMINING REACTIVE POWER EXPANSION CANDIDATES

The ISO has enough experience for suggesting primary locations for reactive power expansion based on system requirements. The investors should determine the best candidate for expansion. The following procedure is used to determine proper candidates for reactive power expansion.

(a) *Determining the primary candidate locations*: ISO proposes some location for reactive expansion based on the system requirements. The locations of the network in which power plants and power stations were installed, are appropriate locations for reactive power expansion, because investment cost for reactive power expansion in these locations are less than other locations.

(b) *Adding new reactive power generators*: in order to determine the proper candidate locations for reactive expansion a new unlimited reactive power generator is added to each primary candidate location.

(c) *Running optimization*: the optimization problem of (1)-(7) are solved for peak load and other probable operating points.

(d) *Identifying the proper candidate locations*: proper candidate locations for reactive power expansion are those that upon adding unlimited reactive generator, acceptable amount of their reactive power capacity is dispatched.

V. COMPUTING ANNUAL EXPANSION PROFIT

From the viewpoint of investors, the best candidate location for investment is the one in which- despite the submitting suitable price in tender- expanded capacity of reactive power is fully dispatched. In order to identify the optimal candidate for reactive power expansion, the annual profit of reactive power expansion at each candidate location should be determined. To this end, Annual Profit-Bid curve is drawn for each candidate and its max is determined. To draw the Annual Profit-Bid curve for the *i*th candidate the following stages should be taken:

- Add a new unlimited reactive generator at *i*th candidate location, while there is no new reactive generator at other candidate locations and existing generators have their own limitations in producing reactive power.

- Increase the bid of new reactive power generator from zero by small steps. In each step compute dispatched value of its reactive power using optimization problem (1)-(7). Increase the bid until its dispatched reactive power vanishes.
- Compute annual profit in each step. Annual profit is equal to annual revenue minus annual investment cost. The annual profit of new reactive generator due to selling reactive power is given by:

$$AP = BID \times Q \times \xi - Q \times C \qquad (8)$$

Where BID is the submitted price for selling reactive power. Q is the generated reactive power. ξ is a coefficient for converting hourly peak load profit to annual profit and C is the annual investment cost per 1 MVar expansion.

- Plot bid and annual profit of different steps to obtain Annual Profit-Bid curve.

If a low bid is submitted for reactive power full reactive capacity is dispatched, but the producer receives low revenue due to low price. If a high bid is submitted for reactive power the reactive capacity will not dispatched and the producer receives zero revenue. Hence revenue and consequently profit has a max. The reactive power corresponding to the max annual profit specifies the optimal amount of reactive power expansion, let's show it by Q_{opt}. The bid corresponding to the max annual profit specifies the optimal bid, let's show it by BID_{opt}. Therefore, annual expansion profit is given by:

$$\mathrm{AEP} = Q_{opt} \times (BID_{opt} \times \xi - C) \qquad (9)$$

Where AEP is the annual expansion profit. AEPPUI, annual expansion profit per unit investment, is equal to:

$$\mathrm{AEPPUI} = (BID_{opt} \times \xi - C) / C \qquad (10)$$

The candidate which has max AEPPUI and its AEP is acceptable is selected as the final plan for expansion.

VI. Selecting the Final Expansion Plan

Due to uncertainty in bid of competitors, obtaining the exact Annual Profit-Bid curve for a generator is not possible, since uncertainty in competitors' bid causes uncertainty in amount of dispatched reactive power and consequently uncertainty in annual expansion profit. In order to take into account uncertainty in competitor's bid in decision making on reactive power expansion, probable occurring scenarios for competitor's bid are identified. For each candidate max AEPPUI is computed under different scenarios. A probability density function is fitted to AEPPUI of each candidate using Kernel method [8]. Kernel method is explained in appendix A. The candidate which has the max average and min variance in AEPPUI is selected as the final plan. This plan has max average profit and less risk in investment and capital return.

VII. Case Study

Consider the 8-bus system shown in Fig 1. Line parameters, generation data, and load data at peak load of planning horizon are given in tables I, II and III.

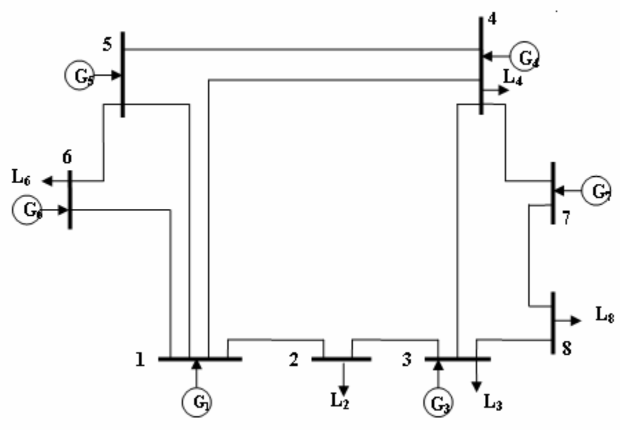

Fig. 1. PJM 8-bus test system

TABLE I
PARAMETERS OF TRANSMISSION LINES OF THE 8-BUS NETWORK

Line No.	From Bus No.	To Bus No.	X (ohm)	Limit (MVA)
1	1	2	0.03	280
2	1	4	0.03	140
3	1	5	0.0065	380
4	2	3	0.01	120
5	3	4	0.03	230
6	4	5	0.03	200
7	5	6	0.02	300
8	6	1	0.025	250
9	7	4	0.015	250
10	7	8	0.022	340
11	8	3	0.018	240

TABLE II
GENERATION DATA OF THE 8-BUS NETWORK

Bus No.	Min MW	Max MW	Min MVar	Max MVar	Bid $/MWh	Bid $/MVarh
1	0	280	0	84	20	4
3	0	520	0	300	25	5
4	0	250	0	150	20	4
5	0	500	0	150	10	2
6	0	400	0	200	20	4
7	0	200	0	60	20	4

TABLE III
LOAD DATA OF THE 8-BUS NETWORK

Name	Bus No.	Max MW	Bid $/MWh	Max MVar	Bid $/MVarh
L_1	1	0	0	90	30
L_2	2	300	35	90	32
L_3	3	300	28	90	35
L_4	4	250	35	75	28
L_5	5	0	0	75	35
L_6	6	250	30	75	20
L_7	7	0	0	75	25
L_8	8	300	25	90	15

Busses which have generator, i.e. busses 1, 3, 4, 5, 6, and 7, are selected as primary candidate locations for reactive power expansion. In this case study since in all primary candidate locations there is generator, instead of adding a new unlimited reactive generator, limit of existing generators is released. After releasing reactive power limits, the optimization problem (1)-(7) is solved for operating point given in tables II and III. Table IV shows the amount of generated reactive power at the primary candidate locations for base case and released reactive power generation limits case.

TABLE IV

GENERATED REACTIVE POWER AT BASE CASE AND RELEASED LIMIT CASE

Generators	base case	released limit case
G_1	84	98.2
G_3	194.5	158.24
G_4	86.9	60.45
G_5	150	166.54
G_6	39.86	0
G_7	60	151.8

Comparing the generated reactive power of different candidates in abovementioned cases shows that the generated reactive power of generators 1, 5, and 7 has been increased, noticeably in released limits case respect to base case. Consequently, these busses are selected as proper candidate locations for reactive power expansion. At the next step, in order to determine the optimal candidate for reactive power expansion, the Annual Profit-Bid curve is drawn for each plan. In order to consider competitor's bid uncertainties the network is divided into two parts. Generators 1, 5, and 6 are located in part A, and 3, 4, and 7 ones in part B. This classification may have different criteria such as generators' proximity to each other, uniform management, etc. Now nine predominant scenarios are defined. The defined scenarios are given in table V:

TABLE V

PREDOMINANT SCENARIOS IN FLUCTUATIONS IN BID MADE BY COMPETITORS' GENERATORS

SEN.NUM	Group:A	Group:B
1	$1.15\times\text{bid}_{\text{base}}$	$1.15\times\text{bid}_{\text{base}}$
2	$1.15\times\text{bid}_{\text{base}}$	bid_{base}
3	$1.15\times\text{bid}_{\text{base}}$	$0.85\times\text{bid}_{\text{base}}$
4	bid_{base}	$1.15\times\text{bid}_{\text{base}}$
5	bid_{base}	bid_{base}
6	bid_{base}	$0.85\times\text{bid}_{\text{base}}$
7	$0.85\times\text{bid}_{\text{base}}$	$1.15\times\text{bid}_{\text{base}}$
8	$0.85\times\text{bid}_{\text{base}}$	bid_{base}
9	$0.85\times\text{bid}_{\text{base}}$	$0.85\times\text{bid}_{\text{base}}$

It is assumed that ISO is applied price cap of 7 ($/MVarh) to reactive energy bid. Annual Profit-Bid curve is drawn for each expansion plan in each scenario using the presented method. Figs 2, 3, and 4 show Annual Profit-Bid curves for each expansion plan in the base case, scenario 5.

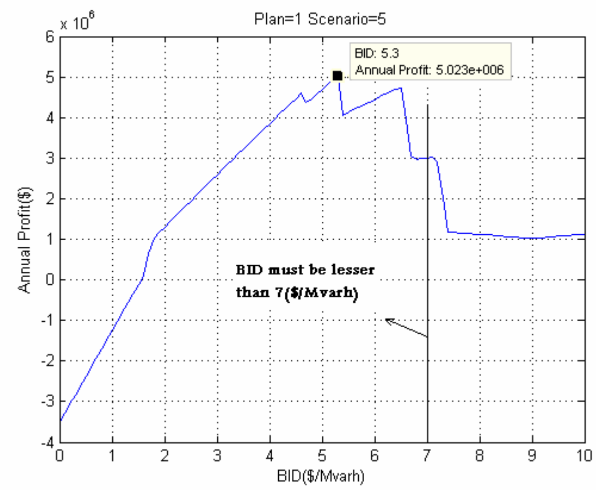

Fig. 2. Annual profit curve for bus 1 in scenario 5 (base case)

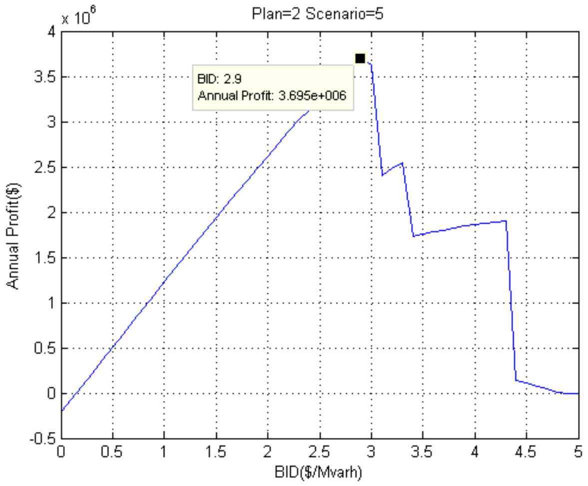

Fig. 3. Annual profit curve for bus 5 in scenario 5 (base case)

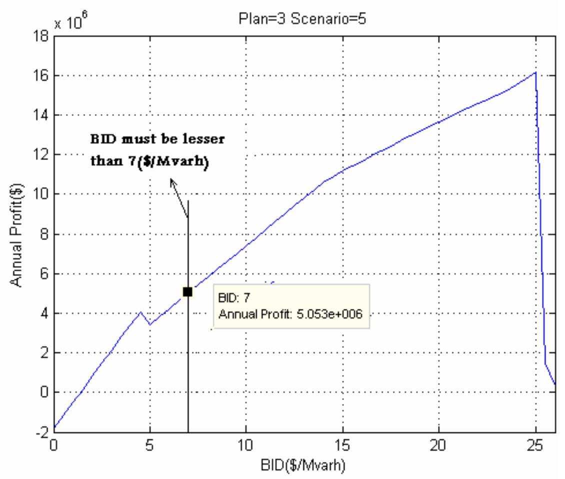

Fig. 4. Annual profit curve for bus 7 in scenario 5 (base case)

Table VI shows the optimal amount of reactive power expansion, optimal bid, and max reactive generation limit before expansion, $Q_{\max}$, for each expansion plan.

TABLE VI

OPTIMAL VALUES OF BID AND REACTIVE POWER PRODUCTION IN BUSES 1, 5, AND 7

	plan1 (Expansion in Bus 1)	plan2 (Expansion in Bus 5)	plan3 (Expansion in Bus 7)
BID_{opt} (\$/ MVarh)	5.3	2.9	7
Q_{opt} (MVar)	127.08	145.91	93.63
$Q_{\max}$ (MVar)	84	150	60

Table VI shows that, optimal amount for reactive power production at bus 5 is less than its max capability before expansion; so expansion in bus 5, plan 2, is unnecessary. Table VII shows the value of AEPPUI for both plan 1 and plan 3 in different scenarios. Average and variance of AEPPUI over different scenarios are given in last rows of table VII.

TABLE VII

ANNUAL EXPANSION PROFIT PER UNIT OF INVESTMENT FOR EACH PLAN IN EACH SCENARIO

	plan1 (Expansion in 1st bus)	Plan3 (Expansion in 7th bus)
1st Scenario	1.533	2.08
2nd Scenario	1.533	2.08
3rd Scenario	1.533	2.08
4th Scenario	1.315	2.05
5th Scenario	1.315	2.05
6th Scenario	1.315	2.05
7th Scenario	1.097	2.01
8th Scenario	1.097	2.01
9th Scenario	1.097	2.01
Average	1.315	2.0467
Variance	0.0317	8.2222e-004

Probability density functions of AEPPUI for two candidates are shown in Fig 5.

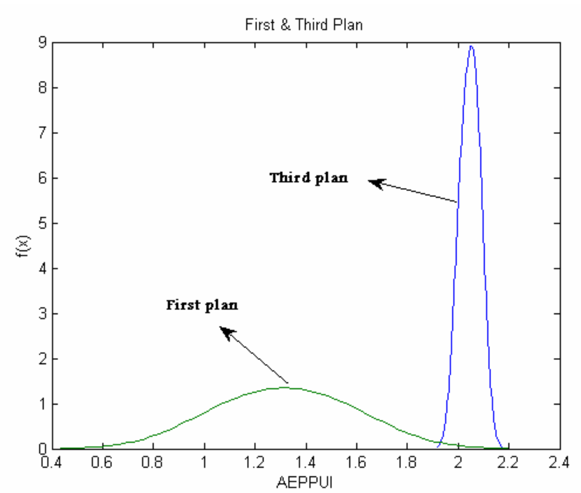

Fig. 5. Probability density function of AEPPUI for first and third plans

Average and variances of AEPPUI of expansion plans 1 and 3 show that investment in plan 3 is economically preferred since it has higher expected value and lower risk. According to table VI reactive power production in bus 7 should be expanded from 60 to 94 MVar. The optimal bid for producer of bus 7 under scenario 5, is 7 (\$/MVarh).

Now expansion in bus 1 is assessed. After investment in bus 7, Annual Profit-Bid curve is drawn for plan 1 in different scenarios. This curve for scenario 5 (base case) is shown in Fig 6.

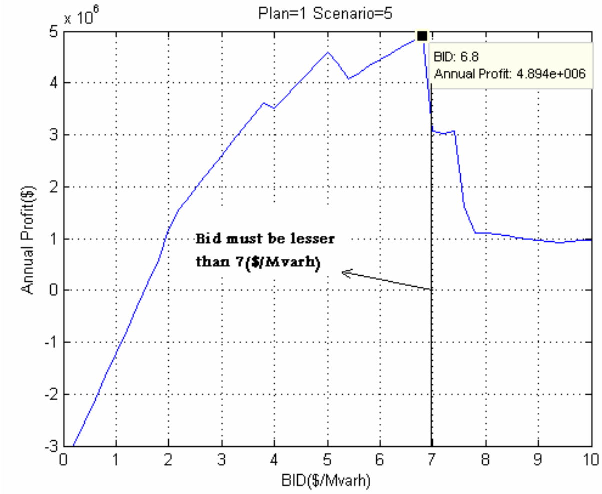

Fig. 6. Annual profit curve for bus 1 in scenario 5 (base case)

Expected value of optimal bid over different scenarios, expected value of optimal amount of reactive power generation over different scenarios, and max generation limit before expansion, $Q_{\max}$, for bus 1 is given in table VIII.

TABLE VIII

OPTIMAL VALUES OF BID AND REACTIVE POWER PRODUCTION IN BUS 1

plan1 (Expansion in bus 1)	
BID_{opt} (\$/ MVarh)	6.8
Q_{opt} (MVar)	82.38
$Q_{\max}$ (MVar)	84

Table VIII shows that after expansion in bus 7, the expected value of optimal amount of reactive power generation at bus 1 is less than its max reactive power generation capability, thus reactive power expansion in bus 1 is unnecessary. It is observed that expansion in reactive power in bus 7 resolves the need to the expansion in reactive power in bus 1.

Due to the reactive power expansion in bus 7, annual operation cost of the system reduces from 260412578 (\$) to 258109920 (\$). This is because of providing the required reactive power by a producer in an appropriate location and/or with suitable bid. Appropriate location is the one which is close to the reactive power consumer and causes reduction in transmission losses.

TABLE IX
ANNUAL PROFIT BEFORE AND AFTER REACTIVE POWER EXPANSION IN BUS 7

Bus number	**Before reactive power expansion in bus 7**		**After reactive power expansion in bus 7**	
	AP($)	**Q(Mvar)**	**AP($)**	**Q(Mvar)**
1	2935296	84	2679156	45
3	8494677	194.47	7687680	176
4	3036263	86.89	3529344	101
5	2620800	150	1869504	107
6	1392725	39.855	4018560	114
7	2096640	60	5025662	93.63

Table IX shows the amount of reactive power generated by different generators in base case before and after reactive power expansion in bus 7. Table IX shows that reactive power expansion in bus 7 causes reduction in reactive power sale in busses 1, 3, and 5 and consequently reduces their profits. In order to prevent this profit reduction, optimal bid of these generators should be determined. In order to determine the optimal bid of a generator, its Annual Profit-Bid curve should be drawn. Fig 7 shows the Annual Profit-Bid curve for bus 5 in base case, scenario 5.

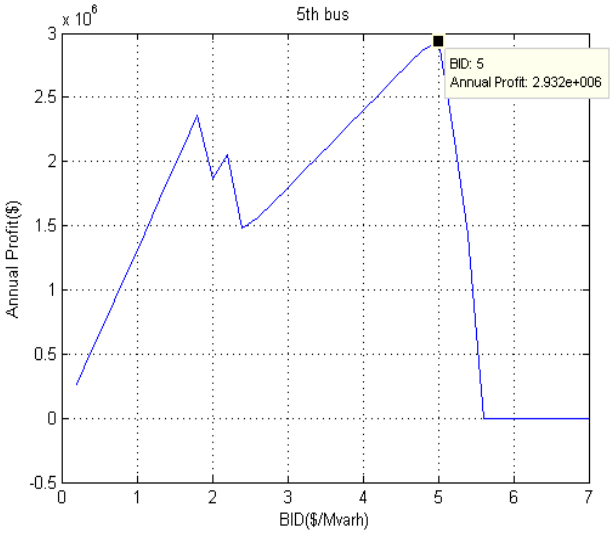

Fig. 7. Annual profit curve for generator of bus 5 in scenario 5 (base case)

Annual Profit-Bid curve shows that if reactive power producer in bus 5 offers 5 ($/MVarh) for reactive power production in base case, then his annual profit will be increased from 1869504 ($) to 2932000($).

VIII. CONCLUSION

In this paper a new method for market oriented reactive power expansion planning and determining the optimal bid for reactive power production is presented. Primary candidates are proposed by ISO and investors determine the optimal candidate by computing annual expansion profit of each candidate. Market oriented reactive power expansion planning expands reactive power of candidates who have appropriate location in the network, and/or submit suitable bids for reactive power production. Expansion the capacity of these candidates will lead to decrease the transmission losses and consequently operational cost of the system.

Appendix A. Estimating the Probability Density Function Using Kernel Method

Probability density function of the random variable Xi can be estimated using Kernel method [8]:

$$f(x) = \frac{1}{Nh}\sum_{i=1}^{N} K(\frac{x - x_i}{h}) \qquad (11)$$

In which N is the number of samples of random variable X, and $K(x)$ is standard normal probability density function. In equation (11), h is Kernel bandwidth or smoothing parameter. This parameter controls the smoothness of probability density function. Increase in h causes smoothness in the probability density function curve.

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X. BIOGRAPHIES

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