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–Brief Paper–

OBSERVER-BASED ADAPTIVE FUZZY CONTROL OF TIME-DELAY UNCERTAIN NONLINEAR SYSTEMS

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ABSTRACT

An observer-based adaptive fuzzy model following controller is proposed for a class of MIMO nonlinear uncertain systems to cope with time-delay, uncertainty in plant structure and disturbances. Based on universal approximation theorem the unknown nonlinear functions are approximated by fuzzy systems, where the premise and the consequent parts of the fuzzy rules are tuned with adaptive schemes. To have more robustness, and at the same time to alleviate chattering, an adaptive discontinuous structure is suggested. Moreover, the availability of the states measurement is not required and an adaptive observer is used to estimate the states. Asymptoic stability of the overall system is ensured using suitable a Lyapunov-Krasovskii functional candidate.

Key Words: Nonlinear time-delay systems, universal approximation, adaptive control, Lyapunov-Krasovskii functional.

I. INTRODUCTION

In most of the recent works in adaptive fuzzy controllers the parameters of the consequent part of the fuzzy controller were assumed free and were tuned by adaptive laws derived using the Lyapunov method, which also guaranteed stability of the system (see, for example, [1, 2]). However, parameters of the premise part of fuzzy controller had to be chosen appropriately. Therefore, this methodology is not able to completely make the designing of fuzzy rules systematic and decrease the number of fuzzy rules. To overcome this problem, recently, in [3–5] some novel methods for designing adaptive fuzzy control with both premise and consequent tuning have been proposed. However, the proposed methods are only valid for a class of single input single output (SISO) nonlinear systems with known or measurable states. But, in practical situations the states of the nonlinear systems are fully or partially unknown because the states are either not available for measurement or the sensors or transducers are very expensive to be used. In addition many plants are modeled as multiple input multiple output (MIMO) nonlinear systems.

It is well known that time delays are frequently encountered in real engineering systems. It has been shown that the existence of time delays usually becomes the source of instability and degrading performance of systems. Therefore, stability analysis and controller synthesis for nonlinear time-delay systems are important both in theory and in practice [6].

In this paper a full adaptive fuzzy observer-based controller is proposed for a class of uncertain time-delay MIMO nonlinear systems with unknown but bounded disturbances. The unknown nonlinear functions are approximated by fuzzy systems based on universal approximation theorem, where the premise and the consequent parts of the fuzzy rules are tuned with adaptive schemes. The proposed approach does not need the availability of the states and uses an adaptive observer to estimate the states. An adaptive

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discontinuous structure is used as a robust control term which also attenuates chattering in control effort. The overall asymptotic stability is guaranteed based on Lyapunov–Krasovskii approach.

II. PROBLEM FORMULATION

Consider a class of time-delay MIMO nonlinear systems in the following form,

$$\begin{aligned}\dot{x}(t) &= (A_1+\Delta A_1(t))x(t)+(A_2+\Delta A_2(t))x(t-\tau)\\ &\quad +B(f(x)+u+d(t,x)) \qquad (1)\\ y &= Cx,\end{aligned}$$

where $f(x)=[f_1(x),\ldots,f_m(x)]^T$ is an unknown continuous nonlinear vector function, $x=[x_1,\ldots,x_n]^T\in\mathbb{R}^n$ is the state vector. $u=[u_1,\ldots,u_m]^T\in\mathbb{R}^m$ and $y=[y_1,\ldots,y_m]^T\in\mathbb{R}^m$ are the input and the output vectors of the system, respectively. A_1, A_2, B and C are known matrices with appropriate dimensions, $x(t-\tau)$ is the time-delay state vector with $\tau>0$ is constant time delay. $\Delta A_1(t)$ and $\Delta A_2(t)$ are corresponding uncertainties that represent parameter variations, which are assumed to satisfy the matching condition, that is, uniformly bounded continuous functions $M_{A_1}(t)$ and $M_{A_2}(t)$ exist such that $\Delta A_1(t)=BM_{A_1}(t)$ and $\Delta A_2(t)=BM_{A_2}(t)$. $d(t,x)=[d_1(t,x),\ldots,d_m(t,x)]^T$ is an unknown but norm bounded external disturbance vector, where this bound is also unknown. Let $x_d=[x_{d1},\ldots,x_{dn}]^T$ be a bounded desired state, $e=x-x_d=[e_1,\ldots,e_n]^T$ be the tracking error and $\hat{x}$ be the estimation of x. Denote $\hat{y}=[\hat{y}_1,\ldots,\hat{y}_p]^T=C\hat{x}$, $\hat{e}=\hat{x}-x_d$ and $\tilde{e}=e-\hat{e}$. The control objective is to make all the signals involved are uniformly bounded and $\lim_{t\to\infty}e(t)=0$. We have the following assumptions:

Assumption 1. There exist matrix L and symmetric positive-definite matrices P_1, S_1 and Q_1 such that

$$\begin{aligned}&P_1(A_1-LC)+(A_1-LC)^TP_1+P_1A_2S_1^{-1}A_2^TP_1+S_1\\ &\quad=-Q_1, \qquad (2)\\ &B^TP_1=C. \qquad (3)\end{aligned}$$

Assumption 2. There exist matrix K and symmetric positive-definite matrices P_2, Q_2 and S_2 which satisfy the following Lyapunov equality:

$$\begin{aligned}&P_2(A_1-BK)+(A_1-BK)^TP_2\\ &\quad+P_2A_2S_2^{-1}A_2^TP_2+S_2=-Q_2. \qquad (4)\end{aligned}$$

III. THE PROPOSED METHOD

Based on the matching condition of the parametric uncertainties the uncertain nonlinear (1) can be rewritten as

$$\begin{aligned}\dot{x}(t) &= A_1x(t)+A_2x(t-\tau)+Bf(x)\\ &\quad+B\eta(t,x,x(t-\tau))+Bu \qquad (5)\\ y &= Cx,\end{aligned}$$

where $\eta(t,x,x(t-\tau))=M_{A_1}(t)x(t)+M_{A_2}(t)x(t-\tau)+d(t,x)$. To approximate $f(x)$ we use the following fuzzy basis function network (FBFN) [3], with free parameter vectors $\hat{W}$, $\hat{\omega}$ and $\hat{c}$:

$$\begin{aligned}&\hat{f}(\hat{x},\hat{W},\hat{\omega},\hat{c})\\ &\quad=[\hat{f}_1(\hat{x},\hat{W}_1,\hat{\omega}_1,\hat{c}_1),\ldots,\hat{f}_m(\hat{x},\hat{W}_m,\hat{\omega}_m,\hat{c}_m)]\\ &\quad=\Phi(\hat{x},\hat{\omega},\hat{c})\hat{W}, \qquad (6)\end{aligned}$$

where

$$\hat{W}=[\hat{W}_1^T,\ldots,\hat{W}_m^T]^T$$

$$\Phi(\hat{x},\hat{\omega},\hat{c})=\mathrm{diag}[\xi_1^T(\hat{x},\hat{\omega}_1,\hat{c}_1),\ldots,\xi_m^T(\hat{x},\hat{\omega}_m,\hat{c}_m)]$$

and $\hat{W}_i^T=[\hat{W}_{i1},\ldots,\hat{W}_{iM}]$, $\hat{\omega}=[\hat{\omega}_1^T,\ldots,\hat{\omega}_m^T]^T$, $\hat{\omega}_i^T=[\hat{\omega}_{i1}^T,\ldots,\hat{\omega}_{iM}^T]$, $\hat{\omega}_{il}=[\hat{\omega}_{il}^1,\ldots,\hat{\omega}_{il}^n]^T$, $\hat{c}=[\hat{c}_1^T,\ldots,\hat{c}_m^T]^T$, $\hat{c}_i^T=[\hat{c}_{i1}^T,\ldots,\hat{c}_{iM}^T]$, $\hat{c}_{il}=[\hat{c}_{il}^1,\ldots,\hat{c}_{il}^n]^T$ which should be tuned appropriately. $\xi_i^T=[\xi_{i1},\ldots,\xi_{iM}]$, each $\xi_{il}=\exp(-\sum_{k=1}^n\hat{\omega}_{il}^{k^2}(\hat{x}_k-\hat{c}_{il}^k)^2)$ and M is the number of fuzzy rules for each entry of $f(x)$. The estimated state $\hat{x}$ comes from an adaptive observer which will be introduced in (17). In this paper, we assume that the fuzzy systems do not violate the universal approximation property on the compact set U_x, which is assumed large enough so that state variables remain within U_x under closed-loop control. Considering this and the boundedness of $M_{A_1}(t)$, $M_{A_2}(t)$ and $d(t,x)$ it can be concluded that there exists a positive constant ψ such that $\|\eta(t,x,x(t-\tau))\|\le\psi$. The desired state is considered to obtain by the following reference model:

$$\dot{x}_d=A_mx_d+A_2x_d(t-\tau)+B_mr, \qquad (7)$$

where $r\in\mathbb{R}^{m\times1}$ is the bounded reference signal, and A_m, B_m are known matrices with appropriate dimensions, that should satisfy the following equalities:

$$A_m=A_1+B(G-K),\quad B_m=BR, \qquad (8)$$

where G and R are matrices with appropriate dimensions. For each entry of $f(x)$ we have:

$$f_i(x)-\hat{f}_i(\hat{x},\hat{W}_i,\hat{\omega}_i,\hat{c}_i)=\xi_i^T(x,\omega_i^*,c_i^*)W_i^*-\xi_i^T(\hat{x},\hat{\omega}_i,\hat{c}_i)\hat{W}_i+\Delta_i, \tag{9}$$

where Δ_i is the approximation error of FBFN for the i-th component of $f(x)$, W_i^*, ω_i^* and c_i^* are ideal parameters for $\hat{W}_i$, $\hat{\omega}_i$ and $\hat{c}_i$, which are assumed to lie in some compact sets. Thus, there exist positive constants b_{W_i}, b_{ω_i} and b_{c_i} such that $\|W_i^*\|\le b_{W_i}$, $\|\omega_i^*\|\le b_{\omega_i}$ and $\|c_i^*\|\le b_{c_i}$. For simplicity considering $\xi_i(x,\omega_i^*,c_i^*)=\xi_i^*$, $\xi_i(\hat{x},\hat{\omega}_i,\hat{c}_i)=\hat{\xi}_i$ and define $\tilde{W}_i=W_i^*-\hat{W}_i$, $\tilde{\xi}_i=\xi_i^*-\hat{\xi}_i$, we have

$$f_i(x)-\hat{f}_i(\hat{x},\hat{W}_i,\hat{\omega}_i,\hat{c}_i)=\tilde{\xi}_i^T\hat{W}_i+\hat{\xi}_i^T\tilde{W}_i+\tilde{\xi}_i^T\tilde{W}_i+\Delta_i. \tag{10}$$

If the vector of Gaussian membership functions is linearized by using Taylor series expansion then $\tilde{\xi}_i$ can be written as:

$$\tilde{\xi}_i=\begin{bmatrix}\tilde{\xi}_{i1}\\ \vdots\\ \tilde{\xi}_{iM}\end{bmatrix}=\begin{bmatrix}\dfrac{\partial\xi_{i1}^*}{\partial\omega_i^*}\\ \vdots\\ \dfrac{\partial\xi_{iM}^*}{\partial\omega_i^*}\end{bmatrix}\Bigg|_{\substack{\omega_i^*=\hat{\omega}_i\\ c_i^*=\hat{c}_i\\ x=\hat{x}}}\tilde{\omega}_i+\begin{bmatrix}\dfrac{\partial\xi_{i1}^*}{\partial c_i^*}\\ \vdots\\ \dfrac{\partial\xi_{iM}^*}{\partial c_i^*}\end{bmatrix}\Bigg|_{\substack{\omega_i^*=\hat{\omega}_i\\ c_i^*=\hat{c}_i\\ x=\hat{x}}}\tilde{c}_i+\begin{bmatrix}\dfrac{\partial\xi_{i1}^*}{\partial x}\\ \vdots\\ \dfrac{\partial\xi_{iM}^*}{\partial x}\end{bmatrix}\Bigg|_{\substack{\omega_i^*=\hat{\omega}_i\\ c_i^*=\hat{c}_i\\ x=\hat{x}}}\tilde{e}+h_i$$
$$=\Lambda_i\tilde{\omega}_i+\Omega_i\tilde{c}_i+\Gamma_i\tilde{e}+h_i,\quad (i=1,\ldots,m), \tag{11}$$

where $\tilde{\omega}_i=\omega_i^*-\hat{\omega}_i$, $\tilde{c}_i=c_i^*-\hat{c}_i$, $\tilde{e}=x-\hat{x}$ and h_i denotes higher order terms. Moreover, we have:

$$\frac{\partial\xi_{il}^*}{\partial\omega_i^*}=\left[\underbrace{0\ \cdots\ 0}_{(l-1)\times n}\ \ \frac{\partial\xi_{il}^*}{\partial\omega_{il}^{*1}}\ \cdots\ \frac{\partial\xi_{il}^*}{\partial\omega_{il}^{*n}}\ \ \underbrace{0\ \cdots\ 0}_{(M-l)\times n}\right]$$

$$\frac{\partial\xi_{il}^*}{\partial c_i^*}=\left[\underbrace{0\ \cdots\ 0}_{(l-1)\times n}\ \ \frac{\partial\xi_{il}^*}{\partial c_{il}^{*1}}\ \cdots\ \frac{\partial\xi_{il}^*}{\partial c_{il}^{*n}}\ \ \underbrace{0\ \cdots\ 0}_{(M-l)\times n}\right]$$

$$\frac{\partial\xi_{il}^*}{\partial x}=\left[\frac{\partial\xi_{il}^*}{\partial x_1}\ \cdots\ \frac{\partial\xi_{il}^*}{\partial x_n}\right].$$

Therefore, using (11) in (10) it can be concluded

$$\begin{aligned}f_i(x)-\hat{f}(\hat{x},\hat{W}_i,\hat{\omega}_i,\hat{c}_i)&=(\hat{\xi}_i^T-\hat{\omega}_i^T\Lambda_i^T-\hat{c}_i^T\Omega_i^T)\tilde{W}_i\\&\quad+(\tilde{\omega}_i^T\Lambda_i^T+\tilde{c}_i^T\Omega_i^T)\hat{W}_i+\varepsilon_i,\end{aligned} \tag{12}$$

where $\varepsilon_i=({\omega_i^*}^T\Lambda_i^T+{c_i^*}^T\Omega_i^T)\tilde{W}_i+(\tilde{e}^T\Gamma_i^T+h_i^T)W_i^*+\Delta_i$. From (11) we have

$$\begin{aligned}\|h_i^T+\tilde{e}^T\Gamma_i^T\|&=\|\tilde{\xi}_i^T-\tilde{\omega}_i^T\Lambda_i^T-\tilde{c}_i^T\Omega_i^T\|\\&\le\ell_1+\ell_2\|\tilde{\omega}_i\|+\ell_3\|\tilde{c}_i\|,\end{aligned} \tag{13}$$

where ℓ_1, ℓ_2 and ℓ_3 are positive constants due to the fact that FBFN and its derivatives are always bounded by constants. From (12)

$$\begin{aligned}\|\varepsilon_i\|&\le\ell_2\|\omega_i^*\|\,\|\tilde{W}_i\|+\ell_3\|c_i^*\|\,\|\tilde{W}_i\|+\ell_1\|W_i^*\|\\&\quad+\ell_2\|W_i^*\|\,\|\tilde{\omega}_i\|+\ell_3\|W_i^*\|\,\|\tilde{c}_i\|+b_{\Delta_i},\end{aligned} \tag{14}$$

where b_{Δ_i} is a positive constant based on universal approximation theorem of FBFN. On the other hand, due to boundedness of ideal parameters we can write $\|\tilde{W}_i\|\le b_{W_i}+\|\hat{W}_i\|$, $\|\tilde{\omega}_i\|\le b_{\omega_i}+\|\hat{\omega}_i\|$ and $\|\tilde{c}_i\|\le b_{c_i}+\|\hat{c}_i\|$, thus (14) can be written as

$$\begin{aligned}\|\varepsilon_i\|&\le[\Re_{i1},\Re_{i2},\Re_{i3},\Re_{i4}].[1,\|\hat{W}_i\|,\|\hat{\omega}_i\|,\|\hat{c}_i\|]^T\\&=\Re_i^T\Upsilon_i,\end{aligned} \tag{15}$$

where $\Re_{i1}=2\ell_2b_{\omega_i}b_{W_i}+2\ell_3b_{c_i}b_{W_i}+\ell_1b_{W_i}+b_{\Delta_i}$, $\Re_{i2}=\ell_2b_{\omega_i}+\ell_3b_{c_i}$, $\Re_{i3}=\ell_2b_{W_i}$, $\Re_{i4}=\ell_3b_{W_i}$ and $\Upsilon_i=[1,\|\hat{W}_i\|,\|\hat{\omega}_i\|,\|\hat{c}_i\|]^T$. Thus,

$$\begin{aligned}f(x)-\hat{f}(\hat{x},\hat{W},\hat{\omega},\hat{c})&=(\Phi(\hat{x},\hat{\omega},\hat{c})-\Theta_1)\tilde{W}+\Theta_2\tilde{\omega}+\Theta_3\tilde{c}+\varepsilon,\end{aligned} \tag{16}$$

where $\Theta_1=\mathrm{diag}[\hat{\omega}_1^T\Lambda_1^T+\hat{c}_1^T\Omega_1^T,\ldots,\hat{\omega}_m^T\Lambda_m^T+\hat{c}_m^T\Omega_m^T]$, $\Theta_2=\mathrm{diag}[\hat{W}_1^T\Lambda_1,\ldots,\hat{W}_m^T\Lambda_m]$, $\Theta_3=\mathrm{diag}[\hat{W}_1^T\Omega_1,\ldots,\hat{W}_m^T\Omega_m]$, $\tilde{W}=W^*-\hat{W}$, $\tilde{\omega}=\omega^*-\hat{\omega}$, $\tilde{c}=c^*-\hat{c}$, and

$\varepsilon = [\varepsilon_1, \ldots, \varepsilon_m]^T$. To estimate the states the following adaptive observer is proposed:

$$\dot{\hat{x}}(t) = A_1\hat{x}(t) + A_2\hat{x}(t-\tau) + B\hat{f}(\hat{x}, \hat{W}, \hat{\omega}, \hat{c}) + Bu + \hat{\theta}_o B(y-\hat{y}) + Bu_c + Bu_r, \tag{17}$$

where $\hat{\theta}_o$, is an adaptive parameter and should be tuned appropriately, u_c and u_r are robust structures as follows:

$$u_c = [u_{c1}, \ldots, u_{cm}]^T \tag{18}$$

$$u_r = \mathrm{sgn}(y-\hat{y})\hat{\psi}, \tag{19}$$

where $u_{ci} = \mathrm{sgn}(y_i - \hat{y}_i)\hat{\Re}_i^T \Upsilon_i$. $\hat{\Re}_i \in \mathbb{R}^4$ and $\hat{\psi} \in \mathbb{R}$ should be adapted appropriately. Therefore, we propose a control law in the following form:

$$u = -K\hat{x}(t) - \hat{f}(\hat{x}, \hat{W}, \hat{\omega}, \hat{c}) + Gx_d + Rr - u_c - u_r. \tag{20}$$

Subtracting (17) and (7), using (8) and control law (20), adding and subtracting $(A_1 - BK)x_d$ leads

$$\dot{\hat{e}} = \dot{\hat{x}} - \dot{x}_d = (A_1 - BK)\hat{e} + A_2\hat{e}(t-\tau) + \hat{\theta}_o B(y-\hat{y}). \tag{21}$$

The observation error dynamics by subtracting (5) and (17), using (16), adding and subtracting $LC\tilde{e}$ with L chosen matrix to satisfy Assumption 1, can be obtained as follows:

$$\begin{aligned}\dot{\tilde{e}} &= \dot{e} - \dot{\hat{e}} = \dot{x} - \dot{\hat{x}} \\ &= (A_1 - LC)\tilde{e} + A_2\tilde{e}(t-\tau) + B(\Phi(\hat{x}, \hat{\omega}, \hat{c}) - \Theta_1)\tilde{W} \\ &\quad + B\Theta_2\tilde{\omega} + B\Theta_3\tilde{c} - \hat{\theta}_o B(y-\hat{y}) - Bu_c - Bu_r \\ &\quad + B\eta(t, x, x(t-\tau)) + B\varepsilon + LC\tilde{e}.\end{aligned} \tag{22}$$

Therefore, the following theorem can be expressed.

Theorem 1. Consider the nonlinear system (1) and control law (20). The closed-loop system signals are bounded and the tracking error converges to zero asymptotically if the following adaptation laws hold:

$$\dot{\hat{W}} = \gamma_1(\Phi^T(\hat{x}, \hat{\omega}, \hat{c}) - \Theta_1^T)(y-\hat{y}), \quad \dot{\hat{\omega}} = \gamma_2\Theta_2^T(y-\hat{y}),$$

$$\dot{\hat{c}} = \gamma_3\Theta_3^T(y-\hat{y}), \quad \dot{\hat{\theta}}_o = \gamma_4\|y-\hat{y}\|^2, \quad \dot{\hat{\psi}} = \gamma_5\|y-\hat{y}\|,$$

$$\dot{\hat{\Re}}_i = \mu_i\|y_i - \hat{y}_i\|\Upsilon_i, \quad i = 1, \ldots, m.$$

Proof. The proof of theorem is omitted here to save space. For detailed proof the interested reader may refer to [7]. □

IV. SIMULATION EXAMPLE

The Chen's chaotic system as a nonlinear time-delay MIMO system is used, with

$$f(x) = \begin{bmatrix} -x_1x_3 \\ x_1x_2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & -3 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0.2 & 0 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_1(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0.8\sin 2t & 1 + 0.3\sin 2t & 0 \\ -1 + 0.5\cos t & 0 & 0.4 \end{bmatrix},$$

$$\Delta A_2(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0.1\cos 3t & 0.2 & 0.1 \\ 0.2\sin t & 0 & 0.4\sin 3t \end{bmatrix},$$

$C = B^T$, and $\tau = 1$. The desired trajectories are obtained from dynamic system (7) with

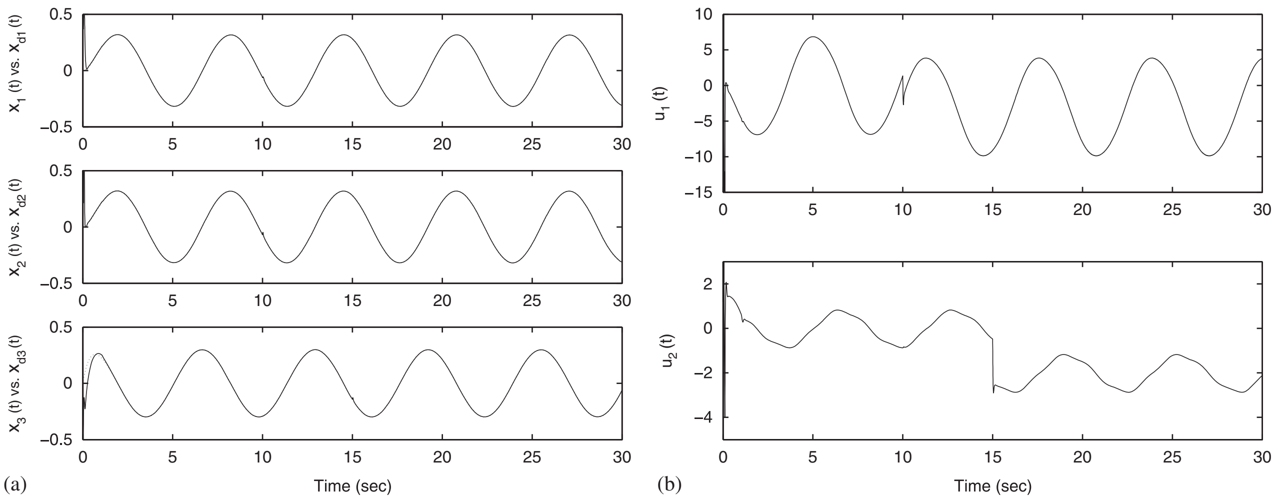
$$A_m = \begin{bmatrix} -35 & 35 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad B_m = B,$$

and without loss of generality the reference signal is chosen $r = [\sin(t), \cos(t)]^T$. $P_1 = \mathrm{diag}[1, 1, 1]$ and the parameters $\gamma_1 = 400$, $\gamma_2 = \gamma_3 = 10$, $\gamma_4 = \gamma_5 = 20$, $\mu_i = 20$, $i = 1, 2$ and $M = 5$ are chosen. The initial condition of the system is $x(0) = [0.5, 0.5, -1]^T$ and the observer is $\hat{x}(0) = [1, 1, -0.5]^T$. Without loss of generality the disturbances are assumed $d_1(t) = 3 + e^{-t}\sin t$ and $d_2(t) = 2 + e^{-3t}\cos 2t$ occur at $t = 10$ sec and $t = 15$ sec, respectively. Fig. 1 shows the results.

Remark 1. To avoid parameter drift and guarantee the remaining of the adaptive parameters in some compact sets the projection operator technique (see [1]) can be utilized.

V. CONCLUSIONS

An output feedback robust controller for time-delay nonlinear uncertain systems is designed based on fuzzy logic systems and adaptive control methods. In the



(a) (b)

Fig. 1. (a) $x(t)$ vs. $x_d(t)$ and (b) control signals.

proposed controller both consequent and premise parts of fuzzy rules have been adjusted via adaptive laws. An adaptive discontinuous structure is also used to make the controller more robust while attenuating chattering effectively. Asymptotic stability of the overall system is ensured by using suitable Lyapunov–Krasovskii functional candidate.

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