Low Complexity Turbo Equalisation for Power Line Communications

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Abstract—**Due to the highly dispersive nature of the power line channel, there has been a strong trend towards the use of orthogonal frequency division multiplexing (OFDM) modulation for wideband power line communications (PLC). However, OFDM modulation has the drawbacks of high peak-to-average power ratio and increased sensitivity to carrier synchronisation errors. On the other hand, single-carrier (SC) modulation techniques do not suffer from these problems and they are widely used in low-rate narrowband PLC. For broadband PLC, however, the strong intersymbol interference (ISI) introduced by the power line channel calls for the use of powerful coding and equalisation techniques, such as turbo coding/equalisation. In this paper, we investigate the performance of the so-called soft interference cancellers (SIC) techniques for turbo equalisation (TE) in PLC applications. The results of simulations over powerline channels with different amounts of memory show that, after performing some number of iterations, the SIC-based TE scheme outperforms a COFDM system that uses the same turbo code, over the entire range of SNR values.**

Keywords— **Power line communications, frequency-selective fading channels, turbo equalisation, soft interference cancellation equalisers**

1. Introduction

The exploitation of the power supply network for providing high-speed data transmission has recently received a lot of attention because of the existence of a vast infrastructure of wires for power distribution already in place. However, despite the enormous potential of power line communications (PLC), the power line channel is a highly dispersive and noisy transmission medium that presents formidable challenges to modem designers.

A modulation scheme suitable for wideband transmissions over channels affected by strong intersymbol interference (ISI) is orthogonal frequency-division multiplexing (OFDM). This multi-carrier (MC) scheme transforms a frequency-selective channel into a set of multiple flat-frequency fading channels. Thus, equalisation of the data with coherent detection amounts simply to a normalisation by a complex scalar, while non-coherent detection does not require any further equalisation.. OFDM, however, has its downsides. This modulation scheme has an increased peak-to-average ratio (PAR) signal (in power) since its output waveform is approximately Gaussian [1]. This large PAR (10-12 dB) necessitates highly linear amplifiers with large back-off and, therefore, higher costs. OFDM also needs the insertion, in each block of symbols, of a cyclic prefix (CP), whose duration depends on the channel memory that is quite long in power line channels. Hence, OFDM is bandwidth-efficient only if block of symbols much longer than the CP can be sent and this might be not possible since the maximum duration of the OFDM symbol is upper-bounded by the coherence time of the channel. In fact, ISI arises again in the form of inter-carrier interference (ICI) because, if the channel response varies over the OFDM symbol, the sub-carriers are no longer orthogonal at the channel output and cannot be properly discriminated by a simple fast Fourier transform (FFT). Besides, carrier synchronisation is crucial, since it affects OFDM in a way that is analogous to the effect of a time-varying channel, and results in ICI.

On the other hand, single-carrier (SC) modulation techniques do not suffer from these problems and they are widely used in low-rate narrowband PLC. For broadband PLC, however, the strong ISI introduced by the power line channel calls for the use of powerful detection and equalisation techniques. Equalisation schemes driven by soft decisions can be used to mitigate the effects of error propagation. A solution along this direction is the employment of turbo equalisation (TE) [2]. The conventional approach to TE uses an optimal soft-input soft-output (SISO) equaliser, such as the maximum *a posteriori* (MAP) equaliser based on the algorithm of Bahl, Cocke, Jelinek, and Raviv (BCJR) [3], but the computational complexity of this algorithm increases exponentially with the length of the channel. This has motivated the development of reduced-complexity alternatives to the BCJR equaliser that can offer good, yet sub-optimal performance. Structures such as the so-called soft interference cancellers (SIC) proposed in [4]-[7] use a linear filter to equalise the received sequence. The output of this filter contains residual ISI that is estimated based on the *a priori* information, and then cancelled.

In this paper, we investigate the performance of SIC techniques for turbo equalisation in PLC applications. The results of simulations over power line channels with impulse responses of different lengths show that, after performing some number of iterations, these schemes outperform a COFDM system that uses the same turbo code, over the entire range of SNR values.

The paper is organised as follows. In Section 2, we introduce the general discrete-time baseband signal model for frequency-selective fading channels. A description of the SIC turbo equalisation technique is presented in Section 3. In Section 4, the performance of the SIC-based TE scheme is evaluated through simulations and compared with the one of a COFDM system that uses the same turbo code and identical block of transmitted symbols. The power line channel is simulated using the parametric multipath model proposed in [8] to obtain the sampled channel impulse response for each PLC scenario. Our conclusions are given in Section 5.

2. Frequency-Selective Channel model

Assuming that all signals and noise are modelled as the complex envelope of the underlying bandpass channel, and that $h(\tau, t)$ is the time-varying channel impulse response from the input of the pulse-shaping filter at the transmitter, through the propagation medium, to the output of the receiver matched-filter, then we define $h(\tau, t)$ as the response at time t to an impulse at time $t - \tau$ [9]. Denoting the continuous time baseband channel impulse response as $h(\tau)$, where the t dependence has been dropped for convenience, and assuming that a sequence of data symbols $s[l]$ (l = 0, 1, 2, …), drawn from a scalar linear constellation, is to be transmitted, the received signal $y(t)$ can be written as

$$\begin{aligned} y(t) &= h(\tau) * \left(\sum_l s[l]\,\delta(t - lT_s) \right) + n(t) \\ &= \sum_l s[l]\,h(t - lT_s) + n(t) \end{aligned} \quad \textbf{(1)}$$

where $*$ denotes the convolution operator, T_s is the symbol duration ($1/T_s \approx B$, the bandwidth of transmission), $\delta(x)$ is the Dirac delta (unit impulse) function, and $n(t)$ is additive white Gaussian noise (AWGN). If this signal is sampled at instants $t = kT_s + \Delta$, (k = 0, 1, 2, …), where Δ is the sampling delay, then the sampled signal response is

$$y(kT_s + \Delta) = \sum_l s[l]\,h((k-l)T_s + \Delta) + n(kT_s + \Delta) \quad \textbf{(2)}$$

which may be rewritten as

$$y(k) = \sum_l s[l]\,h[k - l] + n[k], \qquad k = 0, 1, 2, \ldots \quad \textbf{(3)}$$

$h[l]$ (l = 0, 1, 2, …, L – 1) is the T_s spaced sampled channel, where L is the channel length measured in sampling periods (L represents the span of the ISI on that particular discrete-time channel). The noise samples, $n[k]$, are assumed to be temporally white zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables with variance N_0. The sampled channel $h[l]$ depends on $h(\tau)$ and Δ. The components $h[l]$ will in general be correlated even if the underlying multipath scattering components are uncorrelated. The received signal sample at time index k is given by

$$y[k] = \left[h[L-1] \cdots h[1]\, h[0] \right] \begin{bmatrix} s[k-L+1] \\ \vdots \\ s[k-1] \\ s[k] \end{bmatrix} + n[k] \quad \textbf{(4)}$$

For T successive received signal samples, the input-output relation for frequency selective fading can be expressed as

$$\begin{bmatrix} y[k] \\ \vdots \\ y[k+T-1] \end{bmatrix} = \mathcal{H} \begin{bmatrix} s[k-L+1] \\ \vdots \\ s[k+T-1] \end{bmatrix} + \begin{bmatrix} n[k] \\ \vdots \\ n[k+T-1] \end{bmatrix} \quad \textbf{(5)}$$

where $\mathcal{H}$ is a Toeplitz matrix of dimension T x (T + L – 1), known as the convolution matrix

$$\mathcal{H} = \begin{bmatrix} h[L-1] & \cdots & h[0] & 0 & \cdots & 0 \\ 0 & h[L-1] & \cdots & h[0] & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h[L-1] & \cdots & h[0] \end{bmatrix} \quad \textbf{(6)}$$

3. Single-Carrier Reduced-Complexity Turbo Equalisation

Turbo equalisation (TE) techniques [2] can significantly improve the performance of SC systems in frequency-selective fading environments. Although optimal, the computational complexity of turbo equalisers based on the MAP-BCJR algorithm [3] is manageable only for mildly dispersive wireless channels having a small amount of memory. The complexity of this algorithm increases exponentially with the length of the channel impulse response, which is considerable in the case of power line channels. The need of extending the benefits of TE techniques to highly dispersive channels has motivated the development of reduced-complexity alternatives to the MAP-BCJR equaliser that can offer good, yet sub-optimal performance. These structures, generically known as soft interference cancellers (SIC), have been proposed to replace the trellis-based turbo equaliser by linear-complexity algorithms. In [6], a minimum-mean square error (MMSE) equaliser based on linear filters was derived and it was concluded that several other algorithms, such as the one in [4], could be considered as approximations of this one. For a deeper treatment on the subject of soft interference cancellers, we refer to the works in [4]-[7] and the references therein.

3.1 System Model

At the transmitter, a channel encoder/modulator takes a sequence of binary digits $\{b_k\}$ ($k = 0,..., K_b - 1$) of length K_b, $b_k \in \{0,1\}$, coming from the data source, and outputs a sequence of modulated coded symbols $\{s'_k\}$ ($k = 0,..., K_c - 1$) of length K_c. The symbols $s'_k \in C$, where $C = \{1, -1\}$ is the alphabet of binary phase shift keying (BPSK) modulation. The resulting symbols are then interleaved, and the interleaved sequence $\{s_k\}$ of length K_c, is transmitted over the channel. The impairments of the channel are ISI and additive noise.

We can write the received sample $y[k]$, in equation (3), in an equivalent and more compact form,

$$y_k = \sum_{j=-L_1}^{L_2} h_j s_{k-j} + n_k \tag{7}$$

The discrete-time process $\{y_k\}$ is called a *shift register process* since it can be modelled by a shift register of length L_1+L_2 whose input is the sequence $\mathbf{s}$. The quantities L_1 and L_2+1 denote the lengths of the *anti-causal* and *causal* parts, respectively, of the channel impulse response.

3.2. The SIC-Based Turbo Equaliser

The schematic of a SIC equaliser [4], [6], is shown in Figure 1.

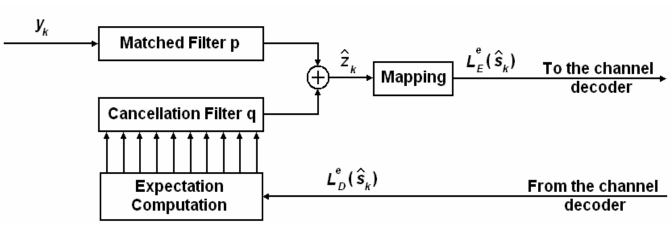

Figure 1. Schematic of a SIC equaliser.

The SIC consists of two filters, i.e., the *matched filter* $\mathbf{p}$ and the *cancellation filter* $\mathbf{q}$, where

$$\mathbf{p} = [p_{-n} \cdots p_0 \cdots p_m]^T \ , \ D = n + m + 1 \tag{8}$$

$$\mathbf{q} = [q_{-R} \cdots q_{-1} \ \ 0 \ \ q_1 \cdots q_S]^T = [\mathbf{q}_f, 0, \mathbf{q}_p \ ]^T \tag{9}$$

and $S = m + L_2$ and $R = L_1 + n$.

The input to the filter $\mathbf{p}$ is the sampled output of the channel y_k, at the symbol rate. The length $D = n + m + 1$ can be considered as an observation window of D samples of the channel output. The input to the cancellation filter $\mathbf{q}$ consists of the expected values of past ($\mathbf{q}_p$) and future ($\mathbf{q}_f$) symbols.

The SISO equaliser output is the log-likelihood ratio (LLR) $L_E^e(\hat{s}_k)$, where the subscript E means that the LLR comes from the equaliser, and the superscript e denotes extrinsic information. This LLR is obtained as

$$\begin{aligned} L_E^e(\hat{s}_k) &= \ln\left(\frac{P(s_k = +1 \mid \hat{z}_k)}{P(s_k = -1 \mid \hat{z}_k)}\right) - \ln\left(\frac{P(s_k = +1)}{P(s_k = -1)}\right) \\ &= \ln\left(\frac{p(\hat{z}_k \mid s_k = +1)}{p(\hat{z}_k \mid s_k = -1)}\right) \end{aligned} \tag{10}$$

The estimate $\hat{z}_k$, which is the output of the SIC, is constructed as the sum of the outputs of the two filters, i.e.,

$$\hat{z}_k = \mathbf{p}^T \mathbf{y}_k + \mathbf{q}_p^T \bar{\mathbf{s}}_{pk} + \mathbf{q}_f^T \bar{\mathbf{s}}_{fk} \tag{11}$$

where $\mathbf{y}_k = [y_{k+n} \cdots y \cdots y_{k-m}]^T$, and $\bar{\mathbf{s}}_{pk}$, $\bar{\mathbf{s}}_{fk}$ are vectors whose entries are the expected values of past and future detected symbols, respectively. The equaliser computes the estimate $\hat{z}_k$ by minimising the mean square error $E\{|s_k - \hat{z}_k|^2\}$, and assuming that the cancellation filter contains correct symbols, the following expressions are obtained for the filters:

$$\mathbf{p} = \frac{1}{\sigma_n^2 + E_h} \mathcal{H}\mathbf{d} \tag{12}$$

$$\mathbf{q}_p = -\mathcal{H}_B^H \mathbf{p} \tag{13}$$

and

$$\mathbf{q}_f = -\mathcal{H}_A^H \mathbf{p} \tag{14}$$

where $E_h = \mathcal{H}\mathbf{d}\mathbf{d}^H \mathcal{H}^H$ is the energy of the channel, σ_n^2 is the noise variance, and matrices $\mathcal{H}_A$, $\mathcal{H}_B$ contain the first R and the last S columns of the D x ($R + S + 1$) convolution matrix $\mathcal{H}$. Note that $R + S + 1 = D + L_1 + L_2$. Matrix $\mathcal{H}$ and vector $\mathbf{d}$ are in turn defined as

$$\mathcal{H} = \begin{bmatrix} h_{-L_1} & \cdots & h_{L_2} & 0 & \cdots & 0 \\ 0 & \ddots & h_{L_2-1} & h_{L_2} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_{-L_1} & \cdots & h_{L_2} \end{bmatrix} \quad \textbf{(15)}$$

$$\mathbf{d} = [\mathbf{0}_{1\times R} \quad 1 \quad \mathbf{0}_{1\times S}]^T \quad \textbf{(16)}$$

In order to perform minimum-mean square error (MMSE) estimation, the statistics $E\{s_k\} = \bar{s}_k$, where $E\{\cdot\}$ denotes the expectation operator, and $\mathrm{Cov}\{s_k, s_k\} = v_k$, where $\mathrm{Cov}\{\cdot,\cdot\}$ denotes the covariance operator, are required. If the symbols s_k are assumed to be equiprobable, as well as independent and identically distributed (IID), $L_D^e(\hat{s}_k) = 0, \forall k$, and $\bar{s}_k = 0$ and $v_k = 1$. For general $L_D^e(\hat{s}_k) \in \mathbb{R}$ (the field of real numbers), where the symbols s_k are not equiprobable, $\bar{s}_k$ and v_k depend on the constellation used and the *a priori* probabilities coming from the channel decoder. For BPSK modulation, it can be seen that [6]

$$\bar{s}_k = \tanh(L_D^e(\hat{s}_k)/2) \quad \textbf{(17)}$$

where the subscript D means that the LLR comes from the channel decoder, and

$$v_k = 1 - |\bar{s}_k|^2 \quad \textbf{(18)}$$

Using (17), the vectors containing past and future symbols: $\bar{\mathbf{s}}_{pk} = [\bar{s}_{k-1} \cdots \bar{s}_{k-S}]^T$ and $\bar{\mathbf{s}}_{fk} = [\bar{s}_{k+R} \cdots \bar{s}_{k+1}]^T$ are calculated for $k = 0, 1, \ldots, K_c - 1$.

The SIC can produce soft outputs in the form of LLR using the assumption that the probability density functions $p(\hat{z}_k \mid s_k = s_i)$, $s_i \in C$, $i = 0, 1$ are Gaussian with $\mu = E\{\hat{z}_k \mid s_k = s_i\}$ and $\sigma^2 = \mathrm{Cov}\{\hat{z}_k, \hat{z}_k \mid s_k = s_i\}$, the mean and covariance, respectively. To this end, the following mapping is used

$$L_E^e(\hat{s}_k) = \ln\left(\frac{p(\hat{z}_k \mid s_k = +1)}{p(\hat{z}_k \mid s_k = -1)}\right) = \frac{2\hat{z}_k\mu}{\sigma^2} \quad \textbf{(19)}$$

The parameters μ and σ^2 can be computed via the following approximate relations [7]

$$\mu = |s_i|, (i = 0, 1) \quad \textbf{(20)}$$

and

$$\sigma^2 = \sigma_n^2 + \bar{v}(\mathbf{q}_p^H \mathbf{q}_p + \mathbf{q}_f^H \mathbf{q}_f) \quad \textbf{(21)}$$

The amplitude of the symbols $|s_i|$ in (20) is equal to 1 for both s_0 and s_1 (BPSK modulation), and $\bar{v}$ in equation (21) is approximated by the following time average [6]

$$\bar{v} = \frac{1}{K_c} \sum_{k=0}^{K_c - 1} 1 - |\bar{s}|^2 \quad \textbf{(22)}$$

4. Simulation Results

To test the performance of the SIC-based turbo equaliser, we simulated the transmission of blocks (or frames) of $K_c = 1024$ BPSK symbols through a power line channel whose frequency transfer function was modelled as in [8]. Using this parametric model, the calculation of the discrete-time channel impulse responses of some typical power line channels is straightforward. Two PLC scenarios were considered for the performance simulations: (a) a channel impulse response with 9 significant taps; and (b) a channel impulse response with 31 significant taps. It was assumed perfect channel state information (CSI) available at the receiver.

For the 9-tap channel, the SIC-based turbo equaliser used the filter parameters $m = n = 15$, while for the 31-tap channel the same equaliser used $m = n = 25$. We note that the effect of employing sufficient filter lengths is to make the channel convolution matrix "tall" (i.e., with almost the same number of rows and columns). This allows good equalisation with almost no residual ISI.

We used a turbo code that comprises two recursive systematic convolutional (RSC) codes of rate 1/3 each, separated by an interleaver. The first encoder is terminated with tails bits. The information and tail bits are scrambled and passed to the second encoder, which is left open without tail bits. The turbo encoder is therefore able to encode blocks of K_b bits continuously without the need of flushing its memory to the zero state. An appropriate puncturing rule allows the turbo code to achieve a final rate of 1/2, so $K_c = 2K_b$. The received bits are decoded using the so-called Log-MAP algorithm. The Log-MAP algorithm is a good choice for the turbo decoder because it avoids the approximations involved in the so-called Max-Log-MAP algorithm and, hence, it provides a good equivalence to MAP, but without its problems of implementation [10]. The performance of the reduced-complexity turbo equaliser for single-carrier systems was compared with the one of a COFDM multi-carrier system. In order to obtain a fair comparison between these two systems, the same turbo encoder/decoder was used for the COFDM scheme. The latter employed 1024 sub-carriers (or tones) to convey the same amount of BPSK symbols as the single-carrier scheme, and a cyclic prefix of length proportional to the channel memory to avoid the need of partial equalisation.

In order to improve the performance of the COFDM scheme, the OFDM detector was designed to pass soft information to the outer channel decoder (on a per-tone basis), in the form of a log-likelihood ratio.

Figure 2 shows the bit error rate (BER) performance of both systems over the 9-tap power line channel using 2 and 5 iterations. In the case of COFDM, it is understood that only the turbo decoder performs the iterations. It was shown that at 5 iterations, there exists a gap of 6dB between the TE and the COFDM, at a BER of 10^{-3}. We attribute this to the fact that, unlike the SIC equaliser, the COFDM system did not interact with the channel decoder in an iterative loop. The exchange of information between the equaliser and the decoder enhanced the performance of the TE.

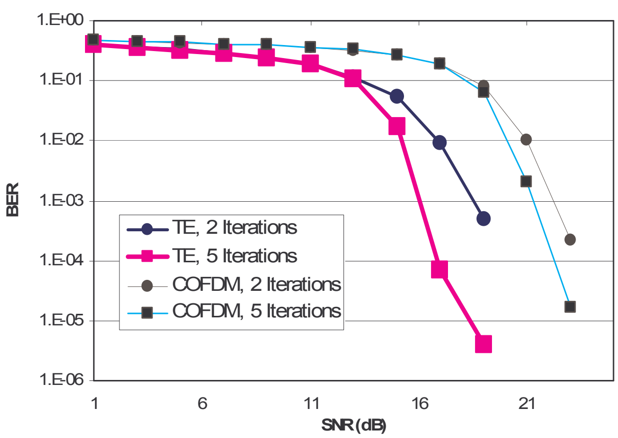

Figure 2. BER performance comparison of the SIC turbo equaliser (TE) and COFDM schemes for two and five iterations over a 9-tap power line channel. For the COFDM system, only the turbo decoder performs iterations.

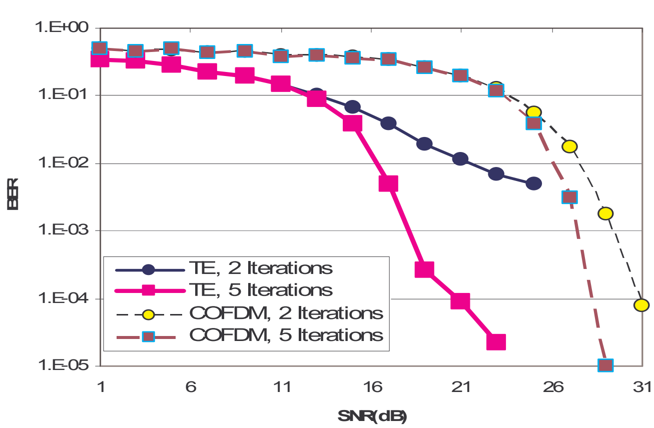

Figure 3. BER performance comparison of the SIC turbo equaliser (TE) and COFDM schemes for two and five iterations over a 31-tap power line channel. For the COFDM system, only the turbo decoder performs iterations.

Included in figure 3 is the BER performance of the TE in a PLC channel with 31 taps. This performance is also compared with a COFDM. It is shown that after 5 iterations, the TE outperformed the COFDM. At a BER of 10-3, the gain is approximately 8dB.

5. Conclusions

A SIC-based turbo equalisation scheme for high rate PLC applications has been investigated. The performance of this scheme is compared with a COFDM scheme, which employs the same decoder. It was shown that the single carrier scheme performed better that the OFDM multi-carrier scheme. This is true for both PLC channel considered. This gain in performance is attributed to the interaction between the decoder and the equaliser; the interaction that the COFDM did not enjoy.

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