Buckling of a Laminated Column with Transverse Crack

[1] S. H. R. Sanei* and [2] J. Rezaeepazhand

[1]Graduate student, [2]Professor, Department of Mechanical Engineering, Ferdowsi University of Mashhad, P.O Box 91775-1111, Mashhad, Iran
*(Tel: (+98) 9155213703, e-mail: hrsanei@gmail.com)

Abstract

Transverse cracks can emerge in composite materials in form of matrix cracking or fiber breaking. Apart from the stress concentration in the vicinity of crack, stability consideration of a cracked beam is incumbent upon any designer. Buckling behavior of an open cracked beam is studied in this paper employing an analytical approach and several numerical examples have been implemented and verified. The conventional analytical approach for cracked beam is to separate beam into two virgin beams and specify two differential equations for the left and right side of the beam, but in this study, one differential equation for the whole beam is assumed which has discontinuity in flexural stiffness, which consequently saves a lot of time and calculation. To investigate the behavior of a cracked beam, the crack is replaced with the linear massless rotational spring joining the two uncracked parts of the structure. In addition, the formulation is extended for the case of composite materials. Moreover, cross-ply laminates with different lay-up are used to monitor the influence of stacking sequence on critical loads of a cracked beam.

Keywords: Critical loads, Transverse Crack, Crack Location, laminated beam, Cross Ply Laminate

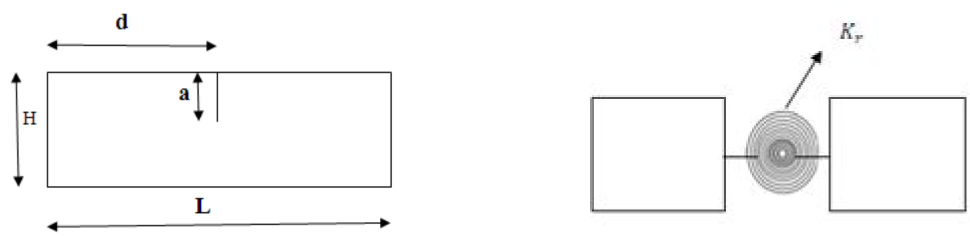
1. Introduction

Composite materials are used in diversity of applications due to their superior merits comparing to their metal counterparts such as high strength to weight ratio, corrosion resistance, damping characteristic, low thermal expansion coefficient and many other features that make composite materials outstanding. Despite the benefits of employing composite materials one should be aware of their pitfalls and downsides. Composite laminates may undergo different defects such as fiber breaking, fiber buckling, matrix fracture, matrix cracking, debonding of layers called delamination or fiber –matrix debonding.

In this study, the static stability of laminated beam in the presence of open transverse crack is investigated. The recent numerous researches in this area is a good testament for its import. Yang et all [1] investigated the natural frequency of laminated composite beam containing several cracks by achieving the overall modulus of the structure. They calculated the stiffness of the cracked beam employing energy method and calculated crack opening displacement by means of boundary value problem. They also studied the layup effect in stiffness of cracked beam. Caddemi and Calio[2] presented exact solution for Euler-Bernouli cracked column. They modeled concentrated crack by Dirac's delta. Ohseop Song et all [3] addressed the vibration of composite cantilever beam with multiple transverse cracks in which they used the concept of the rotational spring as a replacement for transverse cracks. They monitored the first three natural frequencies of a composite beam with single or multi surface cracks versus parameters like fiber orientation, location of crack, number of cracks and fiber volume fraction. Krawczuk et all [4] studied the first natural frequencies of a cracked beam respect to relative crack size using finite element method. They considered the influence of flexural bending deformation emerged because of the existence of crack in the inertia and stiffness matrices. Cracks can also occur at both side of the beam in case of fluctuating loadings. Ostachowicz et all [5] studied the effect of single sided and double sided cracked cantilever beam on natural frequencies. They investigated a beam once with two double sided crack and the other time with two one sided crack. They substitute the cracks with rotational springs and separated the beam into three sub beams with rotational spring at their boundaries. Furthermore, they derived a definition for the stiffness of rotational spring for both double sided and single sided cracks. Inverse problem have been of a great interest to many researchers, by which the location of crack is determined by monitoring the alteration in natural frequencies. Chaudhari et all [6] proposed a modeling for transverse vibration of a cracked beam with linearly variable cross section. They also proved that their method could be effective in prediction of crack location, they benefited from the concept of rotational spring for modeling of the crack. Skrinar [7] presented two approaches for the buckling of cracked beams. In his first approach a polynomial is selected to determine the behavior of the structure which accordingly an expression for the critical load is derived. In his second approach a geometrical stiffness matrix for the finite element of transversely cracked beam is presented. He proved that despite less computational effort comparing to analytical method, the agreement between results is encouraging. Skrinar [8] also formulated finite element of a cracked beam with arbitrary number of transverse cracks. He replaced each crack with a linear rotational spring and computed the effect of flexural bending deformation in the geometrical and stiffness matrices.

1. Analysis

For the case of perfect column, the Euler load is the critical load of a column, but in reality, columns are prone to various imperfections such as notches, crookedness, and cracks. Transverse cracks are the first sign of failure in composite laminates. The stability investigation of composite material has received a great attention, however the stability behavior of laminated structure containing transverse crack is subject to more investigation. The occurrence of crack decreases the stiffness of the structure and consequently reduces the load carrying capacity of the column. To investigate the behavior of a cracked beam, the transverse crack is replaced with massless rotational spring with infinitesimal length as it is depicted in figure 1.



a **b**

Fig.1 a**.** A cracked beam **b.** a beam with rotational spring at the distance d from the left end of the beam

The spring stiffness is dependent of the crack depth, the modulus of an uncracked area, and height of the beam. On the other hand, the spring is representative of the uncracked section at the crack location. There are different methods available in the literature for calculating the stiffness of the rotational spring, which most of them use fracture mechanics to obtain the value for stiffness. In this paper we benefit from the Okamura's definition for spring stiffness [9]. It has been shown [5] that the stiffness of the spring is independent of the mode shape.
We already know that the differential equation of an isotropic beam is

$$\frac{d^4w}{dx^4} + \frac{P}{IE}\frac{d^2w}{dx^2} = 0 \qquad (1)$$

Researchers [3-9], who investigated the behavior of a transversely cracked beam, replaced the open surface crack with rotational spring and separated the cracked beam into two virgin beams with the rotational spring at their interface. Therefore, they had to specify two differential equations one for each beam and consequently two general solutions with 8 unknown coefficients which are presented in eqs.2-4 had to be used. With the contribution of 4 boundary conditions at beam ends and 4 continuity equations at the interface of sub beams they achieved the 8 unknown coefficients.

Differential equation for each beam:

$$(IE)_1 \frac{d^4 w_1}{dx_1{}^4} + P\frac{d^2 w_1}{dx_1{}^2} = 0 \qquad (IE)_2 \frac{d^4 w_2}{dx_2{}^4} + P\frac{d^2 w_2}{dx_2{}^2} = 0 \tag{2}$$

The general solution for each sub beam:

$$w_1(x) = A_1 sin(Kx) + \mathrm{B}_1\cos(Kx) + C_1 x + D_1 \tag{3}$$

$$w_2(x) = A_2 sin(Kx) + \mathrm{B}_2\cos(Kx) + C_2 x + D_2 \tag{4}$$

On contrary to aforementioned formulation, in this paper we use Yavari [10] method to achieve the critical load of a cracked beam. This method enables us to use one differential equation for a beam with arbitrary number of cracks which saves us a good deal of computation. He presented the following differential equation for a beam containing a rotational spring.

$$\frac{d^4 w}{dx^4} + \frac{P}{IE}\frac{d^2 w}{dx^2} = \Theta\delta^{(2)}(x-d) \tag{5}$$

In which Θ is the slope difference at the crack location and δ is Delta function.
Therefore having replaced the transverse crack with rotational spring and having achieved the stiffness of spring, one can use this differential equation for monitoring the buckling behavior of the cracked beam.
Considering $K^2 = \frac{P}{IE}$ and $L(w) = W(S)$
Where L is the Laplace transform operator, P is the compressive force and IE is flexural rigidity.
For the case of simply supported beam $w(0) = w''(0) = 0$ and assuming $A = w'(0)\,, B = w'''(0)$
Therefore,

$$W(s) = \frac{B}{S^2(S^2+K^2)} + \frac{A}{S^2} + \frac{\Theta e^{-ds}}{S^2+K^2} \tag{6}$$

Having inverse Laplace transform, the general equation of a cracked beam at distant d will have the form.

$$w(x) = \left(\frac{B}{K^2} + A\right)x - \frac{B}{k^3}sinkx + \frac{\Theta}{k} \times H(x-d) \times \sin\big(k(x-d)\big) \tag{7}$$

Where $H(x - x_0)$ is a Heaviside function.
By the contribution of boundary conditions of simply supported beam as presented below

$$w(L) = 0 \qquad w''(L) = 0 \qquad w''(d) = \frac{K_r}{EI}\Theta \tag{8}$$

The critical equation would be:

$$\sin(\mathrm{kL}) + \frac{kIE}{LK_r} sin(k(L-d))sin(kd) = 0 \tag{9}$$

As it can be seen from the above equation, when K_r goes large, the buckling equation is identical to the case of a perfect simply supported beam.
For the case of composite materials, the governing equation of a beam in compression which is derived by Wee et all [12] has the following form;

$$\frac{D_{11}A_{11} - B_{11}{}^2}{A_{11}}\frac{d^4 w}{dx^4} + N_x \frac{d^2 w}{dx^2} = 0 \tag{10}$$

Comparison of isotropic and laminated buckling equation presents the effective Flexural stiffness;

$$(EI)_{eff} = \frac{D_{11}A_{11} - {B_{11}}^2}{A_{11}} \tag{11}$$

EI_{eff} indicates that a composite beam can be replaced with an isotropic beam with flexural rigidity equals EI_{eff} to exhibit identical buckling response.

Consequently, for the composite cracked beam, the buckling equation would have the following form:

$$f(kl) = \frac{A_{11}LK_r}{k\left(D_{11}A_{11} - {B_{11}}^2\right)}\sin(\mathrm{kL}) + sin\big(k(L-d)\big)sin(kd) = 0 \tag{12}$$

3. Numerical results

In this section, first a cracked isotropic beam is addressed and then the buckling of a cracked laminated structure is investigated.

In the first case, an isotropic beam is modeled measuring 10m in length, and cross section with dimensions 0.1/0.2m containing a surface crack at distance d. Young modulus is chosen 30 Gpa and 0.3 for Poisson's ratio. The beam is assumed to be simply supported.

In all cases it is assumed that a beam contains a crack with relative size of a/h=0.5 where a is the size of the crack and h is the height of the beam. The relative location of crack is defined as d/L where d is the location of crack from the left side of a beam and L is the length of the beam. The relative location is altered from 0.2 to 0.8 with 0.1 increments.

Table.1 presents the critical value of an isotropic cracked beam which is verified by Skrinar FEM results[7]. As it can be seen from table 1 the agreement with the present method and the finite element result of skrinar is encouraging.

As it can be seen from table 1 due to simply- supported boundary conditions, the critical load is symmetric versus crack location. It is clear from table 1 that as the crack approaches beam centre the critical load decreases. This trend can be explained by the mode shape and the openness of the crack. As we already know, closed cracks are in compression and have no effect on critical load and open crack are responsible for any change in mechanical behavior of cracked beams. Therefore the opener the crack is at a particular mode shape, the more reduction occurs in critical load. Since at the nodal point the crack is closed, the highest critical load is observed at the nodal point.

For the laminated structure, a four- layer and three-layer laminates with the thickness of 0.005m for each layer and the material properties listed in table2 is considered. The critical loads are normalized based on the critical loads of non cracked beams with the same dimensions and properties.

Table1. Critical load of a cracked isotropic beam

d/L	Present Method	Ref(7)	Error (%)
0.2	193240	188565.6	2.42 %
0.3	182210	181479.9	0. 40 %
0.4	176940	176368.3	0. 32 %
0.5	174570	174589.3	0.01 %
0.6	176940	176368.3	0. 32 %
0.7	182210	181479.9	0. 40 %
0.8	193240	188565.6	2.42 %

Table2. Material properties

Glass/epoxy unidirectional	
E_L	42Gpa
E_T	14Gpa
G_{LT}	3.4Gpa
$G_{TT'}$	5.1Gpa
ν_{LT}	0.27
$\nu_{TT'}$	0.37

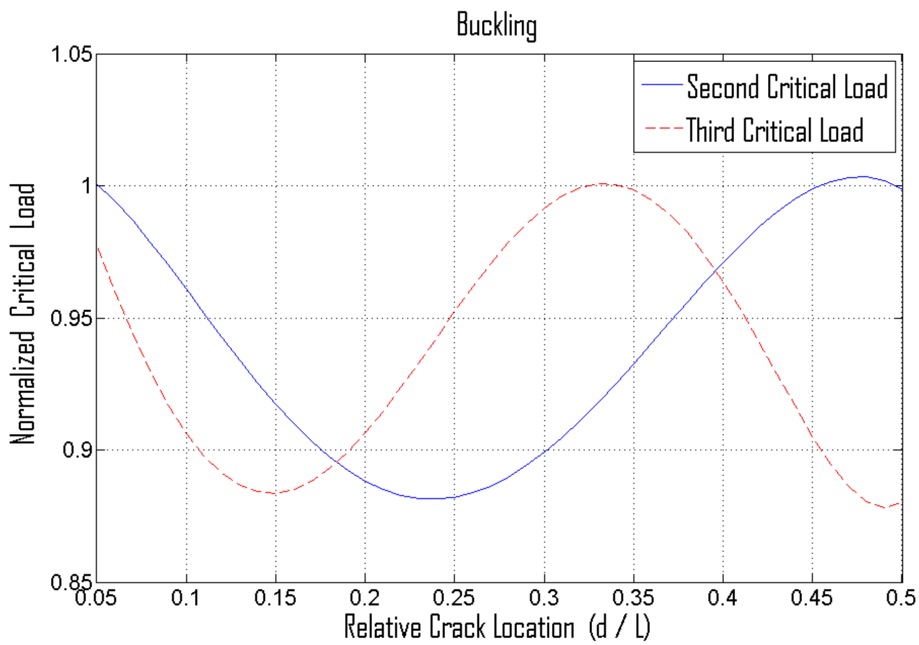


Fig.2. The second and the third normalized critical load versus the crack location.

Fig.2 and table 2 show the significance of crack location in buckling behavior of cracked beams; the first critical load displays a downward trend as the crack approaches beam centre from the beam ends whereas the higher critical loads do not comply with this rule, the second and the third critical load unlike the first critical load don't reveal an absolute downward trend versus the crack location ranging from the beam ends to the centre. As it can been seen from fig.2, the effect of crack on the critical loads is negligible at the nodal point due to the fact that at nodal points cracks are closed. In other words, closed cracks are in compression and consequently have no influence on the behavior of the beam. It is obvious that there are three nodal points in the configuration of the second mode shape, one at the centre and two at the supports, hence at these points crack presence is of no import, and the critical load is identical to that of an uncracked beam. The same fact applies for the third mode shape which can be seen in fig.2. These conclusions for critical loads have already been drawn for natural frequencies [11]; however, making this conclusion for higher critical loads to the best knowledge of authors has not been done in the literature.

For laminated beam, four symmetric cross ply laminates with different lay ups are addressed. The normalized critical load versus the relative crack location (d/L) is plotted which is shown in figure 3. As it can be seen from figure3, similar to the isotropic beam, the critical load of laminated beam shows a downward trend as the crack approaches beam centre. Furthermore, fig.3 shows that, presence of crack in laminates with fiber orientation of 0 in outer part leads to a more reduction in critical load comparing to laminates with fiber orientation of 0 in inner part. This conclusion can be drawn from the comparison of (0/90)s with (90/0)s or (0/90/0) with (90/0/90) . However, it must be brought to a light that, these conclusions are made based on the normalized value of critical load which demonstrates the reduction of beam load carrying capacity, and the effect of fiber lay-up on the absolute value of critical load would be different which fall out of the scope of this paper.

4. Conclusion

In this paper the load-carrying capacity of a cracked beam is tackled. The concept of massless rotational spring with infinitesimal length as a replacement for the crack is used. In this study unlike previous

investigation the beam is not divided into two sub beams with the crack at their interface, but the crack is considered in the differential equation as a jump discontinuity in beam stiffness. By applying this new method, the behavior of beam with several cracks can be achieved using just a single differential equation which saves us a lot of time and calculation.
In this study except the result derived for critical load of isotropic beam which is derived for verification, the other results are normalized to wipe out the influence of dimension and the number of layers on our investigation.
Both for isotropic and laminated structure, the influence of crack on critical load is strongly dependent of how open the crack is at a specific mode shape. Closed cracks which appear at nodal points have no influence on the buckling of the cracked beam. In cross ply laminate, the distribution of layers with fiber orientation of 0 plays a fundamental role; the nearer the 0 layers are to the centre, the less reduction is observed in the critical load comparing to the flawless beam.

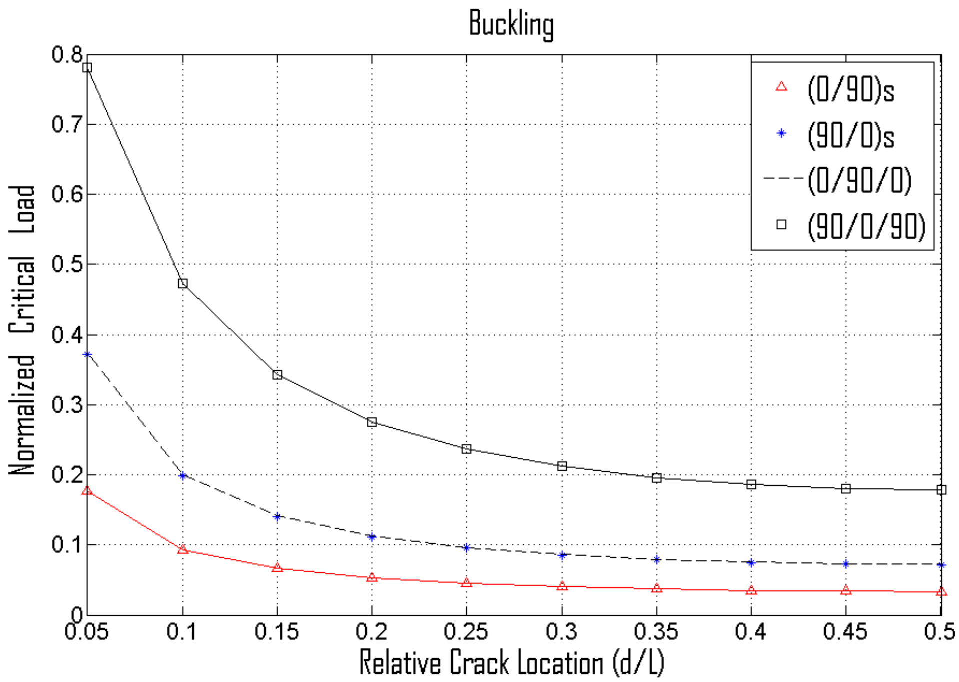


Fig.3. Normalized critical load of a cross ply laminate with different lay-up

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