Vibration of Sandwich Beams with ER Core and Laminated Faces

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Abstract

This study deals with vibration of a sandwich beam with laminated faces and electro-rheological (ER) core. A smart beam element based on Bingham's model is presented. ER layer is adhered to the laminated composite beam to control vibration of the beam. In contrast to most of previous studies in which a Kevin viscoelastic model has been used, in this study behavior of ER core is modeled based on Bingham's equation. Both face layers are considered symmetric. Direct integration method is used to calculate transient response of the beam to an initial excitation. Effects of different parameters such as beam geometry, electric field and stacking sequence on natural frequencies and settling time are investigated. Obtained results show that by increasing the intensity of applied electric field and the thickness of ER core, damping properties of the beam improve and its settling time decreases. Moreover, for symmetric faces by increasing the angle of fiber orientation, natural frequencies are decreased.

Keywords: Electrorheological fluid; Finite element; Laminate; Sandwich beam; Transient response;

1. Introduction

Because of high strength to weight ratio of composite structures, these materials are specially applied in aerospace and automotive industries. Another significant characteristic of composite materials is their anisotropy mechanical properties caused by fibers orientation and stacking sequences. This characteristic lets designers to reach their favorite strength/stiffness in the desire direction of structure. On the other hand, anisotropy properties of composites make the analysis of these materials more complicate and difficult. Due to valuable properties of composites many researchers have focused on flutter, vibration and structural stability of composite structures. Moita et al. [1] studied buckling and vibration of laminated composite structures using a discrete higher-order displacement model. Kameswara et al. [2] proposed an analytical method for evaluating the natural frequencies of laminated sandwich beams using higher-order mixed theory. Subramanian [3] used higher order theories and finite elements to carry out a dynamic analysis of laminated composite beams. Arvin et al. [4] studied free and forced vibration of composite sandwich beam with viscoelastic core.

This study deals with vibration of sandwich beam with laminated faces and ER core. ER fluids are a type of material which their properties change by imposing electric field. When an electric field is applied to ER fluid, its apparent viscosity reversibly changes in a time scale of millisecond. This property has drawn attentions to employ these fluids in active control of vibrating systems. Jia-Yi Yeh et al. [5,6] studied dynamic stability of isotropic and orthotropic sandwich plates with ER core. They used Kevin model to describe viscoelastic behavior of core layer. Narayana and Ganesan [7] have done comparison of viscoelastic damping and electrorheological fluid core damping in composite sandwich skew plates. Rezaeepazhan and Pahlavan [8] investigated transient response of sandwich beam with ER core and metallic faces.

In present study finite element method is employed to investigate vibration of a laminated sandwich beam with ER core. In order to control the amplitude of vibration, an ER layer is adhered to the beam by means of a constraining layer. In contrast to most of previous studies which have used Kevin model, in this study Bingham's model is applied to describe dynamic behavior of ER core. A numerical approach, Direct Integration Method, is employed to obtain transient response of the beam to an initial excitation.

2. Problem explanation and solution method

To investigate vibration response of sandwich beam with laminated faces and ER core the beam shown in Figure 1 is considered. In this beam the ER core is constrained by two laminated elastic faces. As mentioned before, electric field changes the behavior of ER fluid from a Newtonian fluid to a fluid in which polarized particles are aligned in chains. Based on Bingham's model the shear stress in ER layer is defined in terms of shear rate ($\dot{\gamma}$) and electric field (E).

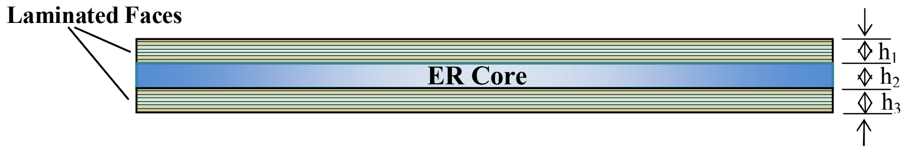

Figure 1: sandwich beam with ER core and laminated faces

$$\tau = \eta\dot{\gamma} + \alpha E^{\beta} \tag{1}$$

α and β are inherent properties of ER fluid, which are usually determined by experiment. For each elastic faces, the relations between deformations and in-plane force and moment are expressed as:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ} \\ \kappa \end{bmatrix} \tag{2}$$

In which, $N = [N_x \quad N_y \quad N_{xy}]^T$, $M = [M_x \quad M_y \quad M_{xy}]^T$, $\varepsilon^{\circ} = [\varepsilon_x^{\circ} \quad \varepsilon_y^{\circ} \quad \gamma_{xy}^{\circ}]^T$, $\kappa = [\kappa_x \quad \kappa_y \quad \kappa_{xy}]^T$ and A, B and D are 3×3 stiffness matrices which their component are define as:

$$A_{ij} = \sum_{k=1}^{n} (Q'_{ij})_k (h_k - h_{k-1}) \tag{3}$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} (Q'_{ij})_k (h_k^2 - h_{k-1}^2) \tag{4}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} (Q'_{ij})_k (h_k^3 - h_{k-1}^3) \tag{5}$$

Where, h_k is the height of the top plane of k^{th} layer form mid-plane of laminate and Q'_{ij} are components of reduced stiffness matrix in beam principle direction. To apply finite element method, the element shown in Figure 2 is considered.

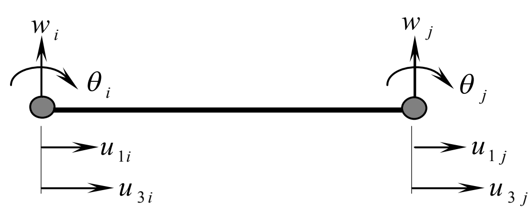

Figure 2: A beam element with two nodes and 4DOF per node

In this element u_1 and u_3 are longitudinal displacement of constraining and base layers in each node, w is transverse displacement and θ is angular rotation of each node. Displacement vector of element is written as:

$$U = \begin{Bmatrix} u_1 \\ u_3 \\ w \end{Bmatrix} = [N(x)]\{q(t)\} \tag{6}$$

Where

$$N = \begin{bmatrix} 1-X & 0 & 0 & 0 & X & 0 & 0 & 0 \\ 0 & 1-X & 0 & 0 & 0 & X & 0 & 0 \\ 0 & 0 & 2X^3 - 3X^2 + 1 & (X^3 - 2X^2 + X)/l & 0 & 0 & -2X^3 + 3X^2 & (X^3 - X^2)/l \end{bmatrix} \tag{7}$$

And $X = x/l$ where l is length of element

Total energy of the element is $\pi=T-V+W$ in which T represents kinetic energy, V represents potential energy of element, and W denotes the work done by external forces. Supposing elastic faces as Euler-Bernoulli beam, the strain energy of elastic layers is obtain as:

$$V_i=\frac{1}{2}\int_v \varepsilon_i^T\sigma_i dv=\frac{1}{2}b\int_l \begin{bmatrix}\varepsilon_i^\circ\\ \kappa\end{bmatrix}^T\begin{bmatrix}A_i & B_i\\ B_i & D_i\end{bmatrix}\begin{bmatrix}\varepsilon_i^\circ\\ \kappa\end{bmatrix}dx$$

$$V_i=\frac{1}{2}b\int_l \{\varepsilon_i^\circ\}^T[A_i]\{\varepsilon_i^\circ\}+\{\varepsilon_i^\circ\}^T[B_i]\{\kappa_i\}+\{\kappa_i\}^T[B_i]\{\varepsilon_i^\circ\}+\{\kappa_i\}^T[D_i]\{\kappa_i\}dx \qquad i=1,3$$

$$V_i=\frac{1}{2}b\int_0^l [q]^T[R_i]^T[A_i][R_i]\{q\}+\{q\}^T[R_i]^T[B_i][S]\{q\}+\{q\}^T[S]^T[B_i][R_i]\{q\}+\{q\}^T[S]^T[D_i][S]\{q\}dx$$

i=1,3 (8)

Where matrices $[R_1]$, $[R_3]$ and $[S]$ are:

$$[R_1]=\begin{bmatrix}\partial/\partial x & 0 & 0\\ 0&0&0\\0&0&0\end{bmatrix}[N],\quad [R_3]=\begin{bmatrix}0 & \partial/\partial x & 0\\ 0&0&0\\0&0&0\end{bmatrix}[N],\quad [S]=\begin{bmatrix}0 & 0 & \partial^2/\partial x^2\\ 0&0&0\\0&0&0\end{bmatrix}[N] \tag{9}$$

In order to obtain the energy lost in core layer, the shear deformation of core layer is needed which is:

$$\gamma_2=\frac{\partial w}{\partial x}+\frac{\partial u_2}{\partial z} \tag{10}$$

Where, u_2 is the longitudinal displacement of the core which can be expressed in terms of u_1 and u_3 [8]. Consequently shear deformation of core layer is obtained as:

$$\gamma_2=\left(\frac{h_1+2h_2+h_3}{2h_2}\right)\frac{\partial w}{\partial x}+\frac{u_1-u_3}{h_2} \tag{11}$$

Hence, the lost energy by ER core based on Bingham's model is achieved by Eq.(12).

$$E_{er}=\int_v\{\gamma_2\}^T\eta\{\dot\gamma_2\}dv+\int_v\{\gamma_2\}^T\tau_E(E)dv=\int_v\{q\}^T[D_2]^T\eta[D_2]\{\dot q\}dv+\int_v\{q\}^T[D_2]^T\tau_E(E)dv \tag{12}$$

$$[D_2]=\frac{1}{2h_2}\begin{bmatrix}2 & -2 & (h_1+2h_2+h_3)\frac{\partial}{\partial x}\end{bmatrix}[N] \tag{13}$$

Neglecting rotary inertia of the elastic layers, the kinetic energy of the elastic layers, due to their longitudinal and transverse displacements is obtained as

$$T_i=\frac{1}{2}\int_v\{\dot q\}^T[Q_i]^T\rho_i[Q_i]\{\dot q\}\ dv \qquad i=1,3 \tag{14}$$

And kinetic energy of the core layer is expressed as:

$$T_2=\frac{1}{2}\int_v\{\dot q\}^T[D_2]^T J_2[D_2]\{\dot q\}dv+\frac{1}{2}\int_v\{\dot q\}^T[Q_2]^T\rho_2[Q_2]\{\dot q\}dv \tag{15}$$

Where, J_2 denotes the mass moment of inertia of core layer and in equations (14) and (15) matrices $[Q_1]$, $[Q_2]$ and $[Q_3]$ are:

$$[Q_1]=\begin{bmatrix}1&0&0\\0&0&0\\0&0&1\end{bmatrix}[N],\quad [Q_2]=\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}[N],\quad [Q_3]=\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}[N] \tag{16}$$

Finally work of external forces (f_{ex}) is obtained as:

$$W=\sum\{U\}^T\{f_{ex}\}=\sum\{q\}^T[N]^T\{f_{ex}\} \tag{17}$$

Applying Hamilton's principle leads to governing equation of motion of the element.

$$[M]\{\ddot q\}+[C]\{\dot q\}+[K_e]\{q\}+\{F_{ER}\}=\{F_{ex}\} \tag{18}$$

Where

$$[M]=\int_v[R_1]^T\rho_1[R_1]\ dv+\int_v[R_3]^T\rho_3[R_3]\ dv+\int_v[R_2]^T\rho_2[R_2]dv+\int_v[D_2]^T J_2[D_2]dv \tag{19}$$

$$[C]=\int_v[D_2]^T\eta[D_2]dv \tag{20}$$

$$[K_e]=b\int_l [R_1]^T[A_1][R_1]+2[R_1]^T[B_3][S]+[S]^T[D_1][S]dx+b\int_l [R_3]^T[A_3][R_3]+2[R_3]^T[B_3][S]+[S]^T[D_3][S]dx \tag{21}$$

$$\{F_{ER}\}=\mathrm{sgn}(\dot{\gamma}).\int_v [D_2]^T \tau_E(E)dv \tag{22}$$

$$\{F_{ex}\}=\sum [N]^T\{f_{ex}\} \tag{23}$$

In order to solve Eq.(18), a numerical approach called direct integration method is employed. Direct integration or explicit integration method, is known as an effective general algorithm for solving dynamic problems. In this method the total solution time is divided into several intervals and the solution is done step by step. In each step the values of displacement, velocity and acceleration are calculated in terms of their values in pervious steps [8]. By applying this approach to Eq.(18), a recursive formula is obtained which calculates vector $\{q\}$ based on its values in the steps before.

$$\{q\}_{i+1}=\left[\frac{1}{(\Delta t)^2}[M]+\frac{1}{2\Delta t}[C]\right]^{-1}\left[\{F_{EX}\}_i-\left([K_e]-\frac{2}{(\Delta t)^2}[M]\right)\{q\}_i-\left(\frac{1}{(\Delta t)^2}[M]-\frac{1}{2\Delta t}[C]\right)\{q\}_{i-1}\right] \tag{24}$$

Where, $\{F_{EX}\}$ is the vector which contains external forces $\{F_{ex}\}$ and $\{F_{ER}\}$. Eq.(24) is a two step recursive equation which in each step, values of vector $\{q\}$ in two steps before are required. Hence, for the first step $\{q\}_{i-1}$ is needed. To resolve this problem, the displacement vector in the step i-1 can be estimated using the initial conditions [8].

$$\{q\}_{-1}=\{q\}_0-\Delta t\{\dot{q}\}_0+\frac{1}{2}(\Delta t)^2\{\ddot{q}\}_0 \tag{25}$$

3. Numerical Results

The mentioned method is applied to investigate vibration response of sandwich beams with ER core and laminated faces. To validate the present method, results are compared with results of other researchers and results of ANSYS finite element software. A three layer sandwich beam with isotropic faces is modeled and its natural frequencies are compared with the results of Haiqing and King [9]. As shown in Table 1 an appropriate accuracy is observed in obtained results. Moreover, a composite cantilever beam of staking sequence [90/0/90] is modeled in ANSYS 11.0 and first five natural frequencies of the beam are obtained. These results are compared in Table.2 with the results of the present FE method. Proper convergence is observed between displayed results of Table 2.

Table 1: Comparison of numerical results with experiments for the first six natural frequencies (Hz).

Mode Number	Present study	Haiqing and King [9]
1	4.6	-
2	12.67	13
3	24.85	22
4	41.07	41
5	61.35	60
6	85.69	84

Table 2: Comparison of first five natural frequencies (Hz) obtained from this study and ANSYS software

Mode Number	Present study	ANSYS 11.0
1	6.477058	6.4194
2	40.59234	40.218
3	113.6849	112.66
4	222.9326	221.33
5	369.1007	368.56

For parametric study, a clamped-free sandwich beam with the length of 600 mm and width of 30 mm is modeled by present finite element model. This beam contains two laminated faces and an ER core. Inherent properties of applied ER fluid are α=36.81 and β=1.52 [10]. Each of elastic faces contains 24 layers with symmetric configuration. Thickness of each layer is equal to 0.15 mm. In all cases, the beam is excited by a 0.1 N.s impulse load applied at the free end of the beam. Results for two kinds of glass/epoxy laminates are obtained. First, influence of electric field on dynamic behavior of the beam is investigated. Since, the effect of ER fluid is appeared as damping force, it does not change the natural frequencies of the system. But it affects the settling time of the beam dramatically. Figure 3 shows influence of electric field on settling time of the beam. Clearly, by increasing the intensity of electric field, settling time of the beam is decreased. It demonstrates that applying electric field strongly improves damping ability of ER layer. Effects of thickness of ER layer on dynamic behavior of the beam are displayed in Figure 4. As shown in Figure 4-(a), by increasing the thickness of core layer natural frequencies of the beam are decreased. It can be explained by increase of equivalent mass of system. Figure 4-(b) shows effects of this parameter on settling time of the beam. By increasing the thickness of ER layer, volume of damping agent increases and consequently enhances the damping properties of the beam. As observed in Figure 4-(b), by increasing the thickness of ER layer, settling time of the beam decreases.

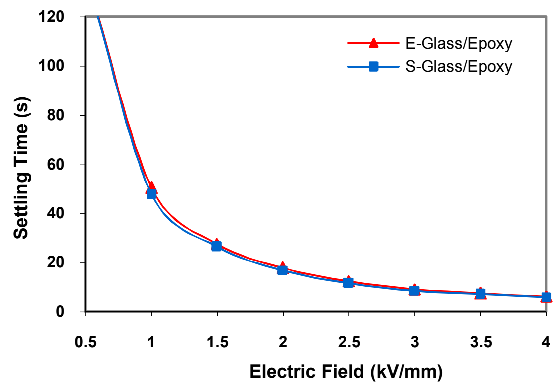

Figure 3: Effects of electric field on settling time of the beam with stacking sequence $[90_6/0_6]_S$

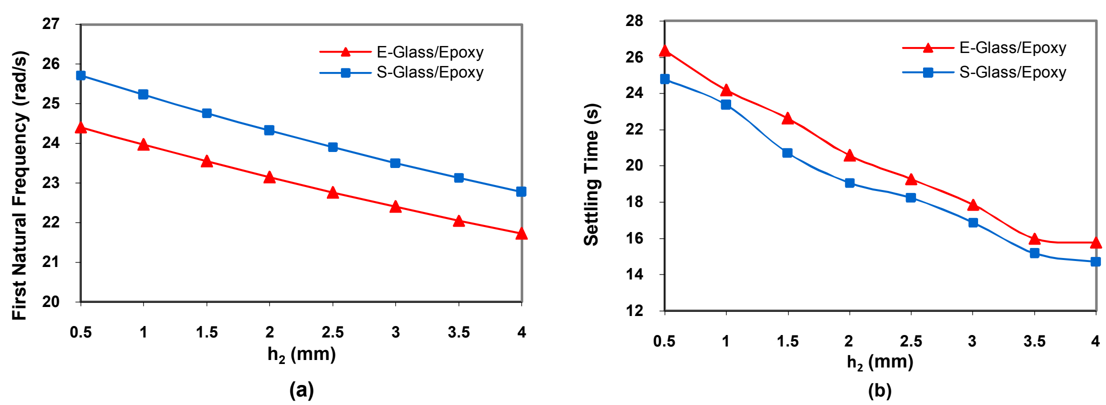

Figure 4: Effects of thickness of ER layer on vibrating behavior of the beam with stacking sequence $[90_6/0_6]_S$

To inspect the influence of fiber orientation, a symmetric laminated model with the stacking sequence of $[\theta_6/0_6]_S$ is considered. Figure 5 illustrates influence of angle θ on dynamic behavior of the beam. As shown in Figure 5-(a) by increasing the angle θ, natural frequencies of the beam is decreased. Actually, increasing the fiber angle reduces the longitudinal stiffness of the beam. As a result, natural frequencies of the beam decrease. This effect also causes the maximum deflection of the beam to be increased. Consequently more time is needed for damping the vibration of the beam. Figure 5-(b) shows that increase in θ, increases the settling time of the beam. Thickness ratio of elastic layers also changes the stiffness of the sandwich beam. To investigate this effect, $[90_{6-n}/0_{6-n}]_S$ and $[90_{6+n}/0_{6+n}]_S$ stacking sequences are considered for the base layer and upper layer of sandwich beam respectively. According to Figures 6-(a) and (b) increasing the thickness ratio of base layer to constraining layer (h_3/h_1), makes the sandwich beam stiffer such that natural frequencies of the beam increase and settling time decreases.

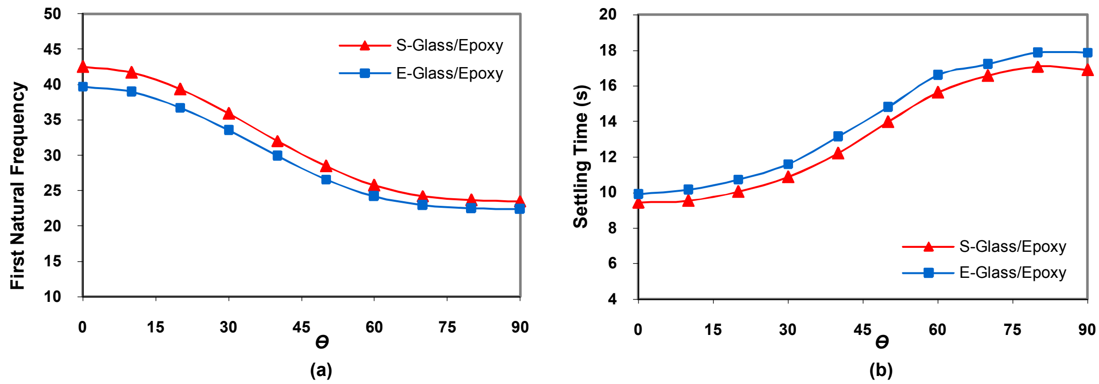

Figure 5: Effects of angle θ on vibrating behavior of the beam with stacking sequence $[\theta_6/0_6]_S$

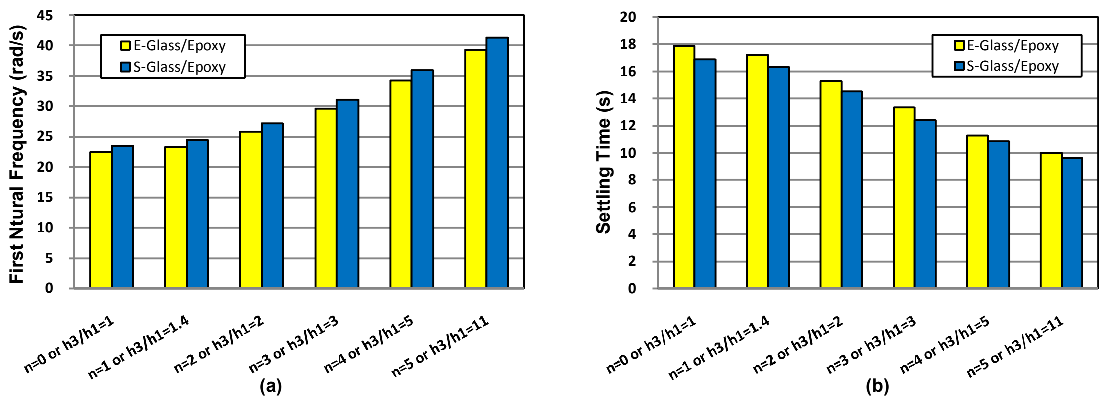

Figure 6: Effects of thickness ratio of elastic layers on vibrating behavior of the beam with stacking sequence ($[90_{6-n}/0_{6-n}]_S$ /ER/$[90_{6+n}/0_{6+n}]_S$).

Conclusion

In this paper a smart beam element based on Bingham's model was proposed to deal with vibration analysis of laminated sandwich beams with ER core. Effects of various parameters were studied. Obtained results showed that the damping ability of the beam increases by increasing the intensity of electric field and also the thickness of ER core. Increasing the fiber angle in sub-layers decreases the longitudinal stiffness of the beam and therefore decreases natural frequencies. The thickness ratio of elastic layers also affects stiffness of the sandwich beam. By increasing thickness ratio of the base layer to the constraining layer, the stiffness of the sandwich beam and consequently its natural frequencies increase.

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