Multiple Access Relay Channel with Orthogonal Components

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***Abstract*—In this paper, we derive an achievable rate region and an outer bound for sum capacity rate for multiple access relay channel with orthogonal components. Superposition block Markov encoding and multiple access encoding and decoding strategies are used to prove the results.**

Keywords- ***Multiple access relay channel, Orthogonal components, Decode and forward strategy, Superposition block markov encoding***

I. INTRODUCTION

The relay channel was first introduced by Van der Meulen [1]. In [2] the capacity of degraded and reversely degraded relay channels and the capacity of the relay channel with feedback as well as upper and lower bounds on the capacity of the general relay channel were established. In [3] the capacity of semi deterministic relay channel , in [4] and [5] the capacity of the relay channel with orthogonal components, in [6] the capacity of modulo-sum relay channel, in [7] the capacity of a class of deterministic relay channel and in [8] capacity of a more general class of relay channels have been determined.

Relaying has been proposed as a means to increase coverage area of wireless networks. Relay nodes in cooperation with the users, act as a distributed multi antenna system. Nowadays, there has been much research on a multi-user extension of the relay channel, e.g. multiple-access relay channel (MARC). In [9] MARC is introduced, where some sources communicate with one single destination with the help of a relay node. An example of such a channel model is the cooperative uplink of some mobile stations to the base station with the help of the relay in a cellular based mobile communication system. Fig. 1 shows a N- source MARC.

Many recent results concerning coding strategies on the MARC can be found in [10]-[12]. An achievable rate region of the MAC with feedback was established in [13].

II. MAIN RESULTS

In this paper, our model is motivated by the practical constraint in wireless communications that a node cannot send and receive at the same time or in the same frequency band.

We divide $X_i, i \in \{1, \dots, N\}$ to orthogonal components (X_{Ri}, X_{Di}) and send these components from the sender to the

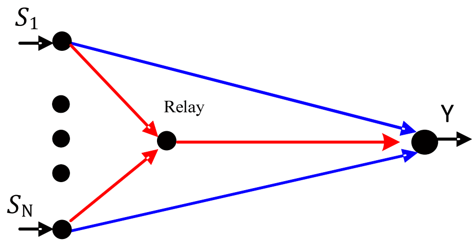

Figure 1. A N- source multiple-access channel

relay receiver (X_{Ri}) and from the sender and relay to the receiver (X_{Di}) . For this model (MARCO), we derive an achievable rate region ($\mathcal{R}_{MARCO}$) and an outer bound**.**

A discrete memoryless multiple access relay channel is said to have orthogonal components if the channel input-output distribution can be expressed as

$$P(y_D, y_R, x_{R1}, \dots, x_{RN}, x_{D1}, \dots, x_{DN}, x_R) = P(y_R|x_{R1}, \dots, x_{RN}, x_R) P(y_D|x_{D1}, \dots, x_{DN}, x_R) \times P(x_R) \prod_{i=1}^{N} P(x_{Ri}|x_R) P(x_{Di}|x_R) \quad (1)$$

where x_R, x_{Ri} and x_{Di}, $i \in \{1, \dots, N\}$, are the inputs, y_D and y_R are the outputs, all with finite alphabets, and $P(y_D, y_R|x_{R1}, \dots, x_{RN}, x_{D1}, \dots, x_{DN}, x_R)$ is the channel probability function.

Now, we express main results as four following theorems.

Theorem 1. An achievable rate region $\mathcal{R}_{MARCO}$ of two-source multiple access relay channels with orthogonal components is given by

$$\mathcal{R}_{MARCO}(R_1, R_2) = \bigcup \{(R_1, R_2)$$

$$0 \le R_1 \le I(X_{R1}; Y_R|X_{R2}, X_R) + I(X_{D1}; Y_D|X_{D2}, X_R) \quad (2)$$

$$0 \le R_2 \le I(X_{R2}; Y_R|X_{R1}, X_R) + I(X_{D2}; Y_D|X_{D1}, X_R) \quad (3)$$

$$0 \le R_1 + R_2 \le I(X_{R1}, X_{R2}; Y_R|X_R) + I(X_{D1}, X_{D2}; Y_D|X_R)\} \quad (4)$$

where the union is taken over

$$P(x_R, x_{R1}, x_{D1}, x_{R2}, x_{D2}) = P(x_R) P(x_{R1}|x_R) P(x_{D1}|x_R) P(x_{R2}|x_R) P(x_{D2}|x_R) \quad (5)$$

Theorem 2. The following is an outer bound on sum capacity rate of two-source multiple access relay channels with orthogonal components

Annual International Conference on Information Theory and Applications (ITA 2011)
Edited by Prof. Gautam Sanyal and Dr. Kalpana Yadav

ISBN: 978-981-08-8113-9
doi:10.5176/978-981-08-8113-9_ITA13

$$0\le R_1+R_2 \le I(X_{R_1},X_{R_2};Y_R|X_R)+I(X_{D_1},X_{D_2};Y_D|X_R) \quad (6)$$

where

$$P(x_R,x_{R1},x_{D1},x_{R2},x_{D2}) = P(x_R)P(x_{R1}|x_R)P(x_{D1}|x_R)P(x_{R2}|x_R)P(x_{D2}|x_R) \quad (7)$$

Theorem 3. $\mathcal{R}_{\text{MARCO}}$ of N-source multiple access relay channels with orthogonal components is contained within the convex hull of the set of rate-tuples $(R_1,\dots,R_N)$ satisfying

$$\mathcal{R}_{\text{MARCO}}(R_1,\dots,R_N)=\bigcup\{(R_1,\dots,R_N)$$

$$0\le \sum_{k\in\zeta} R_k \le I(X_{R(\zeta)};Y_R|X_R)+I(X_{D(\zeta)};Y_D|X_{D(\xi)}X_R)\} \quad (8)$$

where $X_{R(\zeta)}=\{X_{Rk}:k\in\zeta\}$ and $X_{D(\zeta)}=\{X_{Dk}:k\in\zeta\}$, ζ is any subset of $\{1,\cdots,N\}$, ξ is the complement of ζ in $\{1,\cdots,N\}$, and

$$P(x_R,x_{R1},x_{D1},x_{R2},x_{D2},\dots,x_{RN},x_{DN}) = P(x_R)\prod_{i=1}^{N}P(x_{Ri}|x_R)\,P(x_{Di}|x_R) \quad (9)$$

Theorem 4. An outer bound on capacity region of N-source multiple access relay channels with orthogonal components is contained within the convex hull of the set of rate-tuples $(R_1,\dots,R_N)$ satisfying

$$0\le \sum_{k=1}^{N} R_k \le I(X_{R1},\dots,X_{RN};Y_R|X_R)+I(X_{D1},\dots,X_{DN};Y_D|X_R) \quad (10)$$

where

$$P(x_R,x_{R1},x_{D1},x_{R2},x_{D2},\dots,x_{RN},x_{DN}) = P(x_R)\prod_{i=1}^{N}P(x_{Ri}|x_R)\,P(x_{Di}|x_R) \quad (11)$$

III. PROOF OF THE MAIN RESULTS

Proof of Theorem 1:

The two-source multiple access relay channel with orthogonal components is shown in Fig. 2.

Definition: A $\left((2^{nR_1},2^{nR_2}),n\right)$, $R_1=R_{R1}+R_{D1}$, $R_2=R_{R2}+R_{D2}$, code for the multiple access relay channel with orthogonal components consists of two sets of integers $w_1=(w_{R1},w_{D1})\in[1:2^{nR_{R1}}]\times[1:2^{nR_{D1}}]$ and $w_2=(w_{R2},w_{D2})\in[1:2^{nR_{R2}}]\times[1:2^{nR_{D2}}]$, called the message sets; two encoding functions,

$$X_1:\mathcal{W}_1=(\mathcal{W}_{R1},\mathcal{W}_{D1})\to\mathcal{X}_1^n=\mathcal{X}_{R1}^n\times\mathcal{X}_{D1}^n \quad (12)$$

$$X_2:\mathcal{W}_2=(\mathcal{W}_{R2},\mathcal{W}_{D2})\to\mathcal{X}_2^n=\mathcal{X}_{R2}^n\times\mathcal{X}_{D2}^n \quad (13)$$

a set of relay function $\{f_i\}_{i=1}^n$ such that

$$X_{R,i}=f_i\{Y_R^{i-1}\},\qquad 1\le i\le n \quad (14)$$

and a decoding function,

$$g:\mathcal{Y}_D^n\longrightarrow\mathcal{W}_1\times\mathcal{W}_2=(\mathcal{W}_{R1},\mathcal{W}_{D1})\times(\mathcal{W}_{R2},\mathcal{W}_{D2}) \quad (15)$$

Sender 1 chooses an index $w_1=(w_{R1},w_{D1})$ uniformly distributed over $[1:2^{nR_{R1}}]\times[1:2^{nR_{D1}}]$ and sends the corresponding codeword over the channel. Sender 2 does likewise. Assuming that the distribution of messages over the product set $\mathcal{W}_1\times\mathcal{W}_2$ is uniform, we define the average probability of error for the $\left((2^{nR_1},2^{nR_2}),n\right)$ code as follows:

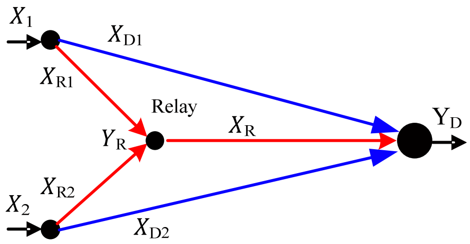

Figure 2. A 2- source multiple-access relay channel with orthogonal components

$$P_e^{(n)}=\frac{1}{2^{n(R_1+R_1)}}\sum_{(w_1,w_2)\in\mathcal{W}_1\times\mathcal{W}_2}\Pr\{g(Y_D^n)\neq(w_1,w_2)|(w_1,w_2)\text{ has been sent}\}$$

A rate pair (R_1,R_2) is said to be achievable for the multiple access relay channel with orthogonal component if there exists a sequence of $\left((2^{nR_1},2^{nR_2}),n\right)$, $R_1=R_{R1}+R_{D1}$, $R_2=R_{R2}+R_{D2}$, codes with $P_e^{(n)}\to 0$.

A. Achievability:

We begin with a brief outline of the proof. We consider B blocks, each of n symbols. We use superposition block Markov coding. A sequence of B messages $w_{1,i}\times w_{2,i}=(w_{R1,i},w_{D1,i})\times(w_{R2,i},w_{D2,i})$, $i\in 1,2,\dots,B$ will be sent over the channel in nB transmissions. In each n-block, $b=1,2,\dots,B$, we use the same set of codebooks:

$$\mathcal{C}=\{x_R^n(v),x_{R1}^n(v,k),x_{R2}^n(v,l),x_{D1}^n(v,t),x_{D2}^n(v,u)\};$$

$$v\in\left[1:2^{n(R_{R1}+R_{R2})}\right],\quad k\in[1:2^{nR_{R1}}],\quad l\in[1:2^{nR_{R2}}],\quad t\in[1:2^{nR_{D1}}],\quad u\in[1:2^{nR_{D2}}]$$

Now we proceed with the proof of achievability using a random coding technique.

Random codebook generation: First fix a choice of $P_{X_R}(x_R)P_{X_{R1}|X_R}(x_{R1}|x_R),P_{X_{R2}|X_R}(x_{R2}|x_R)P_{X_{D_1}}(x_{D1}|x_R)P_{X_{D_2}}(x_{D2}|x_R)$.

1) Generate $2^{n(R_{R1}+R_{R2})}$ independent identically distributed n-sequences x_R^n, each drawn according to $P(x_R^n)=\prod_{i=1}^n P_{X_R}(x_{R,i})$. Index them as $x_R^n(v)$, $v\in\left[1:2^{n(R_{R1}+R_{R2})}\right]$.
2) For each $x_R^n(v)$, generate $2^{nR_{R1}}$ conditionally independent n-sequence x_{R1}^n drawn according to $P(x_{R1}^n|x_R^n(v))=\prod_{i=1}^n P(x_{R1,i}|x_{R,i}(v))$. Index them as $x_{R1}^n(v,k)$, $k\in[1:2^{nR_{R1}}]$.
3) For each $x_R^n(v)$, generate $2^{nR_{R2}}$ conditionally independent n-sequence x_{R2}^n drawn according to $P(x_{R2}^n|x_R^n(v))=\prod_{i=1}^n P\left(x_{R2,i}\middle|x_{R,i}(v)\right)$. Index them as $x_{R2}^n(v,l)$, $l\in[1:2^{nR_{R2}}]$.
4) For each $x_R^n(v)$, generate $2^{nR_{D1}}$ independent identically n-sequence x_{D1}^n, each drawn according to $P(x_{D1}^n|x_R^n(v))=\prod_{i=1}^n P(x_{D1,i}|x_{R,i}(v))$. Index them as $x_{D1}^n(v,t)$, $t\in[1:2^{nR_{D1}}]$

Table 1. Encoding strategy

Block 1	Block 2	...	Block B + 1
$x^n_{R,1}(1,1)$	$x^n_{R,2}(w_{R1,1}, w_{R2,1})$	...	$x^n_{R,B+1}(w_{R1,B}, w_{R2,B})$
$x^n_{R1,1}(1, w_{R1,1})$	$x^n_{R1,2}(w_{R1,1}, w_{R1,2})$	...	$x^n_{R1,B+1}(w_{R1,B}, 1)$
$x^n_{R2,1}(1, w_{R2,1})$	$x^n_{R2,2}(w_{R2,1}, w_{R2,2})$	...	$x^n_{R2,B+1}(w_{R2,B}, 1)$
$x^n_{D1,1}(1, w_{D1,1})$	$x^n_{D1,2}(w_{D1,1}, w_{D1,2})$	...	$x^n_{D1,B+1}(w_{D1,B}, 1)$
$x^n_{D2,1}(1, w_{D2,1})$	$x^n_{D2,2}(w_{D2,1}, w_{D2,2})$	...	$x^n_{D2,B+1}(w_{D2,B}, 1)$

5) For each $x^n_R(v)$, generate $2^{nR_{D2}}$ independent identically n-sequence x^n_{D2}, each drawn according to $P(x^n_{D2}|x^n_R(v)) = \prod_{i=1}^n P(x_{D2,i}|x_{R,i}(v))$. Index them as $x^n_{D2}(v, u)$, $u \in [1: 2^{nR_{D2}}]$.

Encoding: Encoding is performed in B+1 blocks, The coding strategy is shown in Table 1.

1) Source terminals: The messages are splitted into B equally sized blocks $w_{R1,b}$, $w_{R2,b}$, $w_{D1,b}$, $w_{D2,b}$, $b = 1,2, \dots, B$. In block $b = 1,2, \dots, B + 1$, the sender 1 transmit $x^n_{R1,b}(w_{R1,b-1}, w_{R1,b})$ and $x^n_{D1,b}(w_{D1,b})$, where $w_{R1,0} = w_{R1,B+1} = w_{D1,B+1} = 1$. Sender 2 does likewise.

2) Relay Terminal: After the transmission of block b is completed, the relay has seen $y^n_{R,b}$. The relay tries to find a pair $(\tilde{w}_{R1,b}, \tilde{w}_{R2,b})$ such that

$$(x^n_{R1,b}(\hat{w}_{R1,b-1}, \tilde{w}_{R1,b}), x^n_{R2,b}(\hat{w}_{R2,b-1}, \tilde{w}_{R2,b}),$$
$$x^n_{R,b}(\hat{w}_{R1,b-1}, \hat{w}_{R2,b-1}), y^n_{R,b}) \in A^n_\epsilon(X_{R1}, X_{R2}, X_R, Y_R) \quad (16)$$

where $\hat{w}_{R1,b-1}$ and $\hat{w}_{R2,b-1}$ are the relay terminal's estimate of $w_{R1,b-1}$ and $w_{R2,b-1}$, respectively. If one or more such $\tilde{w}_{R1,b}$ and $\tilde{w}_{R2,b}$ are found, then the relay chooses one of them, calls this choice $\hat{w}_{R1,b}$ and $\hat{w}_{R2,b}$ and then transmits $x^n_{R,b+1}(\hat{w}_{R1,b}, \hat{w}_{R2,b})$ in block b+1. If no such $\tilde{w}_{R1,b}$ and $\tilde{w}_{R2,b}$ are found, the relay sets $\hat{w}_{R1,b-1} = 1$ and $\hat{w}_{R2,b-1} = 1$ and then transmits $x^n_{R,b+1}(1,1)$.

3) Sink Terminal: After block b, the receiver has seen $y^n_{D,b-1}$ and $y^n_{D,b}$ and try to find $\tilde{w}_{R1,b-1}$, $\tilde{w}_{R2,b-1}$, $\tilde{w}_{D1,b-1}$ and $\tilde{w}_{D2,b-1}$ such that

$$(x^n_{R,b-1}\left(\hat{\hat{w}}_{R1,b-2}, \hat{\hat{w}}_{R2,b-2}\right), x^n_{D1,b-1}\left(\hat{\hat{w}}_{D1,b-2}, \hat{w}_{D1,b-1}\right),$$
$$x^n_{D2,b-1}\left(\hat{\hat{w}}_{D2,b-2}, \hat{w}_{D2,b-1}\right), y^n_{D,b-1}) \in A^n_\epsilon(X_R, X_{D1}, X_{D2}, Y_D) \quad (17)$$

and

$$(x^n_{R,b}(\tilde{w}_{R1,b-1}, \tilde{w}_{R2,b-1}), y^n_{D,b}) \in A^n_\epsilon(X_R, X_{D1}, X_{D2}, Y_D) \quad (18)$$

If one or more such $\tilde{w}_{D1,b-1}$, $\tilde{w}_{D2,b-1}$, $\tilde{w}_{R1,b-1}$ and $\tilde{w}_{R2,b-1}$ are found, then the sink chooses one of them and puts out these choices $\hat{\hat{w}}_{D1,b-1}$, $\hat{\hat{w}}_{D2,b-1}$ and $\hat{\hat{w}}_{R1,b-1}$ and $\hat{\hat{w}}_{R2,b-1}$, respectively. If no such $\tilde{w}_{D1,b-1}$, $\tilde{w}_{D2,b-1}$, $\tilde{w}_{R1,b-1}$ and $\tilde{w}_{R2,b-1}$ are found, the sink sets $\hat{\hat{w}}_{D1,b-1} = \hat{\hat{w}}_{D2,b-1} = \hat{\hat{w}}_{R1,b-1} = \hat{\hat{w}}_{R2,b-1} = 1$.

Decoding and error Analysis: It can be shown with [2],[14,theorem 15.2.3] that the relay can decode reliably and the receiver can decode with arbitrarily small probability of error if

$$0 \le R_{R1} \le I(X_{R1}; Y_R|X_{R2}, X_R) \quad (19)$$
$$0 \le R_{R2} \le I(X_{R2}; Y_R|X_{R1}, X_R) \quad (20)$$
$$0 \le R_{R1} + R_{R2} \le I(X_{R1}, X_{R2}; Y_R|X_R) \quad (21)$$

and

$$0 \le R_{D1} \le I(X_{D1}; Y_D|X_{D2}, X_R) \quad (22)$$
$$0 \le R_{D2} \le I(X_{D2}; Y_D|X_{D1}, X_R) \quad (23)$$
$$0 \le R_{D1} + R_{D2} \le I(X_{D1}, X_{D2}; Y_D|X_R) \quad (24)$$

Therefore

$$0 \le R_1 \le I(X_{R1}; Y_R|X_{R2}, X_R) + I(X_{D1}; Y_D|X_{D2}, X_R) \quad (25)$$
$$0 \le R_2 \le I(X_{R2}; Y_R|X_{R1}, X_R) + I(X_{D2}; Y_D|X_{D1}, X_R) \quad (26)$$
$$0 \le R_1 + R_2 \le I(X_{R1}, X_{R2}; Y_R|X_R) + I(X_{D1}, X_{D2}; Y_D|X_R) \quad (27)$$

Proof of Theorem 2:

It is possible to estimate w_1, w_2 $((w_{R1}, w_{D1}), (w_{R2}, w_{D2}))$ from the received sequence Y^n with low probability of error. Hence, the conditional entropy of (w_1, w_2) given Y^n must be small. By Fano's inequality,

$$H(W_{R_1}, W_{R_2}|Y^n_R) \le n\epsilon_{1n}$$
$$H(W_{D_1}, W_{D_2}|Y^n_D) \le n\epsilon_{2n}$$

To bound the sum rate, we have

$$n(R_{R_1} + R_{R_2}) = H(W_{R_1}, W_{R_2}) \quad (28)$$
$$= I(W_{R_1}, W_{R_2}; Y^n_R) + H(W_{R_1}, W_{R_2}|Y^n_R) \quad (29)$$
$$\le^a I(W_{R_1}, W_{R_2}; Y^n_R) + n\epsilon_{1n} \quad (30)$$
$$\le^b I(X^n_1(W_{R_1}), X^n_2(W_{R_2}); Y^n_R) + n\epsilon_{1n} \quad (31)$$
$$= H(Y^n_R) - H\left(Y^n_R|X^n_1(W_{R_1}), X^n_2(W_{R_2})\right) + n\epsilon_n \quad (32)$$

$$=^{c} H(Y_R^n) - \sum_{i=1}^{n} H\left(Y_{R,i}|Y_R^{i-1}, X_1^n(W_{R_1}), X_2^n(W_{R_2}), X_{R,i}\right) + n\epsilon_{1n} \quad (33)$$

$$=^{d} H(Y_R^n) - \sum_{i=1}^{n} H\left(Y_{R,i}|X_{R_1,i}, X_{R_2,i}, X_{R,i}\right) + n\epsilon_{1n} \quad (34)$$

$$\leq \sum_{i=1}^{n} H\left(Y_{R,i}|Y_R^{i-1}\right) - \sum_{i=1}^{n} H\left(Y_{R,i}|X_{R_1,i}, X_{R_2,i}, X_{R,i}\right) + n\epsilon_{1n} \quad (35)$$

$$\leq^{e} \sum_{i=1}^{n} H\left(Y_{R,i}|X_{R,i}\right) - \sum_{i=1}^{n} H\left(Y_{R,i}|\, X_{R_1,i}, X_{R_2,i}, X_{R,i}\right) + n\epsilon_{1n} \quad (36)$$

$$= \sum_{i=1}^{n} I\left(X_{R_1,i}, X_{R_2,i}; Y_{R,i}|X_{R,i}\right) + n\epsilon_{1n} \quad (37)$$

where

a) follows from Fano's inequality
b) follows from the data-processing inequality
c) follows from the chain rule and definition of relay function
d) follows from the fact that $Y_{R,i}$ depends only on $X_{R_1,i}, X_{R_2,i}$ and $X_{R,i}$by the memoryless property of the channel
e) follows from the chain rule and definition of relay function and removing conditioning.

$$R_{R_1} + R_{R_2} \leq \frac{1}{n}\sum_{i=1}^{n} I\left(X_{R_1,i}, X_{R_2,i}; Y_{R,i}|X_{R,i}\right) + \epsilon_{1n} \quad (38)$$

$$R_{R_1} + R_{R_2} \leq I\left(X_{R_1}, X_{R_2}; Y_R|X_R\right) \quad (39)$$

Similarly, we have

$$R_{D_1} + R_{D_2} \leq \frac{1}{n}\sum_{i=1}^{n} I\left(X_{D_1,i}, X_{D_2,i}; Y_{D,i}|X_{R,i}\right) + \epsilon_{1n} \quad (40)$$

$$R_{D_1} + R_{D_2} \leq I\left(X_{D_1}, X_{D_2}; Y_D|X_R\right) \quad (41)$$

Therefore proof of theorem 2 is finished.

Proof of Theorem 3, 4:

We will now generalize the result derived from theorem 1, 2 to N senders. We send independent indices $w_1, w_2, \ldots, w_N$ over the channel from the senders $1, \cdots, N$, respectively. The codes, rates, and achievability are all defined in exactly the same way as in the two-source case.

The proof contains no new ideas. There are now $2^N - 1$ terms in the probability of error in the achievability proof and an equal number of inequalities in the proof of the converse. Details are omitted.

IV. CONCLUSION

We have established an achievable rate region and an outer bound for sum capacity rate for the multiple access relay channel with orthogonal components from the sender to the relay receiver and from the sender and relay to the receiver. For the class of multiple access relay channels with orthogonal components discussed here, the optimal strategy is to split the messages into two parts, one is decoded by the relay and sent cooperatively with the senders to the receiver and the other is sent directly to the receiver.

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