Development of a two-phase model for simulating the behavior of reinforced soils

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Abstract

The present paper presents the formulation of a two-phase system applied for reinforced soil media. In a two-phase material, the soil and inclusion are treated as two individual superposed continuous media called matrix and reinforcement phases, respectively, which are connected to each other through perfect bonding condition. The proposed algorithm is aimed to analyze the behavior of reinforced soil structures under operational condition focusing on Geosynthetics-Reinforced-Soil (GRS) walls. The global behavior of such deformable structures is highly dependent to the soil behavior. As a result, accounting for non-linear behavior of matrix phase would be the first step in accurate simulation of GRS structures. Hence, in the existing two-phase system, the linear elastic-perfect plastic soil model is replaced by a relatively simple soil model, which is formulated in Bounding Surface Plasticity framework. For validation of the proposed model, the behavior of several single element reinforced soil samples is simulated and the results are compared with experiment. It is shown that the model is accurately capable of predicting the behavior especially before peak shear strength.

1 INTRODUCTION

In many cases where the soil suffers from insufficient strength or deformability, reinforcing it with inclusions such as steel bars or Geosynthetics layers might be the solution. In such case, the medium can be regarded as a composite which behaves as a homogenous but anisotropic material. In domain of analytical methods of periodic media, de Buhan and Sudret (1999) have proposed a new approach called "Multiphase model", which has been introduced based on virtual work method. This approach explains a macroscopic description of a layered composite as the superposition of individual continuous media called phases. For a reinforced soil as a multiphase material, each point of the geometry is comprised of matrix phase (representative of the soil) and reinforcement phases (representative of axial inclusions). This technique has already been used for analysis of piled-raft group (de Buhan and Sudret, 2000; Hassen and de buhan, 2005), rock-bolted tunnel (de Buhan et al., 2008), and the stability of retaining wall (Thai et al. 2009).

The main objective of the present paper is to present the formulation of the two-phase material by introducing a non-linear elasto-plastic soil model under monotonic loading path. The soil model is for non-cohesive granular soil within the framework of Bounding Surface Plasticity. In this study, perfect bonding is assumed between phases. We evaluate the presented two-phase model by simulating the behavior of several single element reinforced sand samples.

2 DESCRIPTION OF A TWO-PHASE SYSTEM

A reinforced soil mass in the global coordinate system 1-2-3 is shown in Figure 1a. The planar inclusions are placed with an angle of α with horizontal direction (the 1–3 plane). The inclusions have the thickness t placed in a periodic manner with the same spacing h from each other. A local coordinate system x-y-z is assigned for the inclusion. According to Figure 1b, such a medium can be replaced by a two-phase material. From view of macroscopic scale, each geometrical point of such two-phase material is decomposed into matrix phase (labeled as m), and reinforcement phase (labeled as *r*). The statics equations derived from the kinematics of each phase gives the equilibrium equations for each phase (Sudret, 1999; de Buhan and Sudret, 2000):

$$div\sigma_{ij}^{m} + \rho^{m} F_{i}^{m} + I_{i} = 0 \quad (1)$$

for the matrix phase, and

$$div\sigma_{ij}^{r} + \rho^{r} F_{i}^{r} - I_{i} = 0 \quad (2)$$

for the reinforcement phase. σ_{ij}^{m} (resp. σ_{ij}^{r}) stands for the classical Cauchy stress tensor defined in the matrix (resp. reinforcement) phase. The term $\rho^{m}F_{i}^{m}$

(resp. $\rho^r F_i^r$) indicates the body force of the matrix (resp. reinforcement) phase. Finally, $+I_i$ (resp. $-I_i$) is the interaction body force exerted by the reinforcement (resp. matrix) phase on the matrix (resp. reinforcement) phase. These equilibrium equations will be completed by the corresponding stress boundary conditions that are prescribed on the boundary surface of each phase separately.

By summing up Equations 1 and 2, the equilibrium equation for a two-phase material becomes:

$$div\Sigma_{ij} + \rho F_i = 0 \quad (3)$$

where:

$$\Sigma_{ij} = \sigma_{ij}^m + \sigma_{ij}^r,\ \rho F_i = \rho^m F_i^m + \rho^r F_i^r \quad (4)$$

Σ_{ij} represents the global stress tensor with partial stresses of all phases. The term ρF_i, indicates the body force exerted to the whole mass of the two-phase material.

For each phase, the stress-strain relationship is introduced separately:

$$\sigma_{ij}^m = A_{ijkl}^m \varepsilon_{kl}^m,\ \sigma_{ij}^r = A_{ijkl}^r \varepsilon_{kl}^r \quad (5)$$

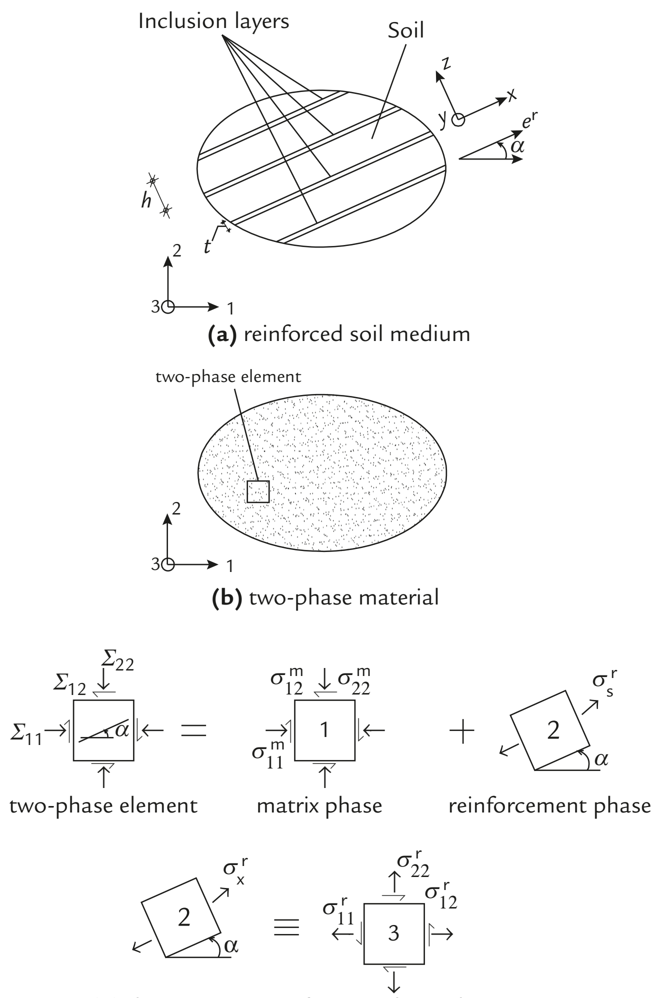

Figure 1 Introduction of a reinforced soil as a two-phase material.

where A_{ijkl}^m (resp. A_{ijkl}^r) indicates the stiffness tensor of the matrix (resp. reinforcement) phase. The evolution of stress state in each phase is governed by its corresponding yield criterion which defines the onset of plastic (irreversible) deformation in the phase. As general and for simplicity, we consider perfect bonding condition between phases. In this case, we have:

$$\in_{ij} = \varepsilon_{ij}^m = \varepsilon_{ij}^r \quad (6)$$

in which $\in_{ij}$ denotes the strain tensor of the two-phase material. By considering such strain compatibility condition, the global stress-strain relationship appears in the following form:

$$\Sigma_{ij} = (A_{ijkl}^m + A_{ijkl}^r)\in_{kl} \quad (7)$$

In the above constitutive equation, the interaction between phases was ignored regarding the detachment of phases. It is interesting to note that the interaction can be considered too in the formulation by applying an extra condition such as frictional strength of the common surface along the inclusion. This contribution is out of the scope of this paper and the reader is referred to Seyedi Hosseininia and Farzaneh (2009).

2.1 Reinforcement Phase

The behavior of the inclusion is described in the local coordinate system x-y-z. It is assumed that the inclusion has an tensile isotropic linear elastic–perfectly plastic behavior with Young modulus (E^{inc}), Poisson's ratio (ν^{inc}), and yield stress (σ_{yield}^{inc}). From elasticity, the incremental stress-strain relationship for a bi-dimensional element has the following form:

$$\begin{Bmatrix} \dot\sigma_x^{inc} \\ \dot\sigma_y^{inc} \end{Bmatrix} = \frac{E^{inc}}{(1-\nu^{inc^2})} \begin{bmatrix} 1 & \nu^{inc} \\ \nu^{inc} & 1 \end{bmatrix} \begin{Bmatrix} \dot\varepsilon_x^{inc} \\ \dot\varepsilon_y^{inc} \end{Bmatrix} \quad (8)$$

σ_x and σ_y are in-plane stresses and ε_x and ε_y are corresponding strains. The over dot denotes the incremental form of the parameters. The yield function of the inclusion (f^{inc}) in a two-dimensional space obeys from Tresca criterion:

$$f^{inc}(\sigma_x^{inc}, \sigma_y^{inc}) = \left|\sigma_x^{inc} - \sigma_y^{inc}\right| \le \left|\sigma_{yield}^{inc}\right| \quad (9)$$

In order to define macroscopic properties of the reinforcement phase in terms of those of the inclusion, the reinforcement volume fraction (χ), which is usually very small, is introduced as the ratio of the inclusion volume (V^{inc}) to that of the soil (V^s):

$$\chi = \frac{V^{inc}}{V^s} = \frac{t}{h} \quad (10)$$

The mechanical properties (without any change in Poisson's ratio) are defined as follows:

$$E^r = \chi E^{inc},\ \sigma_{yield}^r = \chi\sigma_{yield}^{inc} \quad (11)$$

Finally, the stiffness tensor of the reinforcement phase (A_{ijkl}) is defined:

$$A_{ijkl} = E^r(e_i^r \otimes e_j^r \otimes e_k^r \otimes e_l^r) \quad (12)$$

The symbol $\otimes$ indicates dyadic product of vectors.

2.2 Matrix Phase

As mentioned before, we introduce a simple constitutive model for non-cohesive granular soils in such a way that; 1) the soil behavior is simulated with minimum possible number of model parameters for monotonic loading paths; 2) the parameters can be easily measured in a straightforward manner from traditional tests; 3) the formulation can be improved properly for future extensions. The soil constitutive model is introduced within Bounding Surface Plasticity framework (Dafalias and Popov, 1975; Krieg 1975). According to this framework, a yield surface always places inside the other surface called as bounding surface. The magnitude of plastic strain increment is defined as a direct function of distance between current stress state on yield surface from a conjugate (image) stress state defined on the bounding surface. Also, it is noted that the sign of stress and strain components are considered as positive. The formulation is presented in terms of principal stress and strain components.

The total strain increment (shown by dot sign) is supposed to be the sum of elastic and plastic parts:

$$\dot{\varepsilon}_i = \dot{\varepsilon}_i^e + \dot{\varepsilon}_i^p \quad (13)$$

The superscripts *e* and *p* stand for the elastic and plastic parts, respectively.

Elastic Behavior

Incremental elastic strain portion is obtained from the generalized Hooke's law in terms of shear modulus (G) and Poisson's ratio (ν):

$$\dot{\varepsilon}_i^e = \frac{1}{2G(1+\nu)}\left[\dot{\sigma}_i - \nu \sum_{j \neq i} \dot{\sigma}_j\right] \quad (14)$$

According to empirical studies of granular media (e.g., Duncan and Chang, 1970), the shear modulus is considered variable as function of mean effective stress (p):

$$G = G_0\left(\frac{p}{p_r}\right)^n \quad (15)$$

G_0 and n are positive model parameters and p_r is the atmospheric pressure (= 101 kPa).

Plastic Behavior

The yield surface (f) is introduced in the same form of Mohr-Coulomb surface as a wedge-type shape:

$$f(\sigma_1, \sigma_3) = (\sigma_1 - \sigma_3) - Sin\varphi_{mob}(\sigma_1 + \sigma_3) = 0 \quad (16)$$

where ϕ_{mob} is the mobilized friction angle and it is representative of the size of yield surface. σ_1 and σ_3 are major and minor principal stresses, respectively. A similar surface is considered as bounding surface (F^b), where the mobilized friction angle of sand reaches the peak value (φ_{peak}):

$$F^b(\bar{\sigma}_1, \bar{\sigma}_3) = (\bar{\sigma}_1 - \bar{\sigma}_3) - Sin\varphi_{peak}(\bar{\sigma}_1 + \bar{\sigma}_3) = 0 \quad (17)$$

The bar symbol indicates the stress state on the conjugate (image) point over the bounding surface which can be defined by the mapping rule: the conjugate point corresponding to the current stress point is located on the bounding surface with the same effective mean stress (Li and Dafalias, 2000). For soil dilatancy, described as $D = -\dot{\varepsilon}_v^p / \dot{\varepsilon}_q^p$ by Rowe (1962), we consider:

$$D = D_0[Sin\varphi_{mob} - Sin\varphi_{PTL}] \quad (18)$$

where $\dot{\varepsilon}_v^p = \dot{\varepsilon}_1^p + \dot{\varepsilon}_3^p$ and $\dot{\varepsilon}_q^p = \dot{\varepsilon}_1^p - \dot{\varepsilon}_3^p$ represent plastic volumetric and plastic shear (deviatoric) strain increments, respectively. D_0 is a positive model parameter. ϕ_{PTL} is the mobilized friction angle at which the sand behavior is transformed from being contractive to dilative (e.g., Ishihara et al. 1970).

For the plastic modulus, K_p, it is defined in a simple form (Dafalais, 1986b):

$$K_p = h_0 G\left(\frac{Sin\varphi_{peak} - Sin\varphi_{mob}}{Sin\varphi_{mob}}\right) \quad (19)$$

where h_0 is a model parameter.

General Formulation

Referring back to Equation 13 and calculating elastic and plastic parts of strain increments, the general stress-strain relationship can be established:

$$\begin{Bmatrix} \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \\ \dot{\varepsilon}_3 \end{Bmatrix} = \frac{C^{ep}}{2G(1+\nu)K_p} \begin{Bmatrix} \dot{\sigma}_1 \\ \dot{\sigma}_2 \\ \dot{\sigma}_3 \end{Bmatrix} \quad (20)$$

$$C^{ep} = \begin{bmatrix} K_p + (1-D)AG & -\nu K_p & -\nu K_p - (1-D)BG \\ -\nu K_p & K_p & -\nu K_p \\ -\nu K_p - (1+D)AG & -\nu K_p & K_p + (1+D)BG \end{bmatrix}$$

where $A = (1 - \mathrm{Sin}\,\varphi_{mob})$ and $B = (1 + \mathrm{Sin}\,\varphi_{mob})$. There are totally six model parameters including ν, G_0, n, D_0, ϕ_{PTL}, and h_0. The procedure how these parameters can be measured as well as the soil model evaluation is presented in Seyedi Hosseininia (2009) and Seyedi Hosseininia and Farzaneh (2010). In Figure 2, a brief review of the parameter determination is presented.

3 SIMULATION AND EVALUATION

By the proposed formulation, the behavior of several reinforced soil single element is simulated and it is compared with the behavior seen in the laboratory as well as the model with original model. All reinforced soil samples are loaded in a plane strain compression (PSC) apparatus under drained condition.

It is also noted that the aforementioned two-phase model is applicable in analyzing a reinforced soil mass such as reinforced soil retaining wall. One example is mentioned in Seyedi Hosseininia and Farzaneh (2010).

3.1 First Series of Tests

Tatsuoka and Yamauchi (1986) performed a series PSC tests on reinforced dense Toyoura sand. The sand was reinforced by one geotextile layer, placed horizontally at the middle. The sample had the height and length of 75 mm and 80 mm and width of 40 mm. The sample was loaded in a strain control manner with a lateral confining pressure of $\Sigma_1 = 49$ kPa. The geotextile had higher stiffness in its machine direction than the cross direction. The properties are $E^{inc} = 12MPa$ and $\sigma_{\text{yield}}^{\text{inc}} = 42\,kPa$ for machine direction and $E^{inc} = 5MPa$ and $\sigma_{\text{yield}}^{\text{inc}} = 18\,kPa$ for cross direction. In addition, it has the thickness of 4 mm and $\nu^{\text{inc}} = 0$.

The laboratorial as well as simulated behavior of such composite (in terms of stress ratio Σ_2/Σ_1 and volumetric strain along axial strain) is shown in Figure 3 by considering the original soil model as linear elastic-perfectly plastic with Mohr-Coulomb criterion. The soil parameters include Young modulus ($E = 12$ MPa) and Poisson's ratio ($\nu = 0.4$), with dilatancy angle $\Psi = 9.5°$ and friction angle $\phi = 48.3$\ deg;. We have estimated the parameters according to the procedure mentioned in Brinkgreve (2005). As shown, the simulated behavior consists of two strength lines. In addition, the simulation has resulted some discrepancy in predicting the stress and strain paths. The other difference in results is the ultimate strength of the composite. This error originates from the hypothesis of perfect bonding in the simulation which is not true in large deformations in the reinforced soil sample.

The soil behavior is simulated by the proposed soil model (according to the calibration procedure mentioned before) and the values are stated in Table 1. Figure 4 presents the results of simulation and experimental behavior of unreinforced and reinforced soil samples. Despite the previous simple soil model, the results of simulation with non-linear soil model are well fitted on the experimental results for both unreinforced and reinforced samples before the stress ratio of samples reaches the maximum value. This satisfactory agreement can be found both in stress ratio and volumetric strain.

3.2 Second Series of Tests

The results of experimental efforts of McGown et al. (1978) are referred here. The samples were tested with PSC under constant confining pressure of 70 kPa with different types of inclusion. The sand used was dense Leighton Buzzard ($D_r = 65\%$). The unit cell in the tests had the width and height of 102 mm with

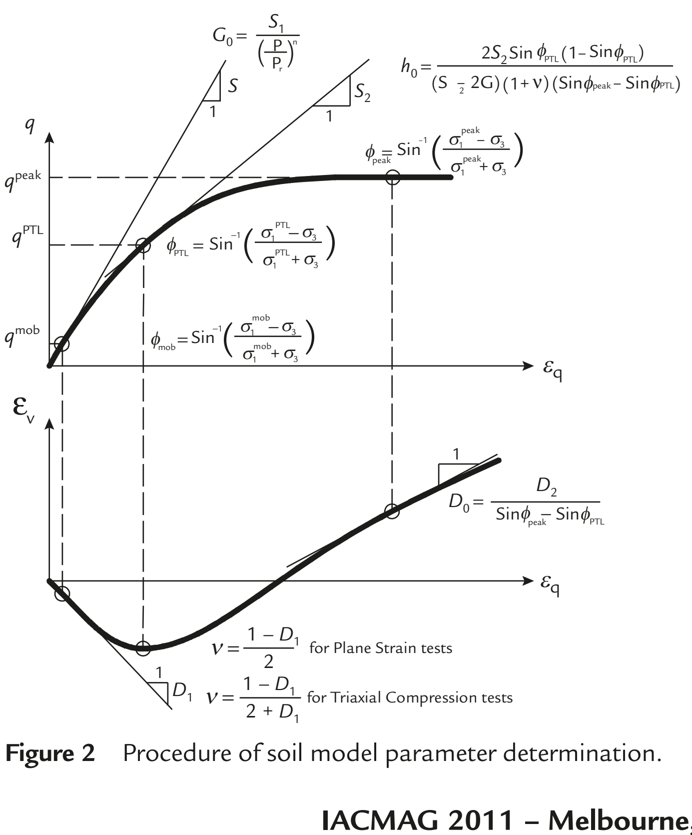

Figure 2 Procedure of soil model parameter determination.

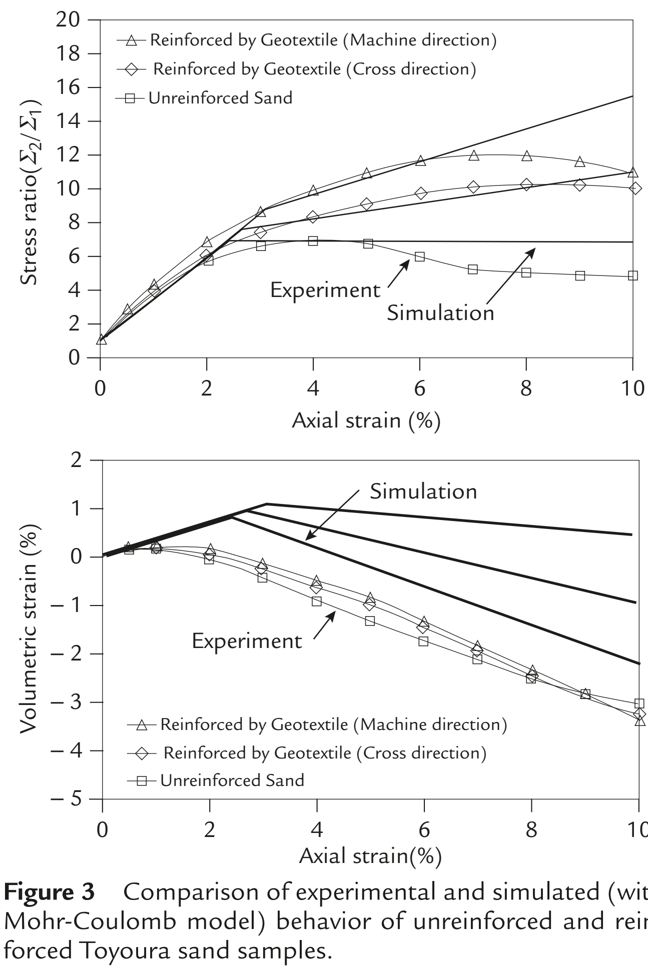

Figure 3 Comparison of experimental and simulated (with Mohr-Coulomb model) behavior of unreinforced and reinforced Toyoura sand samples.

length of 152 mm. One horizontal inclusion layer was used to reinforce the sand.

The inclusions were of three types: (1) non-woven fabric (T140); (2) aluminum mesh, and (3) aluminum foil. The first one was extensible (having large deformability), whilst the latter two ones were of inextensible type. The parameter properties of inclusions as well as of soil with the proposed model are listed in Table 2 and 3, respectively. We have simulated the behavior of samples by considering the linear soil model with $E = 52$ MPa and Poisson's ratio $\nu = 0.3$ and dilatancy angle $\Psi = 21°$. Since no data regarding the strain paths is published in the referred paper, the dilatancy in the non-linear model is assumed to be constant and $D = \text{Sin}\,\Psi$.

The obtained experimental data as well as two sets of perditions of unreinforced and reinforced sand samples are shown in Figure 5. As figured out, the simulation with linear soil model shows several breaks in the curves in addition that all samples have the same initial stiffness. However, the simulated curve of each sample with the proposed soil model is distinguished by their initial slopes. For instance, the slope of sample with T140 is located between unreinforced and aluminum foil samples like that observed in laboratory.

Table 1 Model properties for Toyoura sand

ν (−)	G_0 (MPa)	n (−)	D_0 (−)	ϕ_{PTL} (degree)	h_0 (−)	ϕ_{peak} (degree)
0.4	8	0.5	0.7	27	0.25	48.3

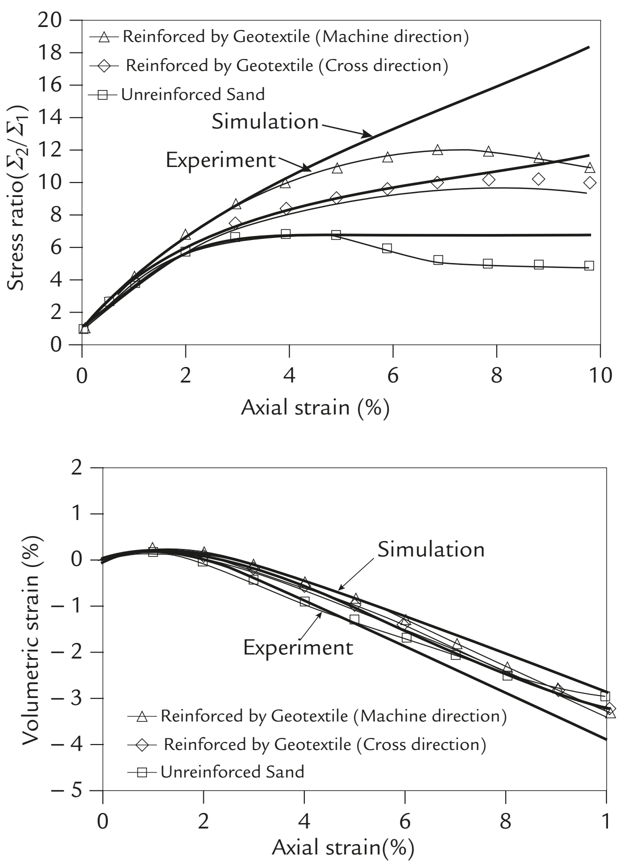

Figure 4 Comparison of experimental and simulated (with proposed non-linear model) behavior of unreinforced and reinforced Toyoura sand samples.

Table 2 Properties of inclusion used in McGown et al. (1978)

Geotextile	E^{inc} (MPa)	ν^{inc} (−)	σ^{inc}_{yield} (MPa)
T140	30	0.0	3.0
Aluminum mesh	200	0.0	4.0
Aluminum foil	560	0.1	1.4

Table 3 Model properties for Leighton Buzzard sand

ν (−)	G_0 (MPa)	n (−)	Ψ (−)	h_0 (−)	ϕ_{peak} (degree)
0.3	63	0.5	21	0.86	51

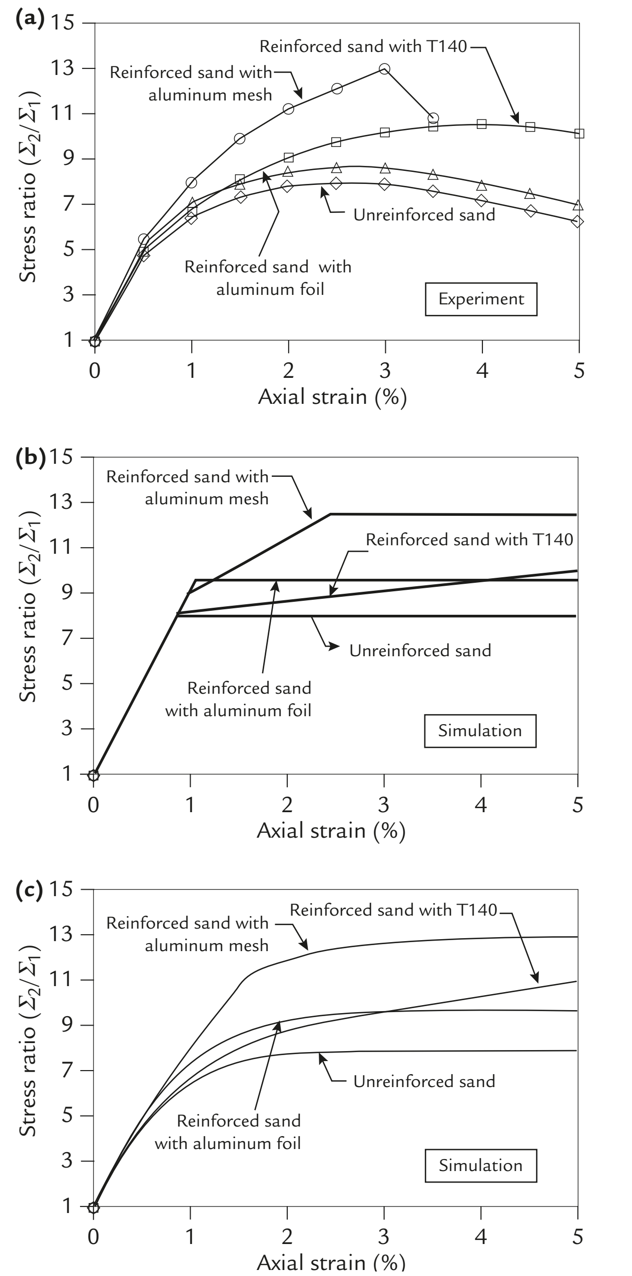

Figure 5 Experimental and simulated behavior of reinforced Leighton Buzzard sand in PSC test.

It would be of interest to note that regardless of the linearity of the soil behavior, the two-phase system can predict the ultimate strength of composite where inextensible inclusions are used in the system.

4 CONCLUSION

In this paper, the formulation of a two-phase material as reinforced soil is explained and the developments including the modifications in the constitutive models of matrix and reinforcement phases are presented too. For the soil, a simplified non-linear elasto-plastic model is introduced in the Bounding Surface Plasticity framework, by which the nonlinearity of soil behavior can be simulated. Substituting this formulation by the original model (the so-called Mohr-Coulomb model) in the two-phase formulation, one can find better agreement between the simulation and the experimental behavior of single element reinforced soil samples. This agreement exists both in the stress and strain paths before the sample reaches the peak strength. This case pertains to the operational condition of Geosynthetics Reinforced Soil (GRS) walls in which the structure do not show rupture, but it deforms under loading condition.

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