A solution procedure for the Generalized Covering Salesman Problem

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Abstract: Given n nodes, the covering salesman problem is to identify the minimum length tour "covering" all the nodes, i.e. the minimum length tour visiting a subset of the n nodes and such that each node not on the tour is within a predetermined distance from the nodes on the tour. In the Generalized Covering Salesman Problem (GCSP) each node i needs to be covered at least k_i times and there is a visiting cost associated with each node. This problem has three variants; in the first case, each node can be visited by the tour at most once, in the second version visiting a node more than once is possible but it is not allowed to stay overnight, and finally, in the third variant the tour can visit each node more than once consecutively. We propose an improvement procedure based on Integer Linear Programming (ILP) techniques. Computational results on benchmark instances from the literature show the effectiveness of the proposed approach.

Keywords: Covering Salesman Problem, Generalized Covering Salesman Problem, Heuristic Procedures, Integer Linear Programming.

1. Introduction

The Traveling Salesman Problem (TSP) is one of the best known problems in Operations Research. Given n nodes, the goal is to find the minimum length tour of the nodes, such that the salesman, starting from a node, visits each node exactly once and returns to the starting node [1]. Defined by Current [2], the Covering Salesman Problem (CSP) is to find the minimum length tour of a subset of n given nodes, such that each node i not on the tour is within a predefined covering distance d_i from a node that is on the tour. Current and Schilling [3] referred to some real world examples, such as routing of rural healthcare delivery teams where the assumption of visiting each city is not valid since it is sufficient for all cities to be near to some stops on the tour (the inhabitants of those cities which are not in the tour are expected to go to their nearest stop). They suggested a heuristic for the CSP in which in the first step the Set Covering Problem (SCP) over the given nodes is solved. Then the algorithm finds the optimal TSP tour of the nodes generated by solving the SCP. Arkin and Hassin [4] introduced a geometric version of the Covering Salesman Problem. In this problem each of the n nodes specifies a compact set in the plan, its neighborhood, within which the salesman should meet the stop [4]. The goal is computing the shortest length tour that intersects all of the neighborhoods and returns to the initial node. They presented simple heuristics for constructing tours for a variety of neighborhood types [4]. Since sometimes in the real world applications some cities need to be covered more than once, and there is a cost for staying in a city for one night, such as the cost of hotel, parking or other fees that commonly occur in practice, Salari et al. [5] introduced the *Generalized Covering Salesman Problem* (GCSP). They divided this problem into three variants: *Binary GCSP*, *Integer GCSP without overnight* and *Integer GCSP with overnight* and developed two local search heuristics, LS1 and LS2, for these variants [5].

2. Problem Definition

Given a directed graph $G=(N,A)$ *with* $N=\{1,2,...,n\}$ and $A=\{<i,j>: i,j\in N\}$ as the node and the arc sets, respectively, each node i can cover a subset of nodes D_i and has a predetermined coverage demand k_i. F_i is the fixed cost associated with node i and the solution is feasible if each node i is covered at least k_i times by the nodes in the tour. The goal is minimizing the total cost including the tour length and the cost associated with visited nodes. In the Binary GCSP, the tour is not allowed to visit a node more than once and after visiting a node the tour must satisfy the remaining coverage demand of that node by visiting other nodes that can cover it. In the GCSP without Overnight, a node can be visited more than once, but overnight stay is not allowed. Therefore, to have a feasible solution, after visiting a node, the tour can return to this node, if necessary, after having visited at least one other node. Finally, Integer GCSP with Overnight is similar to the latter version, but the overnight stay in a node is allowed.

3. PROPOSED HEURISTIC

We present a heuristic improvement procedure for the GCSP based on Integer Linear Programming (ILP) techniques. Given an initial solution to be possibly improved, the procedure iteratively performs the following steps: (a) select several nodes from the current solution, and build the restricted solution obtained from the current one by extracting (i.e., short-cutting) the selected nodes; (b) reallocate the extracted nodes to the restricted solution by solving an ILP problem, in the attempt of finding a new improved solution. This method has been proposed by De Franceschi et al. [6] and deeply investigated by Toth and Tramontani [7] and Salari et al. [8] in the context of the classical VRP and Open VRP, respectively. Here, we consider a simpler version of this procedure, which does not exploit any particular feature of the addressed problem. The method follows a destruct-and-repair paradigm, where the current solution is randomly destroyed (i.e., customers are removed in a random way) and repaired by following ILP techniques.

A. ILP_Improvement Procedure:

Let z be the current feasible solution of the GCSP and F a subset of customers visited in the current solution. We define $z(F)$ as the restricted solution obtained from z by extracting (i.e., by shortcutting) all the nodes in F. We add also to F all the nodes out of the current solution i.e, the nodes which have been covered by those visited in the tour. An insertion point is a potential location in the restricted solution which can be used to insert a new sequence.

The main steps of the procedure are the following:

- Build set F by selecting each node of the tour with a given probability.
- Extract from z the nodes selected in the previous step and build the restricted solution $z(F)$. Add also to F all the nodes out of the current tour i.e, those covered by the nodes visited in the initial tour.
- **Initialization**: For each insertion point i, initialize subset S_i with the possible node subset extracted from i plus the feasible singleton sequence (consisting of one customer belonging to F) having the minimum insertion cost. Initialize the Linear Programming (LP) relaxation of the Reallocation Model (LP-RM) by considering the initial subsets S_i and solve it.
- **Column generation**: For each insertion point i, solve the corresponding column generation problem by means of the *Heuristic Column Generation Procedure* described in the next section, and add to S_i all the sequences s such that the associated variables x_{si}, have a reduced cost under a given threshold $RC_{\max}$.
- Build the corresponding reallocation model and solve it to optimality.

B. Heuristic Column Generation Procedure:

For each insertion point i we use a heuristic approach to solve the corresponding column generation phase. In particular, for each insertion point we generate all the sequences s (by considering a limited neighbourhood of the corresponding insertion point) and if the reduced cost corresponding to the insertion of s into i is less than the given threshold, $RC_{\max}$, we add sequence s to S_i.

4. COMPUTATIONAL RESULTS

Preliminary computational results show that the proposed approach is competitive with the other methods available in the literature. In particular, on average the proposed approach can obtain slightly better results but in a larger CPU time. We are working for improving the quality of the solutions and reducing the CPU time.

5. CONCLUSION

We have proposed an *ILP_Improvement* based heuristic approach for the GCSP. Computational results on benchmark instances from the literature showed that the proposed method can be used as a profitable tool for improving existing GCSP solutions, and that even extremely-good quality solutions found by the other techniques proposed for GCSP can be further improved but in a larger CPU time.

6. REFERENCES

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