

# Generating Snake Robot Concertina Locomotion Using a New Dynamic Curve

Alireza Akbarzadeh, Jalil Safehian, Javad Safehian, and Hadi Kalani

**Abstract**—This paper starts with presenting a novel kinematics modelling approach for a snake-like robot travelling with concertina locomotion. The paper ends with confirmation studies using Webots simulation software. The significant advantage of the proposed kinematics model is in its flexibility to model natural snake robot concertina locomotion. Concertina locomotion refers to a type of motion where parts of the body contract, expand or do not change their shape. To simulate this, first we introduce a mathematical equation, called dynamic function, in which by varying a certain function parameter, body curve during motion is realized. To obtain concertina gait, the snake body is divided into three different modules, head module, tail module and main body module that connects the head to the tail module. Each module forms a specific curve which can be modelled using the proposed dynamic function. At each moment during snake locomotion, the kinematics of different links can be derived by fitting robot links to the body curve. Results indicate concertina locomotion is achieved. The proposed kinematics model represents a new approach to simulation of a snake-like mechanism in order to get basic characteristics of such locomotion and to enable our future research. Several ideas to further obtain natural snake locomotion is also presented.

**Index Terms**—Snake-like robots; Dynamic Curve; Concertina Curve; Body Shape; Concertina gait; Kinematics

## I. INTRODUCTION

Locomotion of snakes and other limbless animals have stimulated research and development of biologically inspired crawling robots. Snake can adapt their locomotion modes, gaits, according to the different grounds and conditions. They have many gaits to choose from and thus can move well on almost all grounds even on water.

Based on studies in exiting literature, snake movement can be divided into four main categories: Serpentine movement; Rectilinear movement; Concertina movement

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and Sidewinding movement. Each movement, also called gait or locomotion, has its own characteristic and is used for different grounds and conditions. In this paper, Concertina locomotion is investigated

The first qualitative research on snake locomotion was made by J. Gray in 1946 [1] and the first snake robot was built by Hirose [2]. Hirose studied kinematics of serpentine gait and proposed a ‘serpenoid curve’ as a means to generate Serpentine locomotion. M. Walter [3], J. Gray, C. Gans [4], [5] are, among many others, biologists specialized in limbless locomotion who tried to explain the principles of the snake locomotion and to model it. However among these works, there aren’t any detailed geometrical approaches which describe body forms and trajectory characteristics.

Klaassen and Paap [6] have presented a new snake-like robot (GMD-SNAKE2) and an algorithm for curvature controlled path calculation. Their work, mathematically is based on an enhancement of the well-known clothoid curve. Saito et al [7] made a snake-like robot without wheels and analyzed the optimally efficient serpentine locomotion. More recently Sh. Hasanzadeh and A. Akbarzadeh [8] presented a novel gait, forward head serpentine (FHS), for a two dimensional snake robot. They use Genetic Algorithm (GA) to find FHS gait parameters. J. Safehian et al.[9] proposed a novel kinematics modeling method for travelling wave locomotion. They also [10] considered kinematics and dynamics of traveling wave locomotion of a snake robot along symmetrical and unsymmetrical body curve. They investigated the effects of friction coefficient, initial winding angle and the unsymmetrical factor on the joint torques.

Snake-like gaits can be divided into two main classes: Snake-like and non snake-like gaits. Serpentine, concertina and sidewinding are three common snake-like gaits which are inspired from real snakes. Non snake-like gaits do not exist in nature but are useful in snake robot motion. However, these gaits are less addressed in literature. Spinning gait, flapping gait and travelling wave [8, 11, 12, 13] are examples of such gaits. Chen [13] analyzed the mechanism of travelling wave locomotion and showed that one period of this locomotion can be divided in four phases. He showed that these phases are based on the number of joints contacting the supporting surface and the resultant friction forces on contacting joints. Chen et al., [14] presented a model for travelling wave locomotion and considered its kinematics and dynamics.

## II. CONCERTINA LOCOMOTION MECHANISM

The word concertina represents a small accordion instrument. This name is used in snake locomotion to

indicate that the snake stretches and contracts its body to move forward. This motion is similar to the motion of the concertina instrument. Concertina movement occurs in snakes and other legless organisms and consists of gripping or anchoring section of the body while pulling/pushing other sections in the direction of movement [1]. In concertina locomotion parts of the body stop while other parts move forward. The sequence repeats and the snake moves forward. The key element of concertina locomotion is the utilization of the difference between higher forces resulting from static coefficient of friction and lower forces resulting from the dynamic coefficient of friction along different parts of the body. Fig. 2 shows a snake moving forward into the pipe by using concertina locomotion. As shown in this figure, the snake keeps the end parts of its body in contact with the pipe wall. Then gradually expand its body.

Forward motion occurs because of the passive action of the ventral scales. The posterior edges of the scales cause the static resistance due to backward motion. This resistance is four or five times greater than the resistance due to forward motion [15]. As shown in Fig. 1, the directions of the scales are so that snake moving forward will face less friction than moving backward. Jayne claims that concertina is seven times less efficient when compared to other kinds of locomotion used by real snakes [16]. However, snakes use concertina only when other options of locomotion are ineffective such as traversing tight spaces with high friction. Due to momentum changes, static friction, and slower speeds, concertina is a relatively inefficient mode of locomotion [17]. However, concertina allows motion not otherwise possible, such as moving along wires and cables as well as through tree branches.

### III. A NOVEL CURVE FITTING TO SNAKE BODY SHAPE

Because the snake body is string like, it can be likened to a curve. The curve attributed to the snake body is called body curve. The spine of the snake is along the body curve. Therefore, links of a snake-like robot that imitate the movement of a real snake may be fit to the body curve.

In this section, for the first time, concertina curve is defined and explained. To do this, the snake body curve is assumed to include several modules. Next, a simple dynamic curve will be defined and fitted to each module.

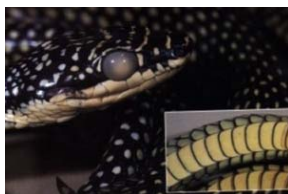


Figure 1. Scales covered the snake body

Consider snake moving in concertina locomotion as shown in Fig. 2. This figure shows 5 stages of progression. The more details of progression from stage 1 to stage 3 are shown in Fig. 3. Consider a frame that encompasses part of a snake body. This frame is shown in Fig. 3. As shown in this figure, a part of the snake body curve within the frame can be likened to the curve shown in diagram on the right side. As snake body changes from stage 1 to stage 2, the

curves in the diagram on the right side also change. The two variations between snake body curve and the other curve, on the right, are in accordance with each other. The two stages, 1 to 2 and 2 to 3 are repeated for the other parts of snake body. Therefore, the snake movement may be simulated using these curves. In what follows, we will introduce a curve which allows kinematics modeling body of a snake in concertina locomotion. The concertina curve is then made by combining several of these curves, each called a *dynamic curve*.

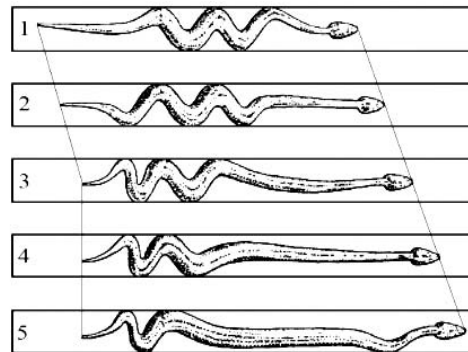


Figure 2. Progression stages of real snake Concertina Locomotion in pipe

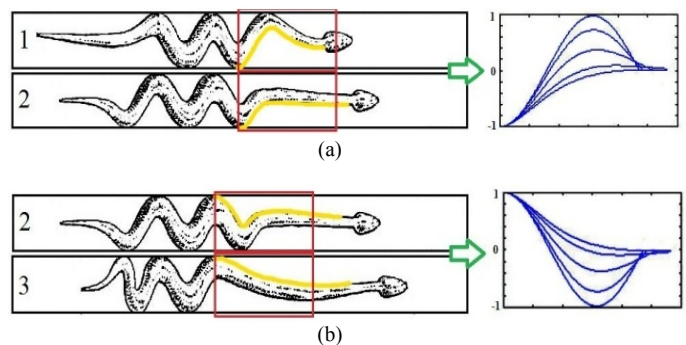


Figure 3. Similarity of snake body with dynamic curve

#### A. Dynamic Curve

Equation for the dynamic curve representing any section of the concertina curve, may be written as,

$$F_{concertina}(x) = \frac{e^{-\alpha x}}{\beta} \sin(\delta x + \theta) \tag{1}$$

$$\theta = \cos^{-1} \gamma, \alpha = \gamma \omega_n, \beta = \sqrt{1 - \gamma^2}, \delta = \omega_n \sqrt{1 - \gamma^2}$$

where  $\gamma = [0,1]$ . The dynamic curve for  $\omega_n = 1$  and different values of  $\gamma$  is shown in Fig. 4.

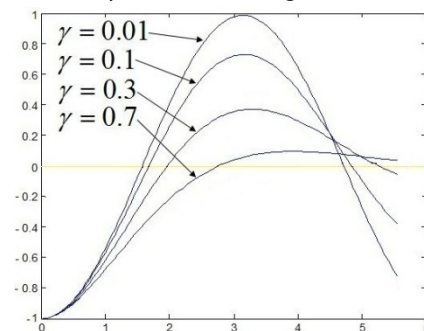


Figure 4. Effect of  $\gamma$  on dynamic curve

As can be seen from this figure, as parameter  $\gamma$  increases the high peak of the curve flattens and appears as though the curve is stretched. The stretched mode of the curve is similar to the case when the snake extends its body onward and thereby advances forward. When  $\gamma = 1$ , the dynamic curve is stretched the most. Referring to (1), when the value of  $\gamma$  move towards zero,  $\gamma \rightarrow 0$ , the curve obtained becomes similar to the serpenoid sine curve. Serpenoid curve is known as the most similar curve to the snake body [2].

Thus far, only a section of a concertina snake body curve is simulated by using the dynamic curve. In the following section, several combination of the dynamic curves are used to simulate the entire snake body curve, the concertina curve.

**B. Composition of Dynamic Curves**

Consider a snake moving in concertina locomotion as shown in Fig. 5. The developed concertina curve, made of connecting several dynamic curves, is also shown in this figure. As can be seen the developed concertina curve closely matches the real snake body curve. Next consider Fig. 6. To develop the concertina curve, the snake body is divided into tail, body and head modules.

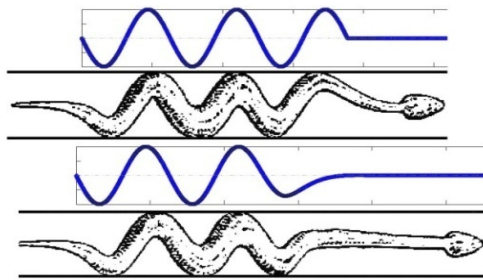


Figure 5. Similarity of snake body curve to concertina curve

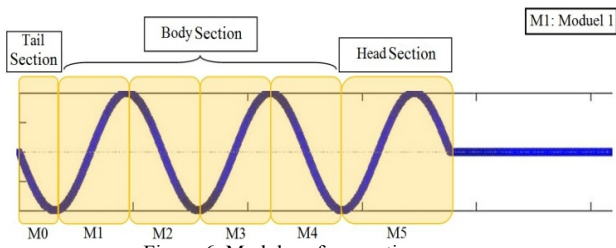


Figure 6. Modules of concertina curve

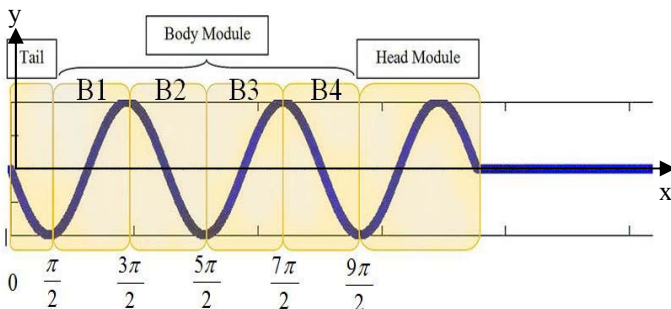


Figure 7. Coordinate system

**A. Tail Section:** The left segment of the snake body is referred to as tail module. A simplified dynamic curve is used to represent the tail section. This function is presented by,

$$y_{Tail\ Module}(x) = -\sin(\omega_n x), \text{ where } x \in [0, \frac{\pi}{2}] \& \omega_n = 1 \quad (2)$$

To define a reference point, we designate point (0,0) as the start point for the tail section. See Fig. 7. Further, the start configuration for the snake body is assumed to be in the contracted mode. This configuration represents the condition where snake is ready to start stretching. Note, the value for  $\omega_n$  is arbitrary. In this paper,  $\omega_n$  is assumed to be constant and equal to 1.

**B. Body Section:** The body section connects tail section to head section. The number of body modules is again arbitrary and can assume any even number. In this paper, the number of body modules considered is four. See Fig. 6 & Fig. 7. If the modules were numbered from left to right, and the number zero is allocated to tail module, then the equation for each body module would be determined as,

$$y_{Body\ Module}^{(i)}(x) = (-1)^{i+1} F_{Concertina}(x - x_{Body\ Module}) \quad (3)$$

where,

$i =$  The number assigned to body module  
 $i = 1, 2, \dots, N_B$  ( $N_B$  : Number of Body Modules)

$$x_{Body\ Module}^i = (\frac{\pi}{2} + (i-1)\pi) / \omega_n$$

$$x_{Body\ Module}^i \leq x \leq x_{Body\ Module}^i + \pi / \omega_n$$

Additionally,  $F_{Concertina}$  is defined by the dynamic curve in (1). The concertina curve made of tail, body and head sections with  $\omega_n = 1$  and  $\gamma = 0$  is shown in Fig. 6.

**C. Head Section:** The dynamic curve for the head module is developed by,

$$y_{Head\ Module}(x) = F_{Concertina}(x - x_{Head\ Module}) \quad (4)$$

Where,

$$x_{Head\ Module} \leq x \leq x_{Head\ Module} + \pi,$$

$$x_{Head\ Module} = \frac{\pi}{2} + N_B \times \pi$$

During locomotion, snakes generally keep their head mostly parallel to ground and straight. Furthermore, the up/down motion of the head is mostly negligible. Therefore, an appropriate dynamic curve for the head module is a curve where the end section of the curve remains on line  $y=0$ . To insure the end of head module remains fixed on the line  $y=0$ , the (4) is modified as following,

$$y_{Head\ Module}(x) = \begin{cases} F_{Concertina}(x - x_{Head\ Module}), & x_{Head\ Module} \leq x \leq x' \\ 0, & x' \leq x \end{cases} \quad (5)$$

where,

$$x_{Head\ Module} = (\frac{\pi}{2} + N_B \times \pi) / \omega_n \ \&$$

$$N_C = 2, (N_C: \text{Intersection Number})$$

Consider (5), the output of head module for values greater than  $x'$  is considered zero. Therefore, one must

identify a suitable value for  $x'$ . To do this, the behavior of the dynamic curve should be examined. Consider Fig. 8. The dynamic curves for interval  $[x=0 \sim x=10]$  and different values of  $\gamma$  are plotted. As can be seen, depending on the value of  $\gamma$ , the dynamic curves intersects the x-axis at different points.

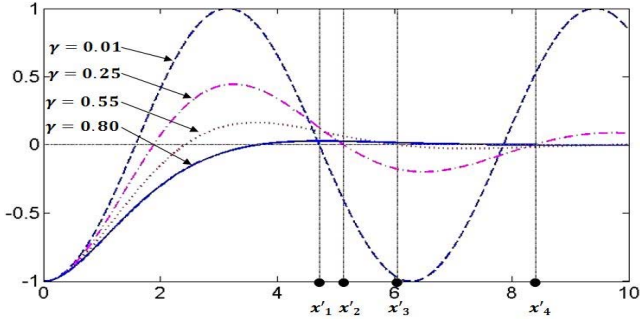


Figure 8. 2nd intersection ( $N_C=2$ ) of the dynamic curve  
( $x'_1=x'\gamma=0.01$ ,  $x'_2=x'\gamma=0.25$ ,  $x'_3=x'\gamma=0.55$ ,  $x'_4=x'\gamma=0.80$ )

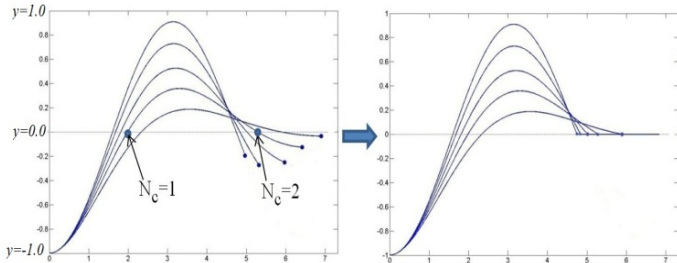


Figure 9. second intersection point of curve with line  $y=0$

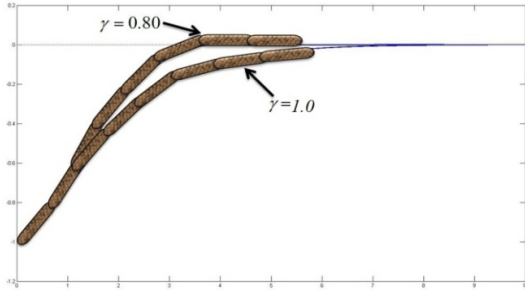


Figure 10. Effect of  $\gamma$  on the head link

To find the  $x'$  point we note,

$$F_{Concertina}(x) = \frac{e^{-\alpha x}}{\beta} \sin(\delta x + \theta) = 0 \rightarrow \delta x + \theta = N_C \pi$$

$$\rightarrow x = \frac{N_C \pi - \theta}{\omega_n \sqrt{1 - \gamma^2}}, \quad (6)$$

where,

$N_C = \text{intersection number}$

$$\theta = \cos^{-1} \gamma, \quad \alpha = \gamma \omega_n, \quad \beta = \sqrt{1 - \gamma^2}, \quad \delta = \omega_n \sqrt{1 - \gamma^2}$$

As stated earlier, the requirement for the head module is to stay flat and without any lateral changes. Part of this requirement is enforced by equating the dynamic curve with value zero. Next, consider Fig. 9. A more natural form of the head module is obtained by selecting the second intersection point,  $N_C=2$ , as a candidate for  $x'$ . Therefore, when  $N_C=2$ , we can write,

$$x' = \frac{N_C \pi - \theta}{\omega_n \sqrt{1 - \gamma^2}} + x_{Head Module} \quad (7)$$

Consider Fig. 8 and Fig. 10, as the value of  $\gamma$  moves toward one, the dynamic curve stretches and intersects the x axis in the infinity ( $x' \rightarrow \infty$ ). In this case, high torque values are exerted on the robot links. Furthermore, the numbers of robot links are limited. To solve this problem, the maximum allowable  $\gamma$  is limited to  $\gamma_{Max}$ . In this paper  $\gamma_{Max} = 0.8$  is selected.

#### IV. EXTENSION & CONTRACTION OF BODY

Thus far, the snake body curve in concertina locomotion is developed by combination of the tail, body and head modules. Next, these modules must change their form in order to generate the concertina locomotion.

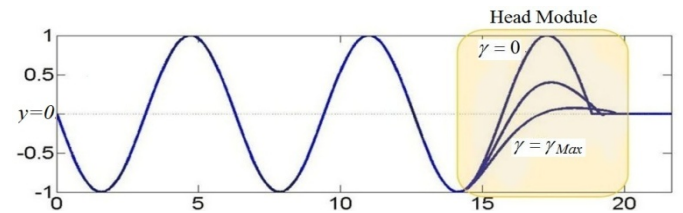


Figure 11. Gradual stretching head module

##### A. Stretching of the snake body curve

Note that all the dynamic curves making the different modules can be changed using only the parameter  $\gamma$  which changes from 0 to  $\gamma_{Max}$ . To model the concertina locomotion, first the head module is stretched followed by the body module and last the tail module. According to the number of modules types, the stages of stretching can be divided into three stages:

**First stage** (stretching of the head module): Parameter changes according to  $\gamma: 0 \rightarrow \gamma_{Max}$ . The head module will then stretch in the interval  $x_{Head Module} \leq x \leq x'$ . The stretching occurs while the other modules remain fixed. See Fig. 11.

**Second stage** (stretching of the body modules): Similar to the stretching of the head module, the  $\gamma$  parameter for each body module is increased according to  $\gamma: 0 \rightarrow \gamma_{Max}$ . Each one of the body modules will stretch in the interval of  $x_{Body Module}^i \leq x \leq x_{Body Module}^i + \pi$ . Consider Fig. 12. Coordination between modules that are stretched,  $\gamma = \gamma_{Max}$ , and those that are being stretched, increasing  $\gamma$  is required. For example, first consider Fig. 12 stages 1 through 4. During these stages, module #5 is stretching while module #4 is fixed. At stage 4, the parameter  $\gamma$  has reached its maximum,  $\gamma = \gamma_{Max}$ . At this point, module #4, begins stretching by increasing its parameter  $\gamma$  according to  $\gamma: 0 \rightarrow \gamma_{Max}$ . Observe stages 4 and 5. Note that, as module #4 is stretching, further stretching of module #5 occurs. To insure proper coordination between these two modules, the total value of the function representing the stretched module, module #5, is multiplied by the corresponding



value of the module being stretched, module #4. The amount of multiplication is equal to the value of the dynamic function at the junction point of the two modules, in this case, point  $P_{4-5}$ .

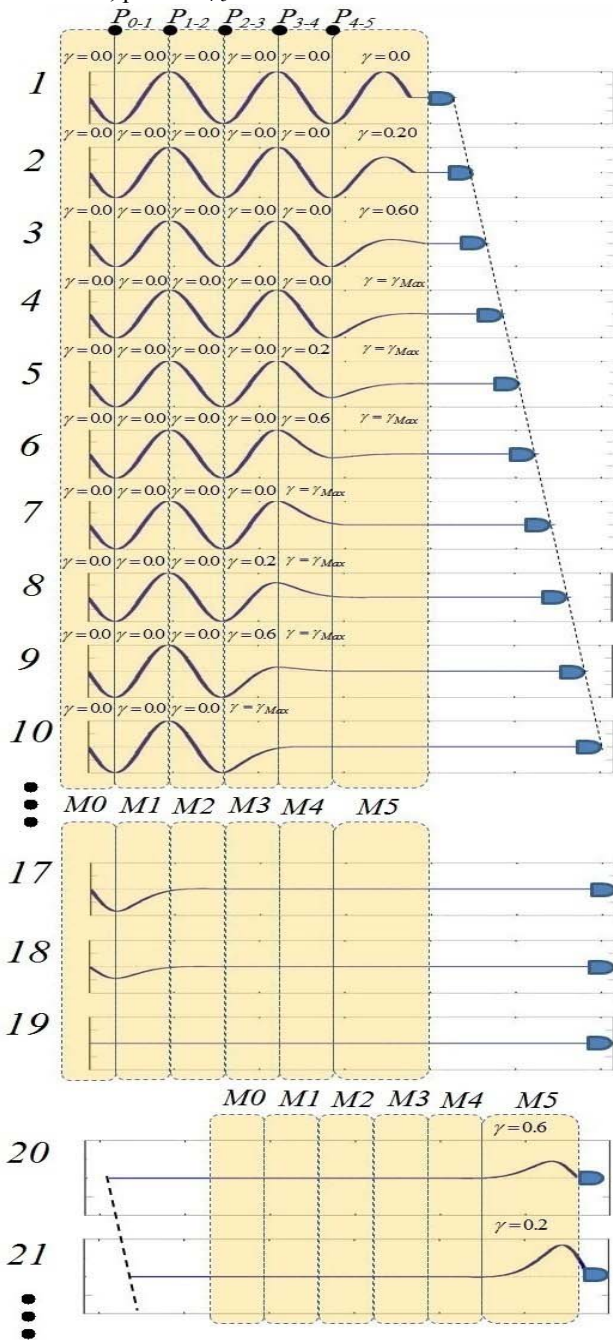


Figure 12. Stages of stretching and Contracting – (Effect of active module on extended module)

The multiplication process continues until the value of the dynamic function being stretched reaches zero. At that point, the dynamic curve being multiplied becomes a straight line.

**Third stage** (stretching of the tail module): When all bodies and the head modules are stretched, all reach  $\gamma_{Max}$ , then the tail module begins its stretching. Note that there are no more modules before to the tail module. Therefore, to stretch the tail, its corresponding dynamic curve is multiplied by numbers smaller than one. The rate of change of this multiplier is equal to the rate of change that was selected for  $\gamma$  when the head and body modules were being stretched. In this paper, the same rate of stretching and later

contracting is selected. However, it should be noted that the rate of stretching/contracting for each module can be set to a different value and thereby creating more natural concertina like motion for the snake. When the tail module is fully stretched, the entire snake body curve will lie flat on a horizontal line.

### B. Contraction of The Snake Body Curve

The contraction cycle begins when the stretching cycle is completed. The cycle is identical to the stretching cycle except that  $\gamma$  starts from its maximum value and goes to zero. The head module will begin contracting using  $\gamma: \gamma_{Max} \rightarrow 0$ . The same procedure is repeated for the remaining modules. Consider Fig 13. The dark part of the snake body is the length that is non moving and the light part of the snake body is the length that is being either stretched or contracted.

Snake stretches its body by anchoring parts of its body, the dark part, mostly the end part of its body. Then the contraction cycle begins by anchoring mostly the front part of its body. Therefore, allowing the body and tail section to be pulled forward.

## V. KINEMATIC

Thus far a new dynamic curve for modeling concertina locomotion is presented. Next, successive links of the robot must be fit to this curve and the corresponding angle for each link should be specified. To do this, first link is fit to the curve by drawing a circle having a radius equal to the length of the robot link [9]. See Fig. 14. The center and the intersection of the circle with the body curve determine begging and end of the first link, respectively. Similarly, center of the next circle is placed at end of the first link and circle is drawn. This identifies begging and ends of the second link. The process is repeated for the remaining links. In this paper, the Secant method [10] is used to obtain the intersection of the circles with the body curve. Upon calculation of all absolute angles, the Five-Point Formula [10] is used to obtain corresponding angle velocity and acceleration.

## VI. MAKING MORE NATURAL OF LOCOMOTION MODELING

Snake-like gaits are the result of long term motion optimization of nature. Therefore, modeling as close to the snake-like locomotion seems to be more desirable. The dynamic curve introduced in this paper enables closer imitation of real snake locomotion. This is made possible by dividing the snake body into several modules and allowing a dynamic curve to model each of these modules. As stated earlier, by allowing different rate of stretching and contracting more natural snake locomotion may be generated. Furthermore, there is no strict requirement on when the modules begin stretching and contacting. For example, in certain situations snake may contract his front part of the body at a much faster rate than the end part. Additional natural snake locomotion is still possible by allowing different parts of the body contract/expand at different times. Refer to Fig. 12 where stages of expansion and contraction are defined. As shown in this figure, links expansion and contraction follow a certain order. However,

again in certain specific situations, it may be more natural and desirable to not necessarily follow the specified expansion and contraction format. For example, consider Fig. 13. As can be seen from this figure two modules stretch simultaneously. In fact, before the full stretching of a module, the next module begins to stretch, then for some moments both modules become active simultaneously.

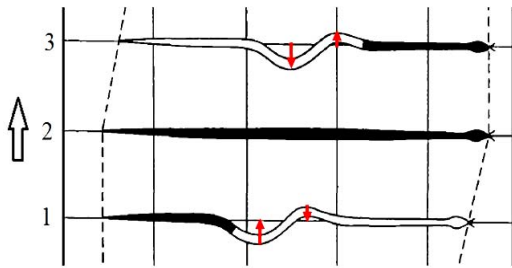


Figure 13. Simultaneity of two active module in snake

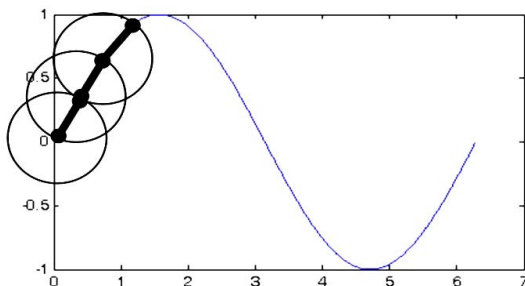


Figure 14. Fitting snake robot links to body curve

## VII. SIMULATION

In this paper Webots™ software is used for simulation. A robot with 26 links is selected. By using the method described in the previous sections, the absolute angles of a concertina locomotion are determined. Using these angles, the 26 link robot is simulated in Webots software. Results indicates concertina locomotion is obtained. Snap shots of simulated snake robot in concertina locomotion is shown in Fig. 15.

## VIII. CONCLUSIONS

In the present research, a novel method for kinematics modeling of snake robot in concertina locomotion is investigated. The kinematics model has the advantage of closely following the concertina locomotion of real snakes. A new dynamic curve for modeling different parts of the snake robot is presented. It is shown that the shape of the dynamic curve can easily be modified using a single parameter  $\alpha$ . The dynamic curve offers significantly high flexibility in imitating natural snake body curves. To obtain concertina gait, the snake body is divided into three different modules, head module, tail module and main body module that connects the head to the tail module. Each module forms a specific curve which is modeled using the proposed dynamic function. Stages of expansion and contraction of the snake in concertina locomotion are specified and graphically shown. At each moment during snake locomotion, the kinematics for different links are derived by fitting links to the body curve. Finally a 26 link snake robot is selected. Absolute joint angles are derived by

fitting the links to the body curve. The joint angles are used as input to simulate the 26 link robot in Webots software. Results indicate concertina locomotion is achieved. The present study should better enable our future research in the area of snake locomotion specifically in characterization of natural like concertina locomotion

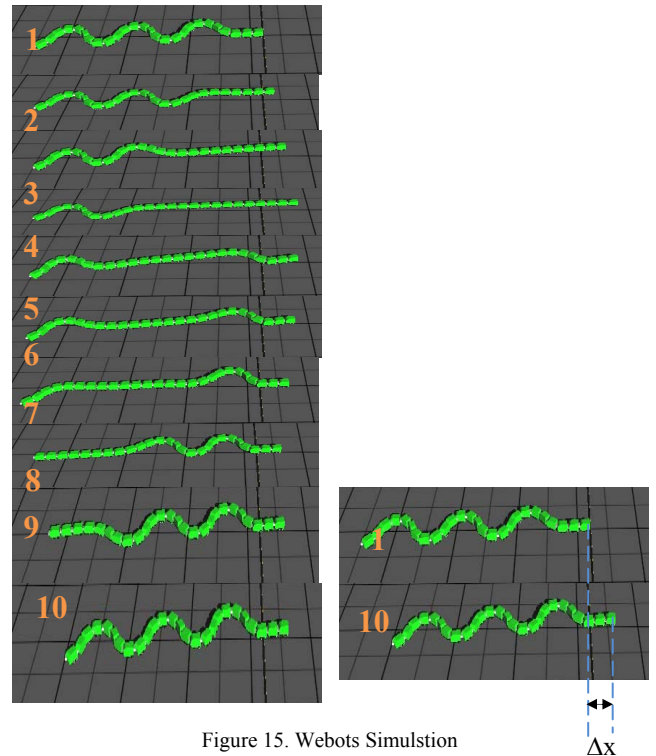


Figure 15. Webots Simulation

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