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ANALYTICAL MODELING OF SQUEEZE FILM DAMPING IN MICROMIRRORS

Hamid Moeenfard
School of Mechanical Engineering
Sharif University of Technology, Tehran, Iran
hamid_moeenfard@mech.sharif.edu

Mohammad Taghi Ahmadian
School of Mechanical Engineering
Sharif University of Technology, Tehran, Iran
ahmadian@mech.sharif.edu

Anoushiravan Farshidianfar
School of Mechanical Engineering
Ferdowsi University of Mashhad, Mashhad, Iran
farshid@ferdowsi.um.ac.ir

ABSTRACT
In the current paper, Extended Kantorovich Method (EKM) has been utilized to analytically solve the problem of squeezed film damping in micromirrors. A one term Galerkin approximation is used and following the extended Kantorovich procedure, the solution of the Reynolds equation which governs the squeezed film damping in micromirrors is reduced to solution of two uncoupled ordinary differential equation which can be solved iteratively with a rapid convergence for finding the pressure distribution underneath the micromirror. It is shown that the EKM results are independent of the initial guess function. It is also shown that since EKM is highly convergent, practically one iterate is sufficient for obtaining a precise response. Furthermore using the presented closed form solutions for the squeezed film damping torque, it is proved that when the tilting angle of the mirror is small, the damping is linear viscous one. Results of this paper can be used for accurate dynamical simulation of micromirrors under the effect of squeezed film damping.

KEYWORDS:
Micromirror, Squeeze film damping, Reynolds equation, Extended Kantorovich Method.

INTRODUCTION
MEMS devices use parallel plate capacitors in which one plate is actuated electrostatically and its movement is detected with capacitive changing. In order to increase the excitation performance as well as the detection sensitivity, the distance between capacitive plates is minimized and the area of the electrodes is maximized. In such a condition, squeeze film damping becomes the most important energy loss mechanism in MEMS. In fact squeeze film damping is the result of massive movement of the trapped gas molecules to out of the space between electrodes which is opposed by the gas viscosity. This mechanism produces some kind of pressure distribution underneath the plate which can act like a damping force or like a spring force.

Currently there are two approaches for modeling the damping mechanism of the microresonators in the rarefied gas ambient. The first approach presented by Veijola et al [1] suggests an effective coefficient of viscosity in which an approximated viscosity coefficient depends on the gas pressure via the Knudsen number of the system. Then by solving the Reynolds equation which governs the squeeze film damping phenomenon and utilizing this empirical coefficient in the solution allow the prediction of the damping effect for different ambient pressures [2]. An alternative approach presented by Christian [3], Bao et al [4] and Hutcherson and Ye [5] is based on free molecular dynamic models developed for a plate vibrating in normal direction to a nearby stationary wall [2]. The mentioned model is based on momentum transfer rate from the vibrating plate to the surrounding gas due to collisions of molecules with the plate.

In recent years, more and more torsion micro-mirrors have been used in a variety of MEMS devices, such as optical displays, light modulator and optical switches. As the squeeze film damping is the key factor to the dynamic performances of the mirror, it has been investigated extensively in recent years. In micromirrors, since the gap distance and the moving speed of the plate are not uniform, the coefficient of the damping torque is a function of the tilting angle and the analysis of the squeeze film air damping of torsion mirrors becomes more difficult than that of a parallel plate actuator [6].

Chang et al [7] modeled the squeeze film damping using the so called modified molecular gas film lubrication equation with the coupling effects of surface roughness and gas rarefaction . Hao et al [8] provided analytical expressions for damping pressure of a rectangular mirror at its balanced position and discussed the influence of design parameters. Wei et al [9] provide a simple expression for the coefficient of damping torque for torsion mirrors. However their result is based on a simple 1D model for a strip mirror at its balanced position. Pan et al [10] presents analytical solutions for the effect of squeeze film damping, on a MEMS torsion mirror using Fourier series solution and the double sine series solution under the assumption of small displacements. For the purpose of verification, they also used a numerical finite difference scheme to obtain squeeze film damping torque and used their analytical and numerical formula to simulate the dynamic response of the micromirror and found out that the two approaches yield almost the same outcomes. They also performed experimental measurements and obtained results that were consistent with those obtained from the analytical and numerical damping models. Excluding the tilting angle which was considered to be small, the main problem of this work was that Pan et al [10] assumes the excitation to be harmonic function of time and so the response would also be a single harmonic function of time, while in real situations where the mirror is actuated electrostatically, the excitation is not only a function of time, but also a strongly nonlinear function of the response, i.e. tilting angle of the mirror. So one cannot assume the response to be a simple harmonic time function. Minikes et al [2] adapted the squeeze film model with artificial viscosity and the molecular dynamics model for the case of a torsion mirror under a wide range of vacuum levels. They employed the green function technique to solve the linearized Reynolds equation. Their method was based on the assumption that the mirror response is a single harmonic function of time which is valid only when the excitation is a single harmonic time function. This kind of excitations does not hold for the electrostatically actuated micromirrors where the excitation is a strongly nonlinear function of the response.

Bao et al [11] proposed an analytical model for calculating the squeeze film air damping of a rectangular torsion mirror at finite normalized tilting angles. Based on Reynolds equation they found damping pressure, damping torque and the coefficient of damping torque as functions of tilting angle and aspect ratio of the micromirror for the two cases of infinite long torsion mirror and rectangular micromirror with finite aspect ratio. In the most general case where the micromirror aspect ratio is finite and its tilting angle is not infinitesimal, they obtained an infinite series for the coefficient of damping torque where the coefficients of the series were complicated integrals with integrands which were explicit functions of normalized tilting angles.

The current paper make use of the Extended Kantorovich Method (EKM) to solve the problem of squeeze film damping in micromirrors. The Kantorovich method occupies a position intermediate between the exact solution of a given problem and solution which is obtained by means of methods of Ritz and Galerkin [12]. Results from extended Kantorovich method are even more accurate. This method is based on Variational principle and reduces the partial differential equation governing the system behavior to a set of uncoupled ordinary differential equations which are solved iteratively with a rapid convergence and the final solution would be independent of the initial guess function.

The Kantorovich method was suggested by Kantorovich and Krylov [13]. Kerr [14] and Kerr and Alexander [15], extended the Kantorovich method by using it as a first step of an iterative procedure and showed that the EKM converges very rapidly to a final form, irrespective of the initial guess function. They [15] used the extended Kantorovich method to analyze a clamped rectangular plate subjected to a uniform lateral load. Cortinez and Laura [16] used the same method for the vibrational analysis of rectangular plates of discontinuously varying thickness. Dalaei and Kerr [12] analyzed clamped rectangular orthotropic plates subjected to a uniform lateral load. Since there was no exact analytical solution for that problem, they tried to derive a closed-form approximate solution of high accuracy which was achieved by the EKM. They found that the convergence of the procedure is very rapid and that the final form of the generated solution is independent of the initial choice. Kerr [17] presented an extended Kantorovich procedure for the solution of the eigenvalue problems. His specific examples were the vibration of rectangular membrane and stability of an elastic rectangular plate compressed in its plane. He showed that for the membrane problem, the generated expressions for the eigenvalues and eigenfunctions are identical with the corresponding exact solution and for the clamped plate compressed uni-axially or bi-axially. Jones and Milne [18] applied the extended Kantorovich method to the vibration analysis of clamped rectangular plates and presented closed-form solutions for the plate mode shapes with high accuracy. They found that the process converges so rapidly that usually two iterates is sufficient to achieve a precise response. Dalaei and Kerr [19] extended what Jones and Milne [18] did and used the Extended Kantorovich method to analyze free vibration of clamped rectangular orthotropic plates. They derived closed-form solutions for system mode shapes and corresponding natural frequencies for the problem which had no exact solution.

EKM has also been used for the solution of problems encountered in MEMS. Ahmadian et al [20] used EKM to solve the problem of static pull-in of electrostatically actuated microplates and found their results in close agreement with experimental data. Behzad et al [21] made use of EKM to find the natural frequencies and modeshapes of electrostatically actuated microplates. Moeenfard et al [22] modeled the static pull-in of the microplates under the effect of capillary force by using EKM.

In this paper the EKM is utilized to solve linearized Reynolds equation to characterize squeezed film damping in micromirrors. The process starts with a one term Galerkin approximation. Following the extended Kantorovich procedure, the response is discretized in x and y directions. Then using an initial guess function for the dicretized response, yields to two uncoupled ordinary differential equations which can be solved iteratively. It is shown that the convergence of this procedure is very rapid and independent of the initial guess function. At the end, the results are delineated in some figures and the effects of geometrical parameters on the squeezed film damping torque in micromirrors are assessed.

Problem Formulation

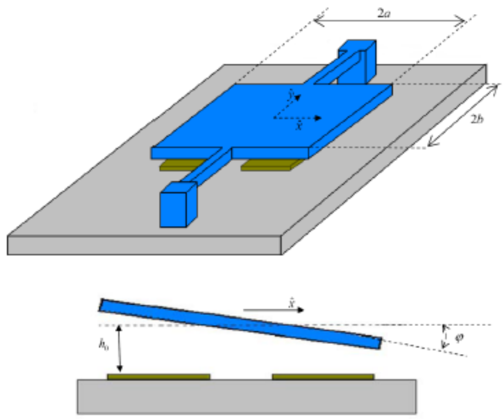

Figure (1): Schematic view of a torsion micromirror.

Figure (1) shows schematic view of a torsion micromirror. As the tilting angle of micromirrors is generally small ($\varphi \le 10^\circ$), the squeeze film air damping of micromirror is approximately governed by the Reynolds equation [6]. For MEMS devices where the inertial effects can be neglected, the Reynolds equation reduces to:

$$\frac{\partial}{\partial \hat{x}}\left(\rho \frac{h^3}{\mu}\frac{\partial P}{\partial \hat{x}}\right)+\frac{\partial}{\partial \hat{y}}\left(\rho \frac{h^3}{\mu}\frac{\partial P}{\partial \hat{y}}\right)=12\frac{\partial (h\rho)}{\partial t} \tag{1}$$

Where ρ is the fluid density, h is the fluid thickness at point $(\hat{x},\hat{y})$, μ is the fluid viscosity, P is the fluid pressure and t is the time. Under the isothermal condition which is the condition which usually arises in MEMS devices, the gas density is directly proportional to its pressure P. So equation (1) can be rewritten in the form:

$$\frac{\partial}{\partial \hat{x}}\left(P \frac{h^3}{\mu}\frac{\partial P}{\partial \hat{x}}\right)+\frac{\partial}{\partial \hat{y}}\left(P \frac{h^3}{\mu}\frac{\partial P}{\partial \hat{y}}\right)=12\frac{\partial (hP)}{\partial t} \tag{2}$$

It has to be noticed that the pressure P is composed of two parts $P = p_a + p$ where p_a is the ambient pressure and p is the relative pressure which is due to the squeezed film effect. For small displacement of the plate around its balance position ($\Delta h << h_0$ and $p << p_a$), equation (2) can be linearized as [6]:

$$\frac{\partial^2 p}{\partial \hat{x}^2}+\frac{\partial^2 p}{\partial \hat{y}^2}-\frac{12\mu}{p_a h^2}\frac{\partial p}{\partial t}=\frac{12\mu}{h^3}\frac{dh}{dt} \tag{3}$$

When $\left(\frac{12\mu}{p_a h^2}\frac{\partial p}{\partial t}\right)\Big/\left(\frac{12\mu}{h^3}\frac{dh}{dt}\right)<<1$ or $\Delta p/p_a << \Delta h/h$, the gas is not appreciably compressed [6]. The mentioned condition is referred as incompressible gas condition, under which the equation (3) is reduced to:

$$\frac{\partial^2 p}{\partial \hat{x}^2}+\frac{\partial^2 p}{\partial \hat{y}^2}=\frac{12\mu}{h^3}\frac{dh}{dt} \tag{4}$$

In electrostatically actuated micromirror, the thickness of the fluid gap is a linear function of the position $\hat{x}$:

$$h = h_0 - \varphi \hat{x} \tag{5}$$

Where h_0 is the initial gap between the mirror plate and the electrodes.

So assuming the torsion beams supporting the mirror do not bend under the effect of the electrostatic force or squeezed film pressure, h_0 would be constant with respect to time. In such a condition, one can conclude:

$$\frac{dh}{dt}=-\dot{\varphi}\hat{x} \tag{6}$$

Substituting equation (6) into equation (4), yields:

$$\frac{\partial^2 p}{\partial \hat{x}^2}+\frac{\partial^2 p}{\partial \hat{y}^2}=-\frac{12\mu}{h^3}\dot{\varphi}\hat{x} \tag{7}$$

The boundary conditions for solving equation (7) are:

$$p(-a,0)=p(a,0)=p(0,-b)=p(0,b)=0 \tag{8}$$

Where a and b are the microplate length in $\hat{x}$ and $\hat{y}$ directions respectively.

If the angle of rotation of the mirror is small enough, then h in equation (7) can be approximated by h_0. In this condition, equation (7) can be written as:

$$\frac{\partial^2 p}{\partial \hat{x}^2}+\frac{\partial^2 p}{\partial \hat{y}^2}=-\frac{12\mu}{h_0^3}\dot{\varphi}\hat{x} \tag{9}$$

Introducing the nondimensionalized variables

$$x=\hat{x}/a, \quad y=\hat{y}/b \tag{10}$$

Equation (9) can be simplified as

$$\frac{\partial^2 p}{\partial x^2}+\alpha^2\frac{\partial^2 p}{\partial y^2}+\beta x=0 \tag{11}$$

Where

$$\alpha = a/b \tag{12}$$

$$\beta = 12\mu a^3 \dot{\varphi}/h^3 \tag{13}$$

The boundary conditions can also be nondimensionalized as:

$$p(-1,0)=p(1,0)=p(0,-1)=p(0,1)=0 \tag{14}$$

For solving equation (11) with EKM, first of all a one-term Galerkin approximation is used as follows:

$$\iint_A \left(\frac{\partial^2 p}{\partial x^2} + \alpha^2 \frac{\partial^2 p}{\partial y^2} + \beta x \right) \delta p(x,y,t)\, dx.dy = 0 \tag{15}$$

Where A is the nondimensionalized mirror area. Then according to the Kantorovich method, it is assumed that:

$$p(x,y,t) = f(x)\, g(y,t) \tag{16}$$

Assuming that $f(x)$ is a previously prescribed known function, then:

$$\delta p(x,y,t) = f(x)\, \delta g(y,t) \tag{17}$$

Substituting equations (16) and (17) into equation (15) yields:

$$\iint_A \left(g(y) \frac{d^2 f(x)}{dx^2} + \alpha^2 f(x) \frac{\partial^2 g(y,t)}{\partial y^2} + \beta x \right) f(x)\, \delta g(y,t)\, dx.dy = 0 \tag{18}$$

Where A is the nondimensionalized mirror area. By rearranging the integration in equation (18), this equation can be rewritten in the form:

$$\int_{-1}^{1} \left[\int_{-1}^{1} \left(f(x) \frac{d^2 f(x)}{dx^2} \right) dx.g(y,t) + \alpha^2 \left(\int_{-1}^{1} f^2(x) dx \right) \frac{\partial^2 g(y,t)}{\partial y^2} + \left(\int_{-1}^{1} (\beta x f(x)) dx \right) \right] \delta g(y,t)\, dy = 0 \tag{19}$$

According to the fundamental lemma of variational calculus, equation (19) would be satisfied if and only if the coefficient of the $\delta g(y,t)$ in equation (19) is zero, or:

$$\left(\alpha^2 \int_{-1}^{1} f^2(x) dx \right) \frac{\partial^2 g(y,t)}{\partial y^2} - \left(\int_{-1}^{1} \left(f(x) \frac{d^2 f(x)}{dx^2} \right) dx \right) g(y,t) + \int_{-1}^{1} \beta x f(x)\, dx = 0 \tag{20}$$

The second integral in equation (20) can be written in its weak form as

$$\int_{-1}^{1} \left(f(x) \frac{d^2 f(x)}{dx^2} \right) dx = -\int_{-1}^{1} \left(\frac{df(x)}{dx} \right)^2 dx \tag{21}$$

So equation (20) can be simplified as:

$$\alpha^2 I_1 \frac{\partial^2 g(y,t)}{\partial y^2} - I_2 g(y,t) = -\beta I_3 \tag{22}$$

Where

$$I_1 = \int_{-1}^{1} f^2(x) dx \tag{23}$$

$$I_2 = \int_{-1}^{1} \left(\frac{df(x)}{dx} \right)^2 dx \tag{24}$$

$$I_3 = \int_{-1}^{1} x f(x) dx \tag{25}$$

The solution of the equation (22) under the boundary conditions $g(-1,t) = g(1,t) = 0$ is as follows:

$$g(y,t) = \beta \frac{I_3}{I_2} \left(1 - \frac{\cosh \gamma y}{\cosh \gamma} \right) \tag{26}$$

Where

$$\gamma = \frac{1}{\alpha} \sqrt{\frac{I_2}{I_1}} \tag{27}$$

If in the formulation of Kantorovich method, it was assumed that the function $g(y,t)$ is a prescribed known function, then by following similar procedure, equation (28) would have been obtained:

$$I_1' \frac{d^2 f(x)}{dx^2} - \alpha^2 I_2' f(x) = -\beta I_3'.x \tag{28}$$

Where

$$I_1' = \int_{-1}^{1} g^2(y,t) dy \tag{29}$$

$$I_2' = \int_{-1}^{1} \left(\frac{\partial g(y,t)}{\partial y} \right)^2 dy \tag{30}$$

$$I_3' = \int_{-1}^{1} g(y,t) dy \tag{31}$$

The solution of this equation under the boundary conditions $f(-1) = f(1) = 0$ is:

$$f(x) = \frac{\beta}{\alpha^2} \frac{I_3'}{I_2'} \left(x - \frac{\sinh \lambda x}{\sinh \lambda} \right) \tag{32}$$

Where

$$\lambda = \alpha \sqrt{\frac{I_2'}{I_1'}} \tag{33}$$

In the extended Kantorovich method, by using an initial guess function $f_0(x)$, equations (22) and (28) can be solved iteratively. Since $f(x)$ and $g(y,t)$ are computed analytically, the integrations in equations (23) to (25) and equations (29) to (31) can also be computed analytically in terms of their values in the previous step of the EKM as follows:

$$I_1'^{n+1} = \beta^2 \left(\frac{I_3^n}{I_2^n} \right)^2 \left(3 - 3\frac{\tanh \gamma_n}{\gamma_n} - (\tanh \gamma_n)^2 \right) \tag{34}$$

$$I_2'^{n+1} = \beta^2 \left(\frac{I_3^n \gamma_n}{I_2^n} \right)^2 \left(-1 + \frac{\tanh \gamma_n}{\gamma_n} + (\tanh \gamma_n)^2 \right) \tag{35}$$

$$I_3'^{n+1} = 2\beta \frac{I_3^n}{I_2^n} \left(1 - \frac{\tanh \gamma_n}{\gamma_n} \right) \tag{36}$$

$$I_1^{n+1} = \beta^2 \left(\frac{I_3'^n}{\alpha^2 I_2'^n} \right)^2 \left(\frac{5}{3} + \frac{4}{\lambda_n^2} - \frac{3}{\lambda_n \tanh \lambda_n} - \frac{1}{(\tanh \lambda_n)^2} \right) \tag{37}$$

$$I_2^{n+1} = \beta^2 \left(\frac{I_3'^n}{\alpha^2 I_2'^n} \right)^2 \left(-(2 + \lambda_n^2) + \left(\frac{\lambda_n}{\tanh \lambda_n} \right)^2 + \frac{\lambda_n}{\tanh \lambda_n} \right) \tag{38}$$

$$I_3^{n+1} = \beta \frac{I_3'^n}{\alpha^2 I_2'^n}\left(\left(\frac{2}{3}+\frac{2}{\lambda_n^2}\right)-\frac{2}{\lambda_n \tanh \lambda_n}\right) \quad (39)$$

Where $I_i'^j$, I_i^j $(1 \le i \le 3)$, λ_j and γ_j are the values of I_i', I_i, λ and γ in the *j*'th step of the EKM and the final solution becomes:

$$p_\infty(x,y,t) = f_\infty(x) g_\infty(y,t) =$$
$$\frac{\beta^2}{\alpha^2}\frac{I_3^\infty I_3'^\infty}{I_2^\infty I_2'^\infty}\left(x - \frac{\sinh \lambda_\infty x}{\sinh \lambda_\infty}\right)\left(1 - \frac{\cosh \gamma_\infty y}{\cosh \gamma_\infty}\right) \quad (40)$$

The convergence of the presented method is so rapid that in most cases, one to two iterates are sufficient for obtaining highly precise pressure distribution.

Results and discussion

To validate the proposed model, an initial guess function is selected as $f_0(x) = x(1-x^2)$. For the case of infinitesimal tilting angle, figure (2) shows the plot of the $f(x)$ and $g(y,t)/\beta$ obtained in first three iterates of the EKM. This figure shows that when the tilting angle of the micromirror is infinitesimal, the pressure distribution is an odd function along x axis.

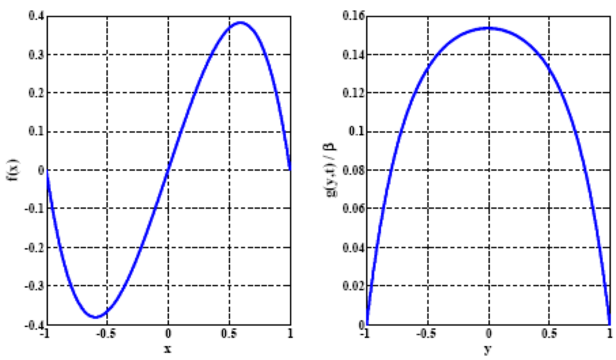

Figure (2): $f(x)$ and $g(y,t)/\beta$ at the first three iterates when $\alpha = 1$.

It can be seen that the convergence of the proposed method is so rapid that the slight difference between the curves of different iterates can't be seen ocularly. So, for the purpose of quantitative analysis of the convergence of the proposed method, table (1) has been prepared. In this table the values of I_1, I_2, I_3, I_1'/β^2, I_2'/β^2 and I_3'/β in different iterates of the EKM has been presented. As it is observed the convergence of the EKM is extremely rapid that the values of the parameters presented in table (1) does not change appreciably in different iterates and practically one iterate is sufficient for obtaining a highly precise response.

Table (1): The values of different parameters at different iterates for $\alpha = 1$ when tilting angle is infinitesimal.

Iterate number	I_1	I_2	I_3	I_1'/β^2	I_2'/β^2	I_3'/β
1	0.1493975748	1.6145042790	0.2674205735	0.0300868186	0.0879521444	0.2307792799
2	0.1493975748	1.6145042790	0.2674205735	0.0300562591	0.0882871363	0.2307819559
3	0.1493975748	1.6145042790	0.2674205735	0.0300561048	0.0882888038	0.2307819560

In order to show that the results of EKM is independent of the initial guess function, the parameters identifying the final solution, that is $\beta I_3^n I_3'^n / (\alpha^2 I_2^n I_2'^n)$, γ_n and λ_n has been listed in table (2) for different initial guess functions. It is observed that despite some of the initial guess functions don't even satisfy the boundary condition(s), their results are practically the same. It has to be noted that the individual functions $f(x)$ and $g(y,t)$ depend on the initial guess function, but their product, that is $p(x,y,t)$ does not.

Table (2): Effect of initial guess function on the final solution when $\alpha = 1$

Initial guess	Parameter	Value
$f_0(x) = x(1-x^2)$ or $f_0(x) = x - 1$ or $f_0(x) = e^x$	$\beta I_3^n I_3'^n / (\alpha^2 I_2^n I_2'^n)$	0.43296400403136
	γ_∞	3.28736423811021
	λ_∞	1.71390400764485

Finally in order to verify the presented model, the squeezed film torque on the micromirror is calculated analytically. In fact the squeezed film torque can be calculated as:

$$T_d(t) = \int_{\hat{y}=-b}^{\hat{y}=b}\int_{\hat{x}=-a}^{\hat{x}=a} \hat{x}.p(\hat{x},\hat{y},t)\,d\hat{x}d\hat{y}$$
$$= \int_{y=-1}^{y=1}\int_{x=-1}^{x=1} ba^2 x.p(x,y,t).dx$$
$$= ba^2 \int_{-1}^{1} x.f(x)dx \int_{-1}^{1} g(y,t)dy$$

Since the initial guess function for $f_0(x)$ is independent of β, I_i $1 \le i \le 3$ is also independent of β and I_i' $1 \le i \le 2$ is some coefficient of β^2 and I_3' is some coefficient of β. So, considering equation (84), $T_d(t)$ would be a linear function of β which implies that when the micromirror tilting angle is very small, the squeezed film damping can be modeled with a linear viscous one.

In order to verify the presented model, in figure (3), the results of the presented method has been compared with the results of Bao et al [11]. It is observed that when the tilting angle is infinitesimal, even a one iterate EKM solution is in a close agreement with previously published results in the literature.

This figure shows that with increasing the microplate aspect ratio, the damping torque is also increased. It has to be noted that for plotting the results presented by Bao et al [11], their suggested relation for damping torque has been used.

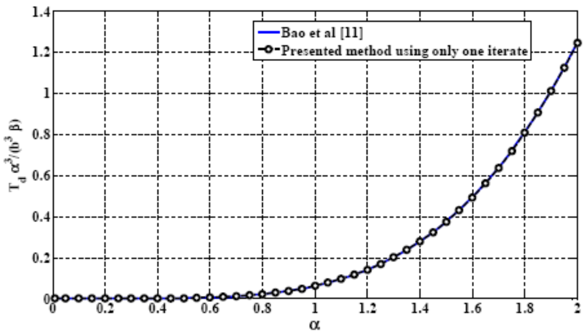

Figure (3): A plot of $g(y,t)/\eta$ and $f(x)$ at different iterates when $\alpha = 1$ and $\sigma = 0.3$.

Conclusions

The current paper made use of the EKM to solve the problem of squeezed film air damping in micromirrors. Using a one-term Galerkin approximation, and following the extended Kantorovich procedure, the solution of the Reynolds equation in micromirrors was reduced to the solution of two uncoupled ordinary differential equation which was solved iteratively with a rapid convergence. It was shown that the presented model is highly convergent and also independent of initial guess function. It was observed that in the case of infinitesimal tilting angle, the damping torque is linear viscous one. The results of this paper can be used to simulate the nonlinear dynamical behavior of micromirrors in presence of squeezed film damping and the method presented by this paper can be viewed as a new efficient approach for solving the problem of squeezed film damping in microstructures.

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