A New Approach In Preliminary Design Of Closed Loop Solar Thermal Systems

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ABSTRACT

In this paper, a model for closed loop systems is presented and an attempt is made to generalized the model to utilize for primary design of any solar active thermal system. This model may be used for a system where the fluid flow is a gas or a liquid.

In this model, two new parameters, namely, the system heat delivery factor and the system heat absorption factor are introduced. These two factors are fully discussed and some equations are developed which determine the values of these factors.

1. INTRODUCTION

Since 1970, there has been a surge of interest and activity in solar heating systems and many thousands of active systems have been designed, installed and operated [1]. The general method of measuring collector performance employed by the manufactures is basically conducted in three parts; The first is determination of instantaneous efficiency with beam radiation nearly normal to the absorber surface. The second, is determination effects of angle of incidence of the solar radiation. The third, is determination of collector time constant, a measure of effective heat capacity. Comparison between various solar collectors can be made by graphs collector efficiency versus performance coefficient.

For most systems, the Hottel – Whillier model [2] is used to determine the efficiency of the solar collector where certain parameters such as number of covered plates, absorption covers, number of pipes, etc. are considered and related to obtain new parameters, namely, transmittance-absorptance product, overall heat removal factor and overall loss coefficient. This paper, introduces a new method for calculation of collector efficiency, utilizing heat exchanger theory.

Yanadori [3] and Gauthier [4] employed this heat exchanger theory to estimate the performance of solar collectors where Number of Transfer Units (NTU) was employed for simplification of their results. The method employed in this paper is different from those mentioned above since the heat transfer surfaces are considered as independent variables.

2. ANALYTICAL MODEL

The schematic diagram of a solar thermal system is shown in Figure (1). Pump, (P_1) , carries a fluid between a collector and a heat exchanger (H_1) . The product of mass flow rate and specific heat $(\dot{m} c_P)$ for this circuit is (C_1) where the heat exchanger (H_1) transfers heat from fluid into the storage system. Pump (P_2) which receives heat from the heat exchanger (H_2) into the heat exchanger (H_3) where heat is utilized for a specific thermal process. Mass flow rate – specific heat product for this circuit is (C_2) . Each of the three heat exchangers employed in this system is specified by a coefficient, E, and each exchanger can be replaced by considering an appropriate effectiveness new factor. The model derived in here is based on the following assumptions:

- 1) The overall area of collector (A_a) and area of receiver (A_r) are considered to be equal.
- 2) The temperature of the storage system is considered to be uniform and homogeneous, even though stratification phenomenon in real storage system occurs.

- 3) The whole solar heat system receives a constant heat load (Q_P) during process at temperature (T_p) while, the system also receives some amount of time dependent solar heat (Q_S) at an ambient temperature (T_a) .
- 4) Solar irradiance is considered to start at sunrise, t = 0, and to reach its maximum value at mid day and goes back to zero at sunset ($t = t_s$).
- 5) Pump (P_1) is assumed to be operational from sunrise until the sunset and remains un-operational for the rest of design period.
- 6) Pump (P_2) can be on or off depending on the receivers heat process (Q_P).

In this model, α' is defined as the overall heat absorption, and U_L is the overall heat loss coefficient. The collector system is specified by an effective factor, E, where $E = F_R A_C U_L / \dot{C}$ and F_R = Heat removal factor which is estimated from Hottel – Whillier model. $\dot{C}_1 = (\dot{m}C_P)_1$, mass flow rate – heat capacity product. Also, C_S is defined as the product of mass and specific heat capacity of the storage system's sensible heat.

3. FORMULATION OF THE ACTIVE SYSTEM WITH FORCED CIRCULATION

 T_i is assumed to be the collector entrance temperature and Q_u the rate of heat transfer from collector to the storage tank. From the energy balance equation for the collector:

$$Q_u = F_{\mathcal{R}} A_c [S - U_L (T_i - T_a)]$$
⁽¹⁾

It is useful to write the energy equation in terms of the heat exchanger effectiveness factor (E_c) as below:

$$E_c = F_R A_c U_L / \dot{C}_1 \tag{2}$$

From Equation (1) and (2):

$$Q_{u} = \dot{C}_{1}E_{c}\left(\frac{S}{U_{L}} + T_{a} - T_{1}\right)$$
(3)

Substituting for α' , as defined before, into Equation (3):

$$Q_u = \dot{C}_1 E_c \left(\frac{\alpha' A_a}{A_r U_L} + T_a - T_1 \right)$$
⁽⁴⁾

From the energy balance for heat exchanger (H_1) :

$$Q_{u} = \dot{C}_{1}E_{1}(T_{2} - T_{s})$$
⁽⁵⁾

Where, E_1 = The heat exchanger effectiveness factor (up to the entrance of the tap), T_2 = The fluid temperature leaving the storage tank, and T_s = The temperature of the storage tank.

The heat transferred from the collector to the storage tank can be written as:

$$Q_u = C_1 (T_2 - T_1)$$
(6)

From Equations (5) and (6):

$$Q_u = \dot{C}_1 \left(\frac{E_1}{1 - E_1}\right) (T_1 - T_s)$$
⁽⁷⁾

Also, from Equations (7) and (4):

$$Q_u = \frac{1}{R_c} \left(\alpha' A_a R_{Lqs} + T_a - T_s \right)$$
⁽⁸⁾

Where

 $R_{c} = \frac{E_{c} + E_{1} - E_{c}E_{1}}{\dot{C}_{1}E_{c}E_{1}}$ (9)

and

$$R_L = 1/A_r U_L \tag{10}$$

For the next step, it is assumed that all heat loss from the storage tank utilized as a heat process. So, Equation (11) can be developed:

$$(mc_p)_s \frac{dT_s}{dt} = Q_u - L - (UA)_s (T_s - T_a)$$
⁽¹¹⁾

By assuming

and

$$Q_P = L - (UA)_s (T_s - T_a)$$

 $(\dot{m}c_P)_s = C_s$

Equation (11) can be expressed as:

$$C_s \frac{dT_s}{dt} = Q_u - Q_P \tag{12}$$

 Q_u can be obtained from Equation (8), during the storage time from t = 0 to $t = t_s$. For the rest of the design period (from $t = t_s$ up to $t = t_d$), $Q_u = 0$. Therefore Equation (13) can be developed as:

$$C_{s} \frac{dT_{s}}{dt} \begin{cases} \frac{1}{R_{c}} (\alpha' A_{a} R_{L} q_{s} + T_{\infty} - T_{s}) - Q_{p} , 0 \le t \le t_{s} \\ -Q_{p} , t_{s} \le t \le t_{d} \end{cases}$$
(13)

Let's assume that t_o is the minimum temperature of the storage tank in which allows Q_p leaves the storage tank without receiving heat from any source except those from the sun. From the energy balance for the heat exchanger H_2 :

$$Q_{P} = \dot{C}_{2}E_{2}(T_{o} - T_{3})$$
⁽¹⁴⁾

Where, T_3 is the fluid temperature leaving the heat exchangers H_3 and entering the heat exchanger H_2 .

Also, from energy balance for the heat exchanger H_3 :

$$Q_{p} = C_{2}E_{3}(T_{4} - T_{p})$$
⁽¹⁵⁾

Where, T_4 is the fluid temperature leaving the heat exchanger H_2 and entering the heat exchanger H_3 . Also,

$$Q_P = \dot{C}_2 (T_4 - T_3) \tag{16}$$

Employing Equation (16) to eliminate T_4 from Equation (15):

$$Q_{P} = \frac{\dot{C}_{2}E_{3}}{1 - E_{3}}(T_{3} - T_{P})$$
⁽¹⁷⁾

Now using Equation (17) to eliminate T_3 from Equation (14):

$$Q_p = \frac{1}{R_p} (T_o - T_p) \tag{18}$$

Where

$$R_{P} = \frac{E_{2} + E_{3} - E_{2}E_{3}}{\dot{C}E_{2}E_{3}}$$
(19)

By utilizing the above equations, a periodic can be designed under steady state conditions in which a constant heat load is undertaken and heat is only supplied from solar energy. The minimum allowed temperature of the storage tank during design procedure is T_o . So, from Equation (18):

$$T_{s}(0) = T_{p} + R_{p}Q_{p}$$

$$T_{s}(t_{d}) = T_{p} + R_{p}Q_{p}$$

$$(20)$$

$$(21)$$

Equation (20) can be employed as a primary condition for Equation (13) and Equation (21) can be used for solving the maximum load heat that the process undergoes (Q_P). Rewriting Equation (21):

$$Q_{P} = \frac{1}{R_{P}} [T_{s}(t_{d}) - T_{P}]$$
(22)
bles are defined:

Now the following integration variables are defined:

$$H_{z} = A_{a} \int_{0}^{z} q_{z} dt$$
⁽²³⁾

$$H_{p} = A_{a} \int_{0} Q_{p} dt \tag{24}$$

$$Y_p = A_a \int_0^a T_p dt \tag{25}$$

$$Y_{\infty} = A_a \int_{0}^{s} T_{\infty} dt \tag{26}$$

These integration variables can be employed for the following dimensionless parameters:

$$\theta_{s} = \frac{T_{s}t_{s} - \gamma_{\infty}}{\alpha' R_{L} H_{s}}$$
⁽²⁷⁾

$$\theta_{p} = \frac{T_{s}t_{s} - \gamma_{\infty}}{\alpha' R_{L} H_{s}}$$
⁽²⁸⁾

$$\phi_{z} = \frac{A_{a}q_{z}t_{z}}{H_{z}} + \frac{T_{\infty}t_{z} - \gamma_{\infty}}{\alpha' R_{L}H_{z}}$$
⁽²⁹⁾

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$$\phi_P = \frac{H_P}{\alpha' H_s} \tag{30}$$

$$F_c = \frac{R_L}{R_c}$$
(31)

$$F_P = \frac{R_L}{R_P}$$
(32)

$$G = \frac{R_L C_s}{t_s}$$
⁽²³⁾

$$\tau = \frac{t}{t_s} \tag{34}$$

(34)

$$\beta = \frac{t_d}{t_s} \tag{35}$$

By writing Equations (13), (20), and (22) in dimensionless form, a complete formulation can be obtained as follows:

$$G = \frac{d\theta_s}{dr} = \begin{cases} F_c = (\phi_s - \theta_s) - \phi_p \beta & 0 \le \tau \le 1\\ -\phi_p / \beta & 1 \le \psi \le \beta \end{cases}$$

$$\theta_s(0) = \frac{\phi_p}{\beta F_p}$$

$$\phi_p = \beta F_p [\theta_s(\beta) - \theta_p]$$
(36)
(37)

3.1 The Solution Method

Integrating Equation (36) from $\tau = 0$ to τ for $0 \le \tau \le 1$, then:

$$\theta_{s}(\tau) = \left[\theta_{s}(0) + \frac{\phi_{p}}{F_{c}} - \psi(0)\right] \exp\left(\frac{-F_{c}\tau}{G}\right) - \frac{\phi_{p}}{\beta F_{c}} + \psi(\tau)$$
(39)

In Equation (39), $\psi(\tau)$ is a specific solution for Equation (40):

$$\frac{G}{F_{c}}\frac{d\psi}{dt} + \psi = \phi_{z}(\tau)$$
⁽⁴⁰⁾

By using Equation (37) and (4) for $0 \le \psi \le 1$:

$$\theta_{s}(\tau) = \left[\theta_{p} + \frac{1}{\beta}\left(\frac{1}{F_{p}} + \frac{1}{F_{c}}\right)\phi_{p} - \psi(0)\right]\exp\left(\frac{-F_{c}\tau}{G}\right) - \frac{\phi_{p}}{\beta F_{c}} + \psi(\tau)$$
(41)

Now, integrating Equation (3), from $\tau = 1$ to τ for $1 \le \tau \le \beta$:

$$\theta_{s}(\tau) = \theta_{s}(1) - \frac{\phi_{p}}{\beta G}(\tau - 1)$$
(42)

Then using Equation (41) for calculating $\theta_{s}(1)$ when $1 \le \tau \le \beta$:

$$\theta_{s}(\tau) = \left[\theta_{p} + \frac{1}{\beta}\left(\frac{1}{F_{p}} + \frac{1}{F_{c}}\right)\phi_{p} - \psi(0)\right]Exp\left(\frac{-F_{c}}{G}\right) + \psi(1) - \frac{\phi_{p}}{\beta}\left(\frac{1}{F_{c}} + \frac{\tau - 1}{G}\right)$$

or $1 \le \tau \le \beta$ (43)

for $1 \le \tau \le \beta$

Now, Equation (43) can be employed to evaluate $\theta_s(\beta)$ and use it in Equation (38):

$$\theta_{p} = \beta F_{p} \left(\left[\theta_{p} + \frac{1}{\beta} \left(\frac{1}{F_{p}} + \frac{1}{F_{c}} \right) \phi_{p} - \psi(0) \right] Exp\left(\frac{-F_{c}}{G} \right) + \psi(1) - \frac{\phi_{p}}{\beta} \left(\frac{1}{F_{c}} + \frac{\beta - 1}{G} \right) - \theta_{p} \right)$$

$$(44)$$

By solving Equation (44) for ϕ_P :

$$\phi_{p} = F_{u}(\alpha_{s} - \theta_{p})$$

$$F_{u} = \beta \left[\frac{1}{F_{p}} + \frac{1}{F_{c}} + \frac{\beta - 1}{G[1 - \exp(-F_{c}/G)]}\right]^{-1}$$

$$\psi(1) - \psi(0)$$
(45)
(46)
(46)

$$\alpha_{s} = \frac{\psi(1) - \psi(0)}{1 - \exp(-F_{c}/G)} + \psi(0)$$
(47)

In here, $\psi(\tau)$ is a specific solution of (3). Rewriting Equation (45) in dimensional form, an expression for the whole received heat process during period can be obtained:

$$H_{p} = F_{u}(\alpha_{s}\alpha'H_{s} - A_{r}U_{L}(\gamma_{p} - \gamma_{\infty}))$$
⁽⁴⁸⁾

Equation (48) gives two variables for the new system, F_u and α_s which are the system heat delivery factor and heat absorption factor, respectively. The delivery heat is defined in Equation (46) but the solution for α_s is rather complicated and needs to be explained in next section.

3.2 The System Heat Absorption Factor

To obtain a numerical value for α_s , it is necessary to know variation of solar irradiance and ambient temperature with time. By knowing S(t) and $T_a(t)$, from Equation (26), the value of $\phi_s(\tau)$ can be calculated. With these data, Equation (3) can be integrated and then by considering some suitable primary conditions $\psi(1)$ and $\psi(0)$ in Equation (47), α_s can be found. It should be noted that α_s is independent of $\psi(0)$ and also $\phi_s(\tau)$ is unique. So α_s is only function of the one dimensional variable G/F_s .

The above method can be utilized to calculate α_s numerically even though it is possible to evaluate α_s analytically. If $\phi_s(\tau)$ is in suitable form, for example, if solar irradiance is in sinusoidal form for $0 \le t \le t_s$, then;

$$q_s(t) = q_a \sin(\omega t)$$

 t_s is the time in which the solar irradiance tends to zero:

$$\omega = \pi / t_s \tag{50}$$

(49)

By substituting from Equation (49) and (50) into Equation (23):

$$H_s = 2A_a q_a t_s / \pi \tag{51}$$

Also from Equation (26), when the ambient temperature is constant:

$$\gamma_{\infty} = T_{\infty} t_s \tag{52}$$

By substituting from Equations (49), (50), (51) and (52) into Equation (29):

$$\phi_s(\tau) = \frac{\pi}{2} \sin(\pi\tau) \tag{53}$$

Now substituting from Equation (53) into Equation (2):

$$\frac{G}{F_c}(\frac{d\psi}{d\tau}) + \psi = \frac{\pi}{2}\sin(\pi\tau)$$
⁽⁵⁴⁾

The solution of Equation (55) is:

$$\psi(\tau) = \frac{m^2 \pi \sin(\pi \tau) - \pi^2 m \cos(\pi \tau)}{2(m^2 + \pi^2)} + \left[\psi(0) + \frac{\pi m}{2(m^2 + \pi^2)}\right] \exp(-m\tau)$$
(55)

Where $m = F_c / G$. $\psi(1)$ can be calculated ($\tau = 1$) from Equation (55):

$$\psi(1) = \frac{\pi^2 m}{2(m^2 + \pi^2)} [1 + \exp(-m)] + \psi(0) \exp(-m)$$
(56)

By employing Equation (56) in Equation (47):

$$\alpha_{s}(G/F_{s}) = \frac{\pi^{2}(G/F_{c})[1 + \exp(-F_{c}/G)]}{2[\pi^{2} + (F_{c}/G)^{2}][1 - \exp(-F_{c}/G)]}$$
(57)

Equation (57) is used whenever the variation of solar irradiance on collector is in sinusoidal form. Phillipes [5] showed that the results of numerical solution for most of solar irradiance variations are nearly the same as those obtainable from Equation (57). Therefore, if the variable data is not enough then Equation (57) can be employed to calculate the system absorption factor, Figure (1).

4. PRESENTATION OF RESULTS

Utilizing equations developed for the system for a given geometry are given in Figures (2), (3), and (4). Figure (2) shows variation of the system heat absorption factor versus G/F_c which is in dimensionless form. This graph is applicable to all systems of this kind and it shows sharp slope up to the point where $G/F_c = 0.6$ and $\alpha_s = 0.95$ and after that the slope slows down and gradually increases. It can be seen that in order to get the value of $\alpha_s \ge 0.95$, G/F_c which is related to mass-specific heat product in the storage tank, should be more than 0.6. Therefore in design procedure, the geometrical conditions of the system should vary in a way that achieve this value. Figure (3) is plotted for a specific system and is gradually applicable to all similar systems. It shows that increasing mass-specific heat of tank product (over a certain amount) does not have any significant effect on the variation of heat load delivery. So, for maximum efficiency point and economical optimization point, the mass-specific heat product should be kept at a certain value. This is achieved by the use of a computer program. For the sample design, this value is about 9000 KJ/C, (Figure 3). Figure (4) is an alternative way of showing Figure (2) in non-dimensional condition. This graph shows the variation of system heat absorption factor versus F_u , which is explained in Equation (46). It can be seen that by increasing the system heat absorption factor, F_u tends to one and if the value of mass-specific heat product increases more than a certain level, F_u variation is not significant. The desired value of mass-specific heat product can be found for this certain value of F_u .

The design procedure adopted in here is to achieve the value $G/F_c \ge 0.6$ and then to obtain F_u from graph (4). By knowing the system heat absorption factor, and using Equation (48), the value of delivered heat can be achieved.

5. CONCLUSIONS

A new method was presented for the preliminary design of a closed loop solar thermal system. A computer program, CSHS.FOR, was employed to calculate the main parameters affecting the system performance, namely, the system heat absorption factor (α_s), mass-specific heat product, heat delivery factor and G/F_c . The results show that for the maximum efficiency point, the mass-specific heat product should be kept at a certain value [$G/F_c > 0.6$ and $\alpha_s > 0.95$]. The method adopted in this paper shows an economical and quick way for designing an active solar heating system, specially when the delivered heat load is high.

NOMENCLATURE

| A_{a} | overall area of collector |
|---------------|----------------------------------------------------------|
| A_{c} | collector area |
| A_r | area of receiver |
| A_{s} | area of storage tank |
| $C_{1,2,S}$ | product of mass and specific heat capacity |
| C_P | specific heat at constant pressure |
| $\dot{C} \ E$ | mass flow rate-heat capacity product effective factor |
| E_{s} | heat exchanger effective factor |
| F_{C} | collector heat removal factor |
| F_{P} | heat removal factor for design period |
| F_{u} | system heat delivery factor |
| G | dimensionless parameter |
| $H_{1,2,3}$ | heat exchanger |
| H_p, H_s | integration variables |
| L | load |
| $P_{1,2}$ | pump |
| Q_P | constant heat load |
| Q_s | solar heat |
| Q_u | rate of heat transferred from collector to storage tank |
| S | solar irradiance |
| T_a | ambient temperature |
| T_i | collector entrance temperature |
| T_o | temperature during design period |
| $T_{1,2,3,4}$ | fluid temperature |
| t | time |
| t_d | design time |
| | |

| t _s | storage time |
|-----------------------------------------------------------------------------------------------------------------------|--------------------------------|
| U_1 | overall heat loss coefficient |
| Y_p, Y_∞ | integration variables |
| α' | overall heat absorption factor |
| α_s | heat absorption factor |
| $\phi_{\scriptscriptstyle P},\phi_{\scriptscriptstyle S},\theta_{\scriptscriptstyle P},\theta_{\scriptscriptstyle S}$ | dimensionless parameters |

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