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Performance Analysis of Optical CDMA Systems Utilizing Optical Encoding in Presence of Interference and Receiver Noises

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Abstract. In this paper, performance of incoherent and asynchronous optical CDMA (OCDMA) systems employing "optical encoding" is investigated and the results are compared with the results obtained when the encoding is performed in electrical domain. These investigations are performed for receiver structures that employ a single hard limiter (SHL), a double optical hard limiter (DHL) and also for the structures without a hard limiter (WHL). In addition, "electrical encoding" OCDMA systems using DHL in their receiver structures are investigated. In this work, conventional optical orthogonal codes are used and Gaussian approximation is assumed for the output of APD. Besides interference, other destructive effects such as APD and thermal noises are also taken into account.
It is shown that when the number of users N is equal to or greater than pulsed laser's modulation extinction ratio M_e, for all receiver structures (SHL, DHL and WHL) "optical encoding" systems are superior to their "electrical encoding" counterparts and this superiority becomes much more apparent in SHL and DHL receivers. We will also show that when $N < M_e$, "optical encoding" WHL systems outperform their "electrical encoding" counterparts. Our results also indicate that the performance of "optical encoding" SHL systems approaches the performance of DHL systems as the transmitted power increases.

Keywords. Optical CDMA, OOC, modulation extinction ratio, APD, strong interference, weak interference.

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1 Introduction

Optical Code Division Multiple Access (OCDMA) has been the subject of interest for many years [1–3]. CDMA technique was originally introduced for radio communication systems as a bandwidth efficient technique with a high ability to combat noise and interference compared with other techniques like TDMA and FDMA. Recent advances in optical devices and optical codes have made CDMA an attractive candidate for optical communication networks. OCDMA can utilize the vast fiber bandwidth and offer the possibility of asynchronous access. High security of CDMA systems and their toleration when the number of users increases and the possibility of asynchronous transmission make these systems highly attractive. However, these systems suffer from multiple access interference (MAI) as the most limiting factor.

Salehi [4] and Chung et al. [5] introduced for the first time a family of (0,1) sequences called optical orthogonal codes (OOC). These codes have the desired autocorrelation and cross-correlation properties allowing asynchronous access and providing easy synchronization techniques [4, 6].

Employing a hard limiter followed by a correlator was first proposed by Salehi et al. [6] to mitigate the MAI effects on incoherent asynchronous OCDMA systems performance. In his work, the correlator receiver was analyzed under the interference limited assumption. Ohtsuki [7] utilized double hard limiters (DHL) to improve the system performance. He also introduced another interference cancellation technique using electro-optic switch and optical hard limiters [8]. As a more practical technique, the concept of chip level detection to decrease the effect of MAI for the above systems was introduced by Shalaby [9].

Coherent OCDMA systems for high capacity optical fiber networks were introduced by Huang et al. [10]. Since synchronization techniques in optical systems are very complicated and costly, incoherent optical systems become very attractive [11]. Intensity modulation with direct detection (IM/DD) technique which is not sensitive to phase or polarity of optical signal is used for incoherent OCDMA systems.

Most OCDMA networks are using ON-OFF Keying (OOK) technique in which for each user, a unique optical sequence called "signature" is transmitted for bit "one" and, in ideal circumstances, an all zero optical sequence is transmitted for bit "zero". Besides OOK, some other signaling techniques such as "equal weight orthogonal" (EWO) and "pulse position modulation" (PPM) have also been proposed for OCDMA systems [12–14].

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Other codes such as prime codes have also been introduced and the system performance utilizing these codes have been investigated [15]. Both [16] and [17] considered shot noise and dark current in their analysis. Zahedi et al. [18] presented a general model based on the photon counting technique in which Poisson distributed shot noise and Gaussian distributed thermal noise with multiuser interference are all taken into account. A performance analysis of incoherent and asynchronous OCDMA systems using OOK has been presented by Kwon [19] for the receiver structures with and without a single hard limiter (SHL). In his work, the encoding is performed in electrical domain i.e. the data is first multiplied (encoded) by a unique signature and the encoded data then modulates a laser (we call this method as "electrical encoding").

In present work, called "optical encoding", the data first modulates a pulsed laser source and the laser output is then encoded by an array of optical delay lines. We will show that, for the same probability of error and in absence of hard limiter, our "optical encoding" system can accommodate more simultaneous users than Kwon's system in which the encoding is performed in electrical domain. Furthermore, we will analyze the performance of "electrical encoding" DHL systems with the same assumptions made in [19]. Moreover, it will be shown that in presence of single and double hard limiters, and when the number of simultaneous users N is equal to or greater than the modulation extinction ratio M_e, the probability of error in "optical encoding" systems is very much lower than their "electrical encoding" counterparts. It is noted that in this work all destructive effects such as MAI, shot noise and thermal noise are taken into account, Gaussian approximation is assumed for APD output and conventional optical orthogonal codes (OOC) are used.

The rest of this paper is organized as follows. In Section 2 the system model is introduced. Section 3 presents the different scenarios for MAI. In Sections 4, 5 and 6, the probability of error is calculated for the receiver structures without hard limiter (WHL), with a single hard limiter (SHL) and with double hard limiters (DHL), respectively. The comparison of our numerical results with "electrical encoding" systems are presented in Section 7 and conclusion is presented in Section 8.

2 System Model

The system model for N users is shown in Figure 1.

The data sequence $b_n \in \{0, 1\}$ corresponding to the n-th user is applied to the n-th pulsed laser source. When b_n is "one", the laser output is a short pulse with duration $T_c = T_b/F$ called "chip time" where T_b is the bit duration and F is the code length (spreading factor). The laser output is split into K (code weight) branches and is then encoded by

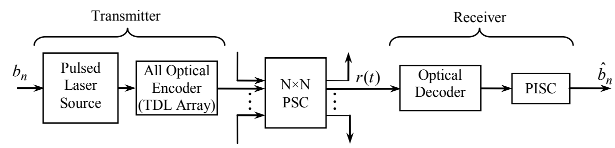

Figure 1. Model of OCDMA system utilizing "optical encoding".

an optical encoder made of a unique array of optical tapped delay lines (TDL) followed by an optical combiner.

The structure of the optical encoder and an example for TDL array is illustrated in Figure 2(a) with $K = 3$ and $F = 7$. Figures 2(b) and 2(c) show the laser output and the corresponding encoded sequence when the input bit is "one".

Since the TDL array is unique for each user, the output sequence corresponding to bit "one" will also be unique. This unique optical sequence of 0's (spaces) and 1's (marks) is called "signature" and is denoted by $C_n(t)$ for the n-th user. As shown in Figure 1, the output of the optical encoder is applied to an $N \times N$ passive star coupler (PSC). When the input bit is "zero", the outputs of laser and encoder are as shown in Figures 2(d) and 2(e), respectively. P_1 and P_0 in Figure 2 denote the powers of mark and space, respectively. Furthermore, an important parameter considered in our analysis is the "modulation extinction ratio" of the pulsed laser defined as $M_e = P_1/P_0$ [19, 20].

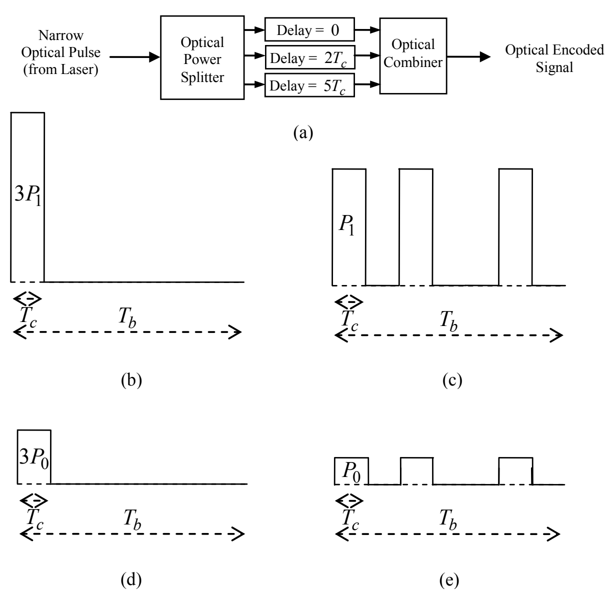

Figure 2. (a) An example of a TDL array (b) Laser output when the input bit is "one" (c) Optical encoded signal when the input bit is "one" (d) Laser output when the input bit is "zero" (e) Optical encoded signal when the input bit is "zero", P_1 and P_0 are the powers of mark and space, respectively.

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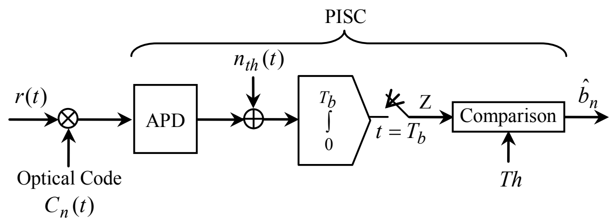

Figure 3. Receiver structure for the n-th user in WHL systems [18].

The receiver structure for the n-th user is shown in Figure 3 in which the received signal $r(t)$ is:

$$r(t) = \sum_{n=1}^{N} S_n(t - \tau_n), \tag{1}$$

where in our asynchronous system, $S_n(t - \tau_n)$ is the optical signal corresponding to the n-th user delayed by τ_n with respect to the first user as reference. $S_n(t)$can be written as:

$$S_n(t) = P_{b_n} C_n(t), \quad n = 1, 2, \ldots, N, \quad 0 \le t \le T_b, \tag{2}$$

where P_{b_n} and $C_n(t)$ are the transmitted power in each mark position and the signature of the n-th user, respectively.

At the receiver, the received signal $r(t)$ is multiplied by the signature of the desired user utilizing, for example, an acousto-optic modulator [18]. The output of the multiplier is then applied to a photo detector followed by an integrate-and-dump, sampler and a comparator (PISC). The APD noise is modeled as a zero mean AWGN denoted by $n_{th}(t)$. The sampled output Z, called "decision variable" is the key parameter in our calculations and is compared with the threshold level Th.

When the transmitted bit is "one", the number of photons absorbed by the APD during a mark chip time is a Poisson distributed r.v. with absorption rate λ_s given by [19]:

$$\lambda_s = \frac{\eta P_1}{h\nu}, \tag{3}$$

where as mentioned before, P_1 is the power of the received mark and the parameters η, h and ν are defined in Table 1. It is clear that when the transmitted bit is "zero", the photon absorption rate during a mark chip time is $\lambda_s / M_e$where λ_s is given in (3). The parameter values given in Table 1 will be used in our calculations.

3 Interference Scenarios

As mentioned before, interference is the most limiting factor in OCDMA systems. Consider a mark m_d in the signature of desired user. m_d can face three different interference scenarios as follows:

Parameter	Symbol	Value
Laser frequency	ν	$2.4242 \cdot 10^{14}$ Hz
APD quantum efficiency	η	0.6
APD mean gain	G	100
Effective ionization ratio	κ_{eff}	0.02
Bulk leakage current	I_b	0.1 nA
Surface leakage current	I_s	10 nA
Modulation extinction ratio	M_e	100
Receiver noise temperature	T_r	1100 K
Receiver load resistance	R_L	1030 Ω
Charge of an electron	e	$1.601 \cdot 10^{-19}$ C
Boltzmann constant	K_b	$1.379 \cdot 10^{-23}$ J/K
Planck's constant	h	$6.626 \cdot 10^{-34}$ J s
Bit rate	$1/T_b$	100 Mbit/s

Table 1. Typical parameter values [19].

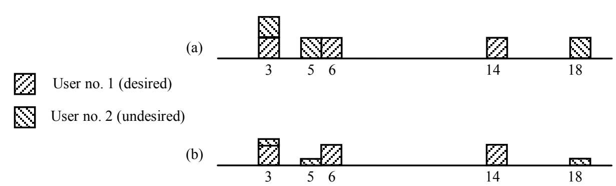

Figure 4. (a) Strong interference, (b) weak interference.

(i) The worst case scenario for interference between m_d and an undesired user occurs when the undesired user transmits "one" and its signature has a mark in the same position as m_d. We call this kind of interference "strong interference".

(ii) The moderate scenario is similar to the previous case except that this time the transmitted bit of the undesired user is "zero". We call this as "weak interference".

(iii) In the best case scenario, the desired and undesired users have no overlapped mark. We call this as "interference free" scenario.

Figures 4(a) and 4(b) illustrate "strong" and "weak" interferences between the desired user (no. 1) and an undesired user (no. 2) at the third chip time, respectively. In Figure 4 it is assumed that the desired user is transmitting "one", $K = 3$ and $F = 19$. In this example, for simplicity, non-OOC and synchronous codes are assumed.

Let I_j^S and I_j^W be the numbers of "strong" and "weak" interfering users at the j-th mark position and at the j-th space position of the desired user respectively. We then define:

$$I^S = \sum_{j=1}^{K} I_j^S, \tag{4}$$

$$I^W = \sum_{j=1}^{K} I_j^W, \tag{5}$$

where I^S and I^W are the total numbers of "strong" and "weak" interfering users respectively and clearly we have $I^S + I^W \leq N-1$. These parameters will be used in our calculations in the following sections.

4 Probability of Bit Error for Optical Encoding WHL Systems

To calculate the probability of bit error (pbe) for an "optical encoding" WHL system the joint probability density function of the random variables I^S and I^W, i.e. $P_{I^S,I^W}(i,j)$, is needed. If p denotes the probability of interference between two codes with weight K and length F, from our definitions in Section 3 the probabilities of "strong interference", "weak interference" and "interference free" between two users will be equal to p, p and $1-2p$, respectively. Thus, $P_{I^S,I^W}(i,j)$ has a trinomial distribution with parameters $N-1$ and $(p, p, 1-2p)$ given by:

$$\begin{aligned} &P_{I^S,I^W}(i,j) \\ &= \binom{N-1}{i, j, N-1-(i+j)} p^i p^j (1-2p)^{N-1-(i+j)} \\ &= \frac{(N-1)!}{i!j!(N-1-(i+j))!} p^{i+j} (1-2p)^{N-1-(i+j)}, \end{aligned} \tag{6}$$

where the probability p has been given by Salehi as $p = K^2/2F$ [4]. The conditional pdf of the Gaussian r.v. Z conditioned on I^S, I^W and the transmitted bit b_n can be written as:

$$\begin{aligned} &P_Z(z|I^S = i, I^W = j, b_n = x) \\ &\quad = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left\{\frac{-(z-\mu_x(i,j))^2}{2\sigma_x^2(i,j)}\right\}, \end{aligned} \tag{7}$$

where $x \in \{0,1\}$ and μ_x, σ_x^2 are mean and variance of the r.v. Z given by [21], respectively:

$$\begin{aligned} \mu_x(i,j) = GT_c &\left[(i+Kx)\lambda_s + (j+1-x)\frac{\lambda_s}{M_e} + F\frac{I_b}{e}\right] \\ &+ T_b \frac{I_s}{e}, \end{aligned} \tag{8}$$

$$\begin{aligned} &\sigma_x^2(i,j) \\ &= G^2 F_e T_c \left[(i+Kx)\lambda_s + (j+1-x)\frac{\lambda_s}{M_e} + F\frac{I_b}{e}\right] \\ &\quad + F\left(T_c \frac{I_s}{e} + \sigma_{\text{th}}^2\right). \end{aligned} \tag{9}$$

F_e is the excess noise factor defined as $F_e = \kappa_{\text{eff}} G + (2 - \frac{1}{G})(1-\kappa_{\text{eff}})$ and κ_{eff} is the effective ionization ratio which is much smaller than 1 for silicon at 800 nm and it is about 0.7 for InGaAs at 1300 nm and 1500 nm [20]. In (9), $\sigma_{\text{th}}^2 = \frac{2K_b T_r T_c}{e^2 R_L}$ is the thermal noise variance of APD.

Probability of bit error conditioned on I^S, I^W, transmitted bit and the threshold level Th is:

$$\begin{aligned} &P_b(e|I^S = i, I^W = j, b_n = x, \text{Th}) \\ &\quad = x + (-1)^x \int_{\text{Th}}^{\infty} P_Z(z|I^S = i, I^W = j, b_n = x) dz, \end{aligned} \tag{10}$$

where, as mentioned before, $x \in \{0,1\}$. From the above equation the pbe in "optical encoding" WHL systems can be obtained as (see Appendix):

$$\begin{aligned} &\text{Pbe}_{(\text{WHL})} = \arg\min_{\text{Th}} \{E_{I^S,I^W,b}\{P_b(e|I^S, I^W, b_n, \text{Th})\}\} \\ &= \arg\min_{\text{Th}} \frac{1}{2}\Bigg\{1 + \sum_{i=0}^{N-1}\sum_{j=0}^{N-1-i} P_{I^S,I^W}(i,j) \\ &\times \left[Q\left(\frac{\text{Th}-\mu_0(i,j)}{\sigma_0(i,j)}\right) - Q\left(\frac{\text{Th}-\mu_1(i,j)}{\sigma_1(i,j)}\right)\right]\Bigg\}, \end{aligned} \tag{11}$$

where $E_y\{a\}$ is the expected value of a with respect to y and $Q(.)$ is the Q-function. Derivation of (11) is presented in the Appendix. It is clear from (11) that the performance of WHL systems depends on both I^S and I^W.

5 Probability of Bit Error for Optical Encoding SHL Systems

Salehi et al. [6] improved the system performance by using a single hard limiter (SHL) at the receiver as shown in Figure 5. The hard limiter removes the interference from undesired users by limiting (clipping) the input power of each receiving pulse to the mark power P_1. Note that if the input power to the HL is less than P_1, output will be zero. It has been shown that the performance of SHL systems depends on the number of nonzero elements in the interference vectors as well as the strong and the weak interferences [13,19].

The results of calculation of pbe in "electrical encoding" SHL systems performed by Kwon [19] shows that in these systems the probability of bit error increase to 1/2 when the number of users $N \geq M_e$. This is due to the fact that in this case, irrespective of the transmitted bit, the accumulated strong or weak interferences on each mark position of

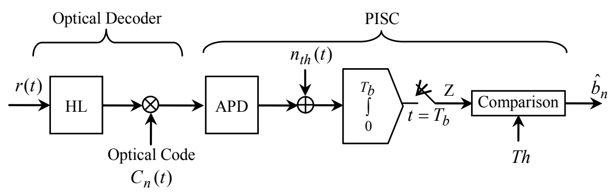

Figure 5. Receiver structure for the n-th user in SHL systems [18].

the desired user exceeds the threshold level P_1 of the hard limiter.

In this section, we calculate the pbe for "optical encoding" SHL systems for both cases $N < M_e$ and $N \geq M_e$. We first consider the case where the desired user transmits "zero". In this case, if $N < M_e$ all weak interferences at any mark position of the desired user are removed by HL and therefore these interferences become ineffective on the pbe. Hence, in this case the conditional pbe is obtained as (see Appendix):

$$\begin{aligned} &P_b(e|b_n = 0, \text{Th}, N < M_e) \\ &\quad = \sum_{i=0}^{N-1} \sum_{m=0}^{\min(K,i)} Q\left(\frac{\text{Th} - \mu(m)}{\sigma(m)}\right) \\ &\quad\quad \times \Pr(|\mathbf{I}^S| = m | I^S = i) P_{I^S}(i), \end{aligned} \tag{12}$$

where

$$\mu(m) = GT_c\left[m\lambda_s + F\frac{I_b}{e}\right] + T_b\frac{I_s}{e}, \tag{13}$$

$$\sigma^2(m) = G^2 F_e T_c\left[m\lambda_s + F\frac{I_b}{e}\right] + F\left(T_c\frac{I_s}{e} + \sigma_{\text{th}}^2\right), \tag{14}$$

$\mathbf{I}^S = [I_1^S, \ldots, I_K^S]$ is the strong interference vector and $|\mathbf{I}^S|$ is the number of nonzero elements in the strong interference vector (for example in the Figure 4(a), $\mathbf{I}^S = [1, 0, 0]$ and $|\mathbf{I}^S| = 1$) . The conditional probability $\Pr(|\mathbf{I}^S| = m | I^S = i)$ has been given in [19]. For $N \geq M_e$ and when the desired user transmits "zero" and the number of weak interfering pulses that coincides with one or more mark positions of the desired user is greater than or equal to $M_e - 1$, weak interferences may degrades the system performance. Note that when the non-negative parameter defined as $\Delta = N - M_e$ increases, the ability of hard limiter to exclude all weak interference patterns decreases. However, for lower values of Δ, i.e. when the number of users is greater than and comparable to M_e, the hard limiter removes almost all weak interference patterns. Hence, in practical situations where the number of users is greater than but not much above the modulation extinction ratio M_e, a near-exact approximation for the conditional pbe of the "optical encoding" SHL system is obtained as:

$$\begin{aligned} P_b(e|b_n = 0, \text{Th}, N \geq M_e) &\simeq \sum_{i=0}^{N-1} \sum_{m=0}^{\min(K,i)} Q\left(\frac{\text{Th} - \mu(m)}{\sigma(m)}\right) \\ &\quad \times \Pr(|\mathbf{I}^S| = m | I^S = i) P_{I^S}(i). \end{aligned} \tag{15}$$

where $\mu(.)$ and $\sigma^2(.)$ are defined in (13) and (14), respectively. We now consider the case where the desired user transmits "one". Since in this case all interferences from undesired users are completely removed by the hard limiter, the conditional pbe is derived similar to (12) as:

$$\begin{aligned} &P_b(e|b_n = 1, \text{Th}, N) \\ &\quad = \sum_{i=0}^{N-1} \left[1 - Q\left(\frac{\text{Th} - \mu(K)}{\sigma(K)}\right)\right] P_{I^S}(i) \\ &\quad = 1 - Q\left(\frac{\text{Th} - \mu(K)}{\sigma(K)}\right). \end{aligned} \tag{16}$$

Finally, assuming equiprobable bits, the average pbe becomes:

$$\begin{aligned} \text{Pbe}_{(\text{SHL})} = \arg\min_{\text{Th}} \frac{1}{2}\{&P_b(\text{Error}|b_n = 0, \text{Th}, N) \\ &+ P_b(\text{Error}|b_n = 1, \text{Th}, N)\}. \end{aligned} \tag{17}$$

The results of our calculations for the pbe of SHL systems based on (12)–(17) will be presented in Section 7.

6 Probability of Bit Error for DHL Systems

6.1 "Optical Encoding" DHL Systems

Ohtsuki et al. [22] and Ohtsuki [7] proposed using a double optical hard limiter in synchronous and asynchronous OCDMA receivers respectively to enhance the system performance. The receiver structure for the n-th user in DHL systems is depicted in Figure 6. In this figure the task of the first hard limiter is similar to the one described for SHL systems and $h_1(t)$ is the impulse response of the passive optical correlator which is followed by the second hard limiter. The optical correlator consists of K optical delay lines inversely matched to the mark positions of the desired user. In other words, the received power during the last chip time of the correlator's output is equal to the sum of its input mark position powers. The stray pulses produced by the correlator are removed by the second HL with threshold level P_1. The rest of the receiver is similar to WHL receivers.

In "optical encoding" DHL systems and when the number of undesired users is less than $K(M_e - 1)$, the probability of bit error conditioned on transmitted bits "one" and

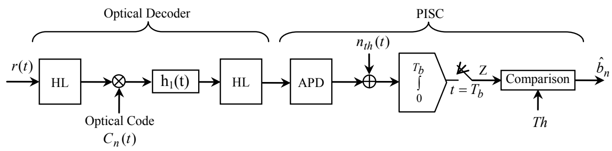

Figure 6. Receiver structure for the n-th user in DHL systems [18].

"zero" can be obtained in a similar manner used for (12) and are given by:

$$P_b(e|b_n = 1, \mathrm{Th}, N) = 1 - Q\left(\frac{\mathrm{Th} - \mu(K)}{\sigma(K)}\right). \quad (18)$$

and

$$\begin{aligned} &P_b(e|b_n = 0, \mathrm{Th}, N < K(M_e - 1) + 1) \\ &= \sum_{i=0}^{N-1} \sum_{m=0}^{\min(K,i)} Q\left(\frac{\mathrm{Th} - \mu(m)}{\sigma(m)}\right) \\ &\quad \times \Pr\left(|\mathbf{I}^S| = m \,\middle|\, I^S = i\right) P_{I^S}(i) \\ &= \sum_{i=K}^{N-1} Q\left(\frac{\mathrm{Th} - \mu(K)}{\sigma(K)}\right) \\ &\quad \times \Pr(|\mathbf{I}^S| = K | I^S = i) P_{I^S}(i) \\ &\quad + \sum_{i=0}^{N-1} \sum_{m=0}^{\min(K-1,i)} Q\left(\frac{\mathrm{Th} - \mu(0)}{\sigma(0)}\right) \\ &\quad \times \Pr(|\mathbf{I}^S| = m | I^S = i) P_{I^S}(i). \end{aligned} \quad (19)$$

Note that, as mentioned before, when the transmitted bit is "zero" weak interferences do not degrade the system performance. For equiprobable transmitted bits, the average pbe is similar to (17). Our numerical results for pbe in "optical encoding" DHL systems based on (17), (18) and (19) will be presented in Section 7.

It is noteworthy that in noise free circumstances, the performances of SHL and DHL OCDMA systems are identical. This is due to the fact that in noise free conditions and when the desired user transmits "zero", an error can only occur in these systems if the received power in every mark position of desired user exceeds P_1. It is obvious that no error can occur in noise free SHL and DHL systems when the transmitted bit is "one".

6.2 "Electrical Encoding" DHL Systems

In this section, the performance of "electrical encoding" DHL systems under the assumptions mentioned in Section 1 is presented. We first consider the case where the number of users is less than the laser's modulation extinction ratio, i.e. $N < M_e$. In this case, the conditional probabilities of bit error in "electrical encoding" DHL systems are:

$$\begin{aligned} &P_b(e|b_n = 0, \mathrm{Th}, N < M_e) \\ &= \sum_{i=0}^{N-1} \sum_{m=0}^{\min(K-1,i)} Q\left(\frac{\mathrm{Th} - \mu(0)}{\sigma(0)}\right) \\ &\quad \times \Pr(|\mathbf{I}^S| = m | I^S = i) P_{I^S}(i) \\ &\quad + \sum_{i=K}^{N-1} Q\left(\frac{\mathrm{Th} - \mu(K)}{\sigma(K)}\right) \\ &\quad \times \Pr(|\mathbf{I}^S| = K | I^S = i) P_{I^S}(i) \end{aligned} \quad (20)$$

and

$$P_b(e|b_n = 1, \mathrm{Th}, N) = 1 - Q\left(\frac{\mathrm{Th} - \mu(K)}{\sigma(K)}\right). \quad (21)$$

From the above equations, the average probability of bit error in "electrical encoding" DHL systems can be computed from (17).

We now consider the case where the number of users is greater than or equal to M_e. In this case, $\mu(0)$ in the first term of (20) is replaced by $\mu(K)$, hence, the conditional pbe becomes:

$$P_b(e|b_n = 0, \mathrm{Th}, N \geq M_e) = Q\left(\frac{\mathrm{Th} - \mu(K)}{\sigma(K)}\right). \quad (22)$$

The average pbe of "electrical encoding" DHL systems when $N \geq M_e$ is obtained by substituting (21) and (22) into (17). This leads to an average pbe equal to $1/2$.

7 Numerical Results

In this section, our numerical results for WHL, SHL and DHL OCDMA systems are presented. In these calculations equiprobable bits are assumed and the parameter values given in Table 1 are used. Note that, throughout this section, by "mean transmitted power", P_{T_c}, in horizontal axis we mean the average power of the pulsed laser output, i.e. $P_{T_c} = (E_1 + E_0)/2T_c$ where E_1 and E_0 are the output energies of the laser corresponding to transmitted bits "one" and "zero" respectively. The pbe for "optical encoding" and "electrical encoding" WHL OCDMA systems versus mean transmitted power in dBW are compared in Figure 7 for $N = 100$, $K = 4$ and $F = 1201$. From this figure, as expected, the "optical encoding" WHL system outperforms its "electrical encoding" counterpart. For example, for error probabilities around 5×10^{-3}, the "optical encoding" WHL system saves about 2.5 dBW power with respect to "electrical encoding" systems. From another point of view, the pbe in "optical encoding" WHL system is nearly half of

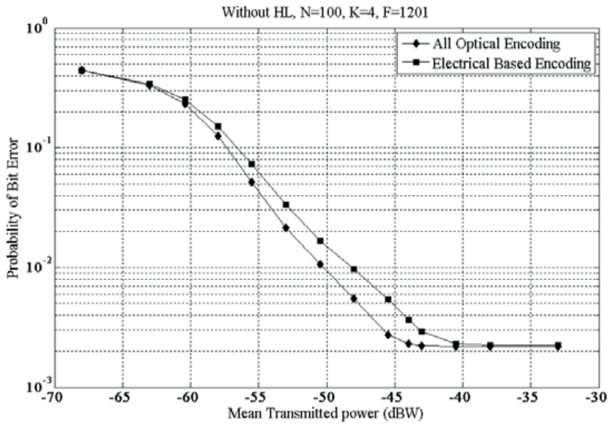

Figure 7. Comparison of pbe for "optical encoding" and "electrical encoding" WHL for $K = 4$, $F = 1201$ and $N = 100$ users.

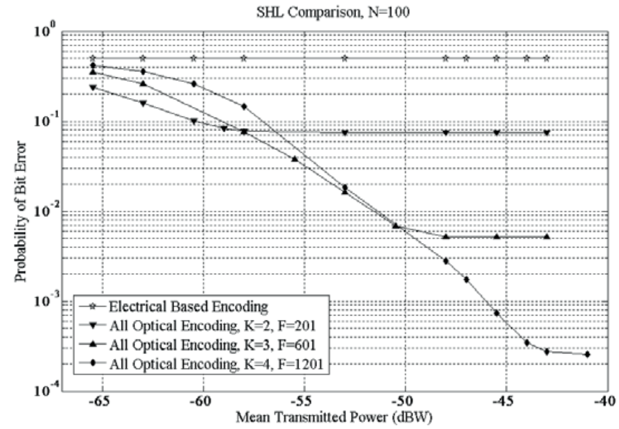

Figure 9. Comparison of pbe for "optical encoding" and "electrical encoding" SHL with both K and F as parameters, and $N = 100$ users.

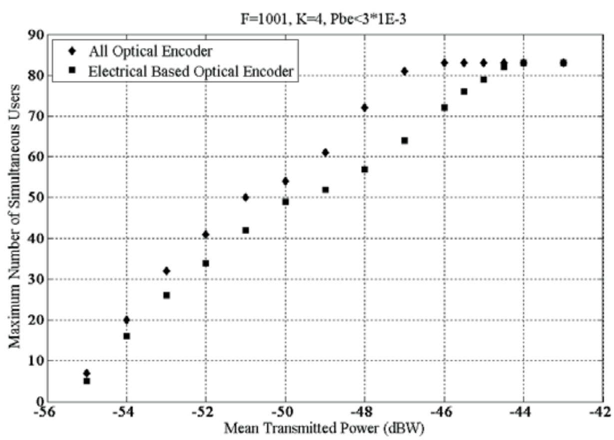

Figure 8. Comparison of the maximum number of simultaneous users in "optical encoding" and "electrical encoding" WHL systems for pbe $< 3 \times 10^{-3}$ when $K = 4$ and $F = 1001$.

the pbe in "electrical encoding" system for mean transmitted power equal to -45.5 dBW. Another comparison is presented in Figure 8 which compares the maximum number of simultaneous users that these systems can accommodate for probability of errors below 3×10^{-3}. It is observed that for mean transmitted power equal to -47 dBW, the "optical encoding" WHL system can support 26 percent more users than its "electrical encoding" counterpart.

Figures 9 and 10 illustrate the pbe of "optical encoding" SHL and DHL systems for three different values of K and F when the number of users $N = 100$. As can be seen, irrespective of the values of K and F, when $N \geq M_e$ the pbe in "electrical encoding" SHL and DHL systems are equal to $\frac{1}{2}$!. From these figures it is also concluded that in high power regimes, i.e. when the transmitted power is high enough to combat noise, the system performance can be improved by increasing the value of K. On the contrary, in low power regimes, increasing K decreases the system performance. This is because in low transmitted powers, when K increases the power in each mark is decreased leading to a lower value for SNR and resulting a lower performance. Figure 11 shows the pbe of OCDMA systems for WHL, SHL and DHL receivers. As can be seen from this figure SHL and DHL systems have equal pbe floors. This is because at high transmitted powers, similar to noise free circumstances, the behaviors of these systems are identical. However, in DHL systems less power is required to achieve the pbe floor compared to SHL. From Figure 11, when $K = 4$, $F = 1201$ and $N = 100$, the lowest value of pbe in both systems is about 2.5×10^{-4} while the minimum transmitted powers required to achieve this value are -43 dBW for SHL and -52 dBW for DHL. It is noted that, similar to the results reported by Kwon [19] for "electrical encoding" systems, in low power "optical encoding" OCDMA systems, presence of a SHL in the receiver structure degrades the system performance.

Figure 12 compares the performance of "optical encoding" and "electrical encoding" SHL systems for different number of simultaneous users. Similar comparison has been drawn for DHL systems in Figure 13. Both comparisons have been made for a mean transmitted power equal to -45.5 dBW, $F = 1201$ and $K = \lfloor \frac{1}{2}(1 + \sqrt{1 + 4(F-1)/N}) \rfloor$ where $\lfloor . \rfloor$ denotes the floor function. Note that when OOCs are employed this value of K minimizes the pbe [19]. As can be seen, in "electrical encoding" SHL systems when the number of simultaneous users reaches M_e, the probability of error increases considerably (more than 1000 times!), while, in "optical encoding" SHL systems pbe varies smoothly. As can be seen form Figure 13, the general behavior of pbe versus number of simultaneous users in DHL systems is similar to the previous case except that in this case, the variation of prob-

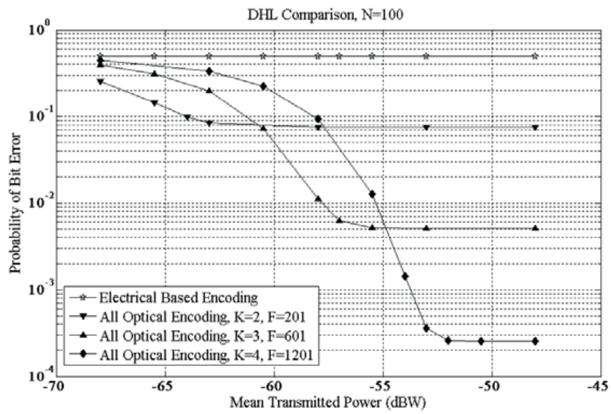

Figure 10. Comparison of pbe for "optical encoding" and "electrical encoding" DHL with both K and F as parameters, and $N = 100$ users.

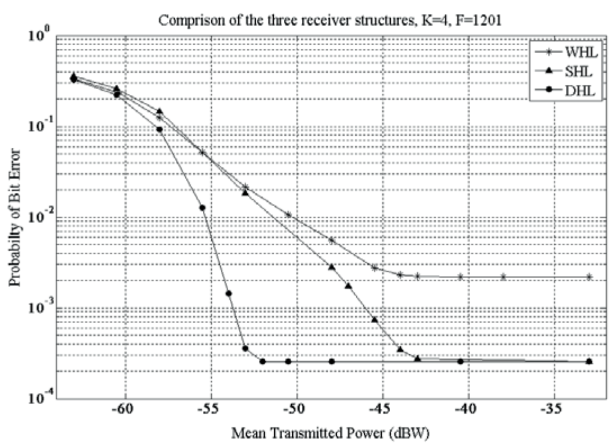

Figure 11. Comparison of pbe versus mean transmitted power in OCDMA systems using WHL, SHL and DHL receivers when "optical encoding" is employed for $F = 1201$ and K = 4.

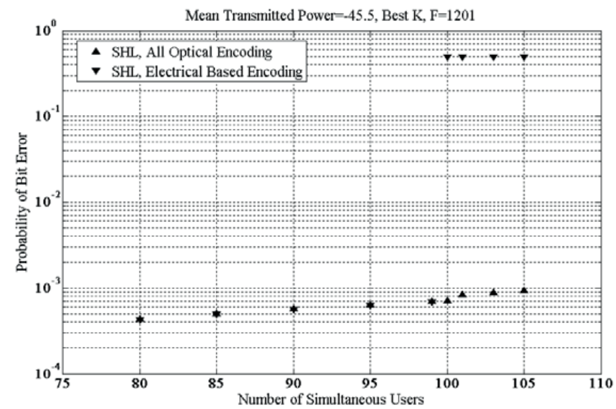

Figure 12. Comparison of pbe for "optical encoding" and "electrical encoding" SHL systems with respect to N as parameter for F = 1201 and mean transmitted power of −45.5 dBW.

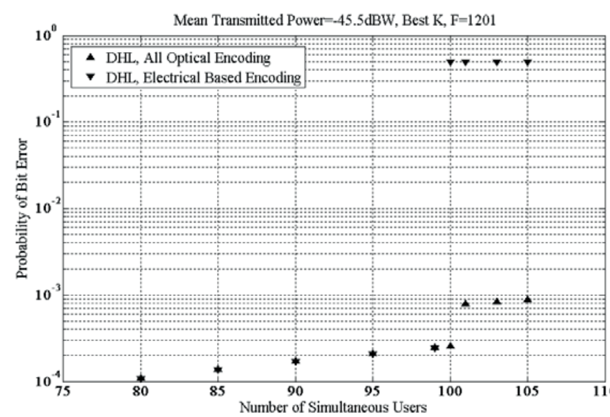

Figure 13. Comparison of pbe for "optical encoding" and "electrical encoding" DHL systems with respect to N as parameter for F = 1201 and mean transmitted power of −45.5 dBW.

ability of error when the number of simultaneous users is around M_e is not as smooth as the previous case. It is noted that in Figures 12 and 13 the value of K decreases from 4 to 3 when the number of users exceeds 100.

8 Conclusion

In this paper, performance of incoherent and asynchronous OCDMA systems employing "optical encoding" and OOCs was investigated and compared with its "electrical encoding" counterpart. Our investigations were performed for three common receiver structures (WHL, SHL and DHLs) and destructive factors such as MAI and APD noises were included. Our numerical results were compared with the corresponding "electrical encoding" OCDMA systems. The comparison between "optical encoding" and "electrical encoding" WHL systems indicates that the pbe in former system is better; for example it was demonstrated that for bit error probabilities less than 3×10^{-3} and mean transmitted power −47 dBW, the "optical encoding" WHL system can support 26 percent more users than its "electrical encoding" counterpart.

It was also concluded that "electrical encoding" SHL and DHLs systems are identical to their "optical encoding" counterparts when $N < M_e$. It was also demonstrated that by increasing N above M_e, the performance of "electrical encoding" SHL and DHLs systems dramatically decreases to $1/2$, while the performance of corresponding "optical encoding" counterparts decreases smoothly. It was also shown that in high power regimes, the floor of pbe in "optical encoding" SHL and DHL systems are the same and it was demonstrated that in low power regimes, systems using WHL outperforms ones using SHL.

Appendix

In this appendix, the details of derivations in (11) and (12) are given. For (11), assuming equiprobable bits, the expression $E_{I^S,I^W,b}\{P_b(e|I^S, I^W, b_n, \mathrm{Th})\}$ can be calculated as:

$$\begin{aligned}
&E_{I^S,I^W,b}\left\{\frac{1}{2}\Pr\left(Z \geq \mathrm{Th}|I^S, I^W, b_n = 0, \mathrm{Th}\right)\right.\\
&\qquad\left.+\frac{1}{2}\Pr\left(Z < \mathrm{Th}|I^S, I^W, b_n = 1, \mathrm{Th}\right)\right\}\\
&= \frac{1}{2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1-i} P_{I^S,I^W}(i,j)\\
&\quad\times \Pr(Z \geq \mathrm{Th}|I^S = i, I^W = j, b_n = 0, \mathrm{Th})\\
&\quad+\frac{1}{2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1-i} P_{I^S,I^W}(i,j) \qquad (.1)\\
&\quad\times \Pr\left(Z < \mathrm{Th}|I^S = i, I^W = j, b_n = 1, \mathrm{Th}\right)\\
&= \frac{1}{2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1-i} P_{I^S,I^W}(i,j)\,Q\left(\frac{\mathrm{Th}-\mu_0(i,j)}{\sigma_0(i,j)}\right)\\
&+\frac{1}{2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1-i} P_{I^S,I^W}(i,j)\left[1-Q\left(\frac{\mathrm{Th}-\mu_1(i,j)}{\sigma_1(i,j)}\right)\right],
\end{aligned}$$

which directly leads to (11). For (12), as mentioned in Section 5, when $N < M_e$ all weak interferences at any mark position of the desired user are removed by HL and therefore these interferences become ineffective on the pbe. Hence, the joint pdf of $P_{I^S,I^W}(i,j)$ reduces to $P_{I^S}(i)$. Given $N < M_e$, the left side of this equation can be calculated as:

$$\begin{aligned}
&\Pr(Z \geq \mathrm{Th}|b_n = 0, \mathrm{Th})\\
&= \sum_{i=0}^{N-1}\sum_{j=0}^{N-1-i}\Pr\left(Z \geq \mathrm{Th}|I^S = i, I^W = j, b_n = 0, \mathrm{Th}\right)\\
&\quad\times P_{I^S,I^W}(i,j)\\
&= \sum_{i=0}^{N-1}\Pr(Z \geq \mathrm{Th}|I^S = i, b_n = 0, \mathrm{Th})P_{I^S}(i)\\
&= \sum_{i=0}^{N-1}\sum_{m=0}^{\min(K,i)}\Pr(Z \geq \mathrm{Th}|I^S = i, |\mathbf{I}^S| = m, b_n = 0, \mathrm{Th})\\
&\quad\times \Pr(|\mathbf{I}^S| = m|I^S = i)P_{I^S}(i) \qquad (.2)\\
&= \sum_{i=0}^{N-1}\sum_{m=0}^{\min(K,i)} Q\left(\frac{\mathrm{Th}-\mu(m)}{\sigma(m)}\right)\\
&\quad\times \Pr(|\mathbf{I}^S| = m|I^S = i)P_{I^S}(i).
\end{aligned}$$

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