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ORIGINAL ARTICLE

Cylindrical and spherical electron acoustic solitary waves in the presence of superthermal hot electrons

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Abstract Propagation of cylindrical and spherical electron-acoustic solitary waves in unmagnetized plasmas consisting of cold electron fluid, hot electrons obeying a superthermal distribution and stationary ions are investigated. The standard reductive perturbation method is employed to derive the cylindrical/spherical Korteweg-de-Vries equation which governs the dynamics of electron-acoustic solitons. The effects of nonplanar geometry and superthermal hot electrons on the behavior of cylindrical and spherical electron acoustic soliton and its structure are also studied using numerical simulations.

Keywords Electron acoustic · Cylindrical and spherical solitary waves · KdV equation · Superthermal electrons

1 Introduction

Electron acoustic waves (EAWs) are one of the basic wave processes in plasmas and they have been studied for several decades. EAWs can be created in a two-temperature (cold and hot) electron plasma. Multispecies models were originally used for laser-plasma interaction but there are several similar situations. The evidence of two populations of electrons in laboratory and space plasmas has already been reported. The observations (Parks et al. 1984; Onsager et al. 1993) in the plasma sheet boundary layer have shown that there exist two types of electrons, namely background plasma electrons and cold electron beams having energies of the order of few eV to few hundreds of eV. Intense broadband electrostatic noise is commonly observed in such these plasma sheet boundary layer of the Earth's magnetosphere (Gurnett et al. 1976). Matsumoto et al. (1991) have shown that broadband electrostatic noise emissions in the plasma sheet boundary layers are not continuous noise but consist of electrostatic impulsive solitary waves. Polar cap boundary layer (Tsurutani et al. 1998), the magnetosheath (Pickett et al. 2003), the bow shock (Bale et al. 1998), and strong currents associated with the auroral acceleration region (Ergun et al. 1998) are other examples of plasmas consisting of two and three similar particle population. The EAWs are typically high frequency waves in comparison with the ion plasma frequency. Therefore ions remain stationary and form a neutralized background. The phase speed of the EAW lies between the cold and hot electron thermal velocities, so that the Landau damping effects are ignored for the consistency of fluid theory in two electron population plasmas. Motivated by these observations, we examine the generation of small amplitude solitons in a plasma with two components namely, cold electron beam and background plasma electrons. Watanabe et al. (1977) used a linear electrostatic Vlasov dispersion equation to show that electron acoustic waves can be destabilized in such plasma. Later on, Yu and Shukla (1983) and Gary et al. (1985) obtained a dispersion relation for EAWs in a two (electron-ion) and three (two-temperature electrons and ions) component plasmas, respectively. The electron-acoustic solitary wave (EASW) (as same as other localized waves in nonlinear me-

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dia) is a localized nonlinear wave phenomena which arises due to a delicate balance between nonlinearity and dispersion. EASWs have been studied both theoretically (Schamel 2000) and numerically (Valentini et al. 2006). These waves have been observed in experiments with pure electron plasmas (Kabantsev et al. 2006) and in laser-produced plasmas (Sircombe et al. 2006) and also have been studied in related subjects using numerical simulations (Ghizzo et al. 2006). The propagation of EASWs in a plasma system has been studied by several investigators in unmagnetized two electron plasmas (Mace et al. 1991; Dubouloz et al. 1991; Chatterjee and Roychoudhury 1995; Berthomier et al. 2000; Mamun and Shukla 2002; Mamun et al. 2002; Clarmann et al. 2002) as well as in magnetized plasmas (Mace and Hellberg 2001; Berthomier et al. 2003; Shukla et al. 2004). Energetic electron distributions are observed in the different regions of the magnetosphere. Gill et al. (2006) studied small amplitude EASWs in a plasma with nonthermal electrons. Recently, Shewy (2007) studied the higher order solution of EASWs with nonthermal electrons. However, numerous observations of space plasmas (Vasyliunas 1968; Leubner 1982; Armstrong et al. 1983) are often characterized by a particle distribution function with high energy tail and they may thus deviate from the Maxwellian. Superthermal particles may arise due to the effect of external forces acting on the natural space environment plasmas or to wave-particle interaction. Plasmas with an excess of superthermal (non-Maxwellian) electrons are generally characterized by a long tail in the high energy region. To model such space plasmas, generalized Lorentzian of k-distribution has been found to be appropriate rather than the Maxwellian distribution (Hasegawa et al. 1985; Thorne and Summers 1991; Summers and Thorne 1991, 1994; Mace and Hellberg 1995). Kappa distribution has been used by several authors (Hellberg and Mace 2002; Podesta 2005; Abbasi and Pajouh 2007; Baluku and Hellberg 2008; Hellberg et al. 2009; Sultana et al. 2010; Baluku et al. 2010) in studying the effect of Landau damping on various plasma modes. "Superthermal" plasma behavior was observed in various experimental plasma contexts, such as laser matter interactions or plasma turbulence (Magni et al. 2005). At very large values of the spectral index k, the velocity distribution function approaches a Maxwellian distribution, while for low values of k, they represent a "hard" spectrum with a strong non-Maxwellian tail having a power-law form at high speeds. Direct measurement of the k distribution in association with the electrostatic solitary structures is not available, however; studies of electron flux spectra in the auroral region where solitary waves are often observed have shown that k rather than Maxwellian fitting gives a better fit to the observed distribution (Olsson and Janhunen 1998). Numerous observations of space plasmas (Feldman et al. 1973; Formisano et al. 1973; Scudder et al. 1981; Marsch et al. 1982) indicate clearly the presence of superthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. The latter may arise due to the effect of external forces acting on the natural space environment plasmas or to the wave-particle interaction which ultimately leads to kappa-like distributions. To study the electron acoustic solitary waves in the nonplanar geometry with radial symmetry we consider unmagnetized plasmas, whose constituents are cold electron fluid, hot electrons obeying a superthermal distribution and stationary ions. In this paper, we try to show how the electron acoustic solitary waves in cylindrical and spherical geometries differ qualitatively from that in one-dimensional planar geometry and how hot superthermal electrons affect on them. The manuscript is organized as follows: We present the basic equations and derive the cylindrical/spherical KdV equation in Sect. 2. Our results are presented and discussed in Sect. 3. A summary of obtained results is given in Sect. 4.

2 Basic equations and derivation of the KdV equation

We consider homogeneous, unmagnetized plasmas consisting of a cold electron fluid, hot electrons obeying a superthermal distribution and stationary ions. In two temperature (cold and hot) electron plasmas, electron acoustic waves can be created due to conservation of equilibrium charge density $n_{e0h} + n_{e0c} = n_{i0}$. It is basically an acoustic (electrostatic) wave in which the inertia is provided by the cold electrons and the restoring force comes from the pressure of the hot electrons. The ions are stationary and provide only the background charge neutrality. This means that the ion dynamics does not influence the electron acoustic waves because the EA wave frequency is much larger than the ion plasma frequency. The nonlinear dynamics of the electron acoustic solitary waves is governed by the continuity and motion equations for cold electrons, and the Poisson's equation

$$\begin{aligned}
&\frac{\partial n_c}{\partial t} + \frac{1}{r^m}\frac{\partial}{\partial r}(r^m n_c u_c) = 0 \\
&\frac{\partial u_c}{\partial t} + u_c\frac{\partial u_c}{\partial r} - \alpha\frac{\partial \phi}{\partial r} = 0 \qquad (1)\\
&\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m\frac{\partial \phi}{\partial r}\right) = \frac{1}{\alpha}n_c + n_h - \left(1+\frac{1}{\alpha}\right)
\end{aligned}$$

where $m = 0$, for one-dimensional geometry and $m = 1, 2$ for cylindrical and spherical geometries, respectively. In the above equations, n_c (n_h) is the cold (hot) electron number density normalized by its equilibrium value n_{c0} (n_{h0}), u_c is the cold electron fluid velocity normalized by $C_e = (k_B T_h/\alpha m_e)^{1/2}$, ϕ is the electrostatic wave potential normalized by $k_B T_h/e$, k_B is the Boltzmann's constant, e and m_e are the electron charge and its mass respectively and

$\alpha = n_{h0}/n_{c0}$. The time and space variables are in units of the cold electron plasma period ω_{pc}^{-1} and the hot electron Debye radius λ_{Dh}, respectively. n_h is the superthermal hot electron density and it is given by Younsi and Tribeche (2010)

$$n_h = \left(1 - \frac{\phi}{k - 1/2}\right)^{-k-\frac{1}{2}} \tag{2}$$

The parameter κ shapes predominantly the superthermal tail of the distribution (Tribeche and Boubakour 2009) and the normalization is provided for any value of the spectral index $\kappa > 1/2$ (Boubakour et al. 2009). In the limit $\kappa \to \infty$, (2) reduces to the well known Maxwell-Boltzmann electron density. Low values of k represent distributions with a relatively large component of particles with speed greater than the thermal speed ("superthermal particles") and an associated reduction in "thermal" particles, as one observes in a "hard" spectrum. Such a very hard spectrum, with an extreme accelerated superthermal component, may be found near very strong shocks associated with Fermi acceleration (Mace and Hellberg 1995).

Now, we study the small but infinite amplitude waves in plasmas with superthermal electrons by using the reductive perturbation method. Firstly, we introduce the stretched coordinates as, $\tau = \varepsilon^{\frac{3}{2}} t$, $\xi = -\varepsilon^{\frac{1}{2}}(r + \lambda t)$, where ε is a small dimensionless expansion parameter and λ is the wave speed normalized by C_e. Secondly, dependent variables are expanded as follows,

$$\left[\begin{array}{l} n_c = 1 + \varepsilon n_{1c} + \varepsilon^2 n_{2c} + \cdots \\ u_c = \varepsilon u_{1c} + \varepsilon^2 u_{2c} + \cdots \\ \phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \cdots \end{array}\right. \tag{3}$$

Substituting (3) into (1) and collecting the terms in different powers of ε the following equations can be obtained at the lower order of ε

$$n_{1c} = \frac{-\alpha\phi_1}{\lambda^2}, \qquad u_{1c} = \frac{-\alpha\phi_1}{\lambda}, \qquad \frac{1}{\lambda^2} = \frac{2k+1}{2k-1} \tag{4}$$

To the next higher order in ε, we obtain a set of equations,

$$\begin{aligned} &\frac{\partial n_{1c}}{\partial \tau} - \lambda \frac{\partial n_{2c}}{\partial \xi} + \frac{\partial}{\partial \xi}(u_{2c} + n_{1c}u_{1c}) + \frac{m u_{1c}}{\lambda \tau} = 0 \\ &\frac{\partial u_{1c}}{\partial \tau} - \lambda \frac{\partial u_{2c}}{\partial \xi} + u_{1c}\frac{\partial u_{1c}}{\partial \xi} - \alpha \frac{\partial \phi_2}{\partial \xi} = 0 \\ &\frac{\partial^2 \phi_1}{\partial \xi^2} - \frac{n_{2c}}{\alpha} - \left[\frac{2k+1}{2k-1}\phi_2 + \frac{(2k+1)(2k+3)}{(2k-1)^2}\phi_1^2\right] = 0 \end{aligned} \tag{5}$$

Finally from (4) and (5) the cylindrical/spherical KdV equation yields

$$\frac{\partial \phi_1}{\partial \tau} + \frac{m}{2\tau}\phi_1 - A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{6}$$

where the coefficients are

$$A = \left[\frac{3\alpha}{2\lambda} + \frac{2k+3}{2k-1}\lambda\right], \qquad B = \frac{\lambda^3}{2} \tag{7}$$

Equation (6) is the cylindrical/spherical KdV equation describing the nonlinear propagation of the electron acoustic solitary waves in plasma consisting superthermal hot electrons and stationary ions. In this equation A and B are the nonlinear coefficient and dispersive terms.

3 Numerical results and discussion

There is not known exact analytical solution for the modified KdV (6) when the geometrical effect is taken into account ($m \neq 0$). Therefore, we have numerically solved equation (6) and have studied the effects of superthermal electrons on the propagation of electron acoustic solitary waves. In the numerical procedure the modified KdV equation was advanced in time with a standard fourth-order Runge-Kuttah method (Press et al. 1992) with a time step of 10^{-4}. The spatial derivatives were approximated with centered finite difference approximations using spatial grid spacing of 0.1 (Maxon and Viecelli 1974a, 1974b). At large values of $|\tau|$ (e.g., $\tau = -14$) the spherical and cylindrical solitary waves are similar to one dimensional solitary waves in flat geometry. In this situation the term $\frac{m}{2\tau}\phi_1$, is no longer dominant and we have usual KdV equation which has solitary wave solution

$$\phi_1 = \phi_0 \sec h^2\left(\frac{\chi}{w}\right) \tag{8}$$

where $\phi_0 = \frac{3u}{A}$ is the amplitude and $w = 2\sqrt{\frac{B}{u}}$ is the width of the solitary wave while u is a constant velocity and $\chi = \xi - u\tau$ is stationary space variable.

Solution (8) can be used as an initial condition for numerical simulation of (6) starts from larger values of $|\tau|$. However, as the value of $|\tau|$ decreases, the term $\frac{m}{2\tau}\phi_1$ becomes dominant and both spherical and cylindrical solitary waves are differ from one dimensional solitary wave. Thus evolution of solitary wave can be investigated using the results of numerical solutions.

Figure 1 shows evolution of solitary solution in cylindrical geometry with initial values of $\alpha = 0.5$, $u = 0.1$ and different values of k. Soliton amplitude increases when k increases. As it was mentioned before, in the limit $k \to \infty$, superthermal distribution reduces to the Maxwell-Boltzmann distribution. Thus, in the presence of hot superthermal electrons, the amplitude of solitons decreases. On the other hand, both amplitude and also width of the solitary wave increases in the presence of greater excess of superthermal hot electrons. This means that the soliton energy increases

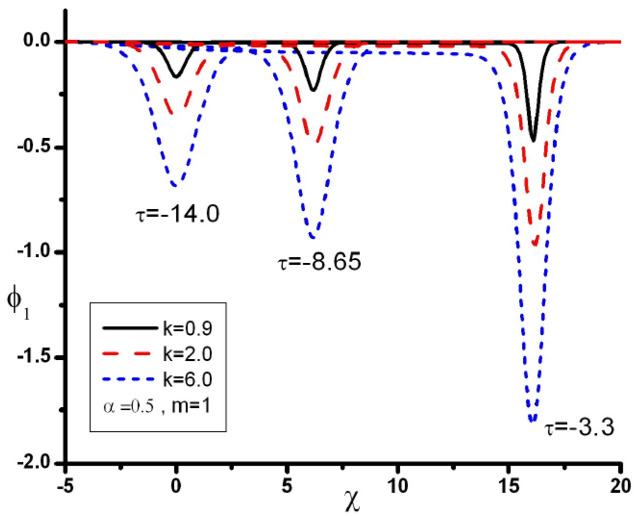

Fig. 1 Time evolution of cylindrical solitary wave ($m = 1$) amplitude (ϕ_1) versus spatial coordinate χ at times $\tau = -14$, $\tau = -8.65$, and $\tau = -3.3$, for different values of $k = 0.2$, 2 and 6 with $\alpha = 0.5$

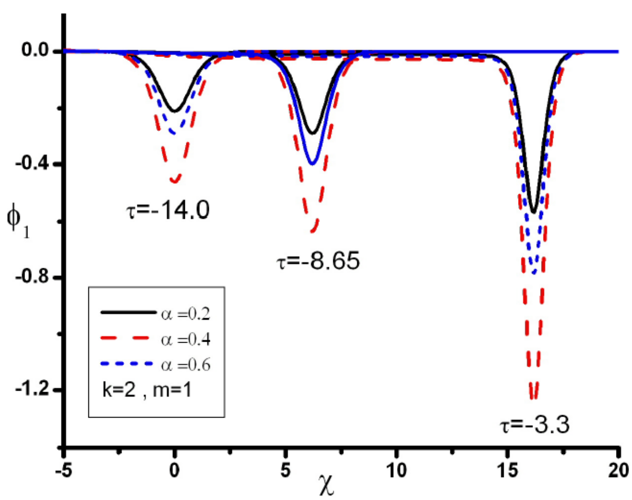

Fig. 2 Time evolution of cylindrical solitary waves ($m = 1$), ϕ_1 versus spatial coordinate χ at times $\tau = -14$, $\tau = -8.65$, and $\tau = -3.3$, for different values of $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.6$ with $k = 2$

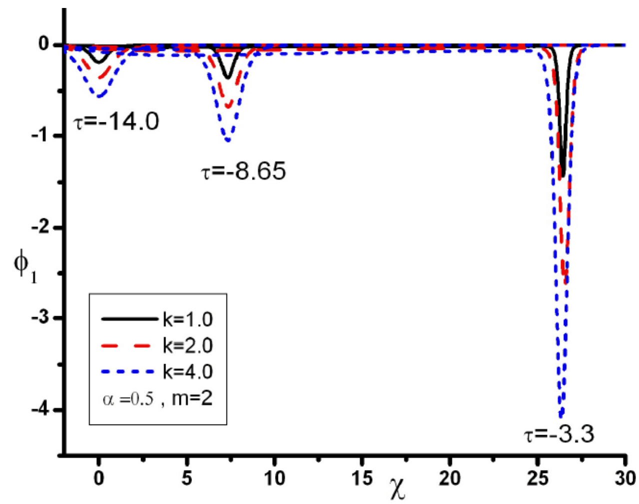

Fig. 3 Time evolution of spherical solitary waves ($m = 2$), ϕ_1 versus spatial coordinate χ at times $\tau = -14$, $\tau = -8.65$, and $\tau = -3.3$, for different values of $k = 1.0$, 2.0 and 4.0 with $\alpha = 0.5$

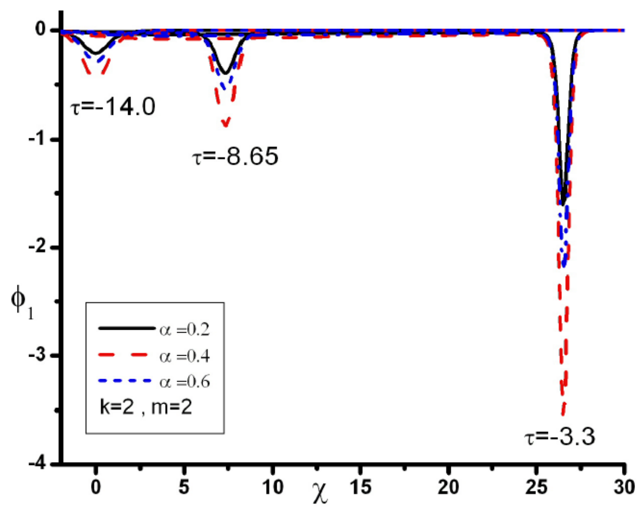

Fig. 4 Time evolution of cylindrical solitary waves ($m = 2$), ϕ_1 versus spatial coordinate ξ at times $\tau = -14$, $\tau = -8.65$, and $\tau = -3.3$, for different values of $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.6$ with $k = 2$

when the population of superthermal hot electrons (the value of parameter k) increases (decreases). It is known that the velocity of the KdV solitons is proportional with their amplitude. As Fig. 1 presents, soliton amplitude and therefore its velocity is not constant and it increases when the term $\frac{m}{2\tau}\phi_1$ become larger (smaller values of $|\tau|$). This means that soliton moves under influence of a kind of external force as we look at the soliton as a localized lump of energy.

Figure 2 demonstrates solitary wave profiles as functions of χ at different times with some values of α. As solitons in flat geometry, the soliton amplitude increases with an increasing α. Also soliton velocity becomes greater with smaller values of $|\tau|$. Soliton velocity changes in time almost independent of the value of α as Fig. 2 clearly presents. Note that the soliton width has not sensible change as α increases. Therefore the soliton becomes narrow when the ratio of electron temperatures increases. Figures 3 and 4 demonstrate similar situations of Figs. 1 and 2 for spherical geometry ($m = 2$) respectively. Both Figs. 3 and 4 indicate that soliton velocity increases as time ($|\tau|$) decreases. The rate of velocity increase in the spherical geometry is very greater than what we can find in cylindrical geometry. All the Figs. 1–4 strongly confirm that parameters "k" and α have not considerable effect in velocity change. Figure 5 compares behavior of soliton in cylindrical and spherical geometries. This figure demonstrates a very clear view of what happening in different geometries. For great values of $|\tau|$ there is no difference between solitons in dif-

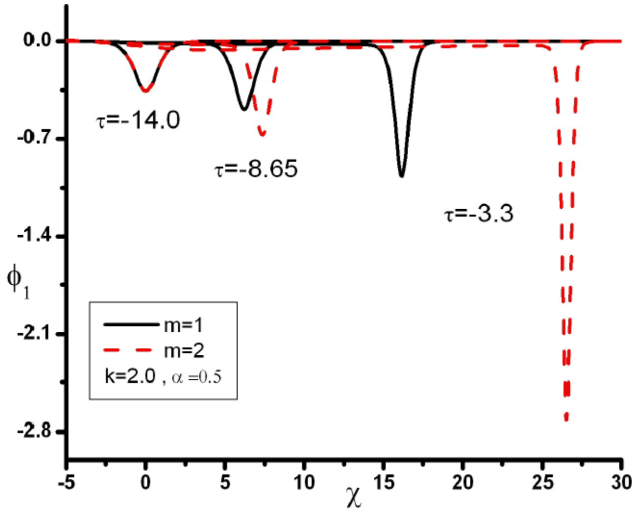

Fig. 5 Time evolution of spherical (*dashed line*) and cylindrical (*solid line*) solitary waves, ϕ_1 versus spatial coordinate χ at times $\tau = -14$, $\tau = -8.65$, and $\tau = -3.3$, for $\alpha = 0.5$ and $k = 2$

ferent geometries. It is clear because in the large values of $|\tau|$ the geometrical effects are negligible. Soliton amplitude (width) increases (decreases) when $|\tau|$ decreases. On the other hand the soliton velocity increases as $|\tau|$ decreases in both cylindrical and spherical geometries. In the spherical geometry solitons move under the influence of stronger force and therefore its amplitude (and thus its velocity) have bigger change. This situation is occurred almost independent of the other parameters like the superthermal electron population (k) or the ratio of the energy of hot and cold electrons. More interesting result is that the kappa distribution is not able to flip the negative amplitude to positive amplitude. All the Figures indicate that the kappa distribution has only quantitative (and not a qualitative) effects on the soliton existence domains and only negative potential solitons can be created (Sahu 2010).

The soliton velocity increases during the evolution in both cylindrical and spherical geometries, as Figs. 1–5 clearly present. There are two different reasons for this phenomenon. Consider a situation with fixed values for the parameters "k" and α. Soliton velocity increases in time as the term $\frac{m}{2\tau}\phi_1$ becomes bigger and bigger. This term in the spherical geometry ($m = 2$) is bigger than that in the cylindrical geometry ($m = 1$). Therefore acceleration in the spherical geometry is greater than acceleration in the cylindrical geometry. All the figures show that the soliton amplitude increases in time too.

On the other hand the soliton amplitude increases when the parameter "k" increases as Figs. 1 and 3 present. Therefore the soliton velocity increases with an increasing value of the parameter "k". This is the second reason for increasing the soliton velocity (and therefore its amplitude). Note that we have to compare soliton profiles with different values of the parameter "k" *in the same time*, for finding this situation.

Figures 1 and 3 clearly show that the dominant reason for increasing the soliton velocity is the term $\frac{m}{2\tau}\phi_1$ (geometry effect) and the effect of the parameter "k" is very smaller than the geometry effect. Thus the peaks of the solitons with different values of "k" are located in very near positions in every instant of time as one can find in the Figs. 1 and 3.

4 Conclusion

We have addressed the problem of nonlinear electron-acoustic oscillations in unmagnetized collisionless plasmas comprising cold fluid electrons, superthermal hot electrons and stationary ions. It was shown that the reductive perturbation method results nonlinear waves in this situation which can be described by solitary waves of the cylindrical/spherical Korteweg-de-Vries equation. Solitons move under the influence of a kind of force in the spherical and cylindrical geometry. The soliton velocity (and thus its amplitude) increases in non planar geometries. This situation is occurred almost independent of the other parameters like (k) or α. Our investigations presented that the soliton velocity increases as $|\tau|$ decreases in both cylindrical and spherical geometries. This effect is more noticeable for spherical geometry. Also it has been shown that the soliton amplitude (and its velocity) increases and also it becomes narrow when as $|\tau|$ decreases. The effects of superthermal electrons on the spherical and cylindrical solitary wave structure have been studied too. We found that an increase of the parameter k increases the amplitude of cylindrical and spherical solitary waves. The soliton energy increases when the population of superthermal hot electrons increases. The same changes can be found for the parameter "α" too. It is shown that the soliton amplitude increases when the ratio of hot electrons to cold electrons increases. But the soliton width has not considerable change with α. Therefore one can conclude that the soliton amplitude increases when α increases. In general one can conclude that the kappa distribution has only quantitative effects on the soliton existence domains and only negative potential solitons can be created in the medium. Our results may be useful in understanding the wide relevance of nonlinear features of localized electroacoustic structures in different regions of the magnetosphere (Singh and Lakhina 2004). In further studies one can pay attention on the role of relativistic particles in the media which can strongly affect on the behavior of nonlinear waves.

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References

Abbasi, H., Pajouh, H.H.: Phys. Plasmas **14**, 012307 (2007)
Armstrong, T.P., Paonessa, M.T., Bell, E.V., Krimigis, S.M.: J. Geophys. Res. **88**, 893 (1983). doi:10.1029/JA088iA11p08893
Bale, S.D., Kellogg, P.J., Larson, D.E., Lin, R.P., Goetz, K., Lepping, R.P.: Geophys. Res. Lett. **25**, 2929 (1998)
Baluku, T.K., Hellberg, M.A.: Phys. Plasmas **15**, 123705 (2008)
Baluku, T.K., Hellberg, M.A., Kourakis, I., Saini, N.S.: Phys. Plasmas **17**, 053702 (2010)
Berthomier, M., Pottelette, R., Malingre, M., Khotyaintsev, Y.: Phys. Plasmas **7**, 2987 (2000)
Berthomier, M., Pottelette, R., Muschietti, L., Roth, I., Carlson, C.W.: Geophys. Res. Lett. **30**, 2148 (2003). doi:10.1029/2003GL018491
Boubakour, N., Tribeche, M., Aoutou, K.: Phys. Scr. **79**, 065503 (2009)
Chatterjee, P., Roychoudhury, R.K.: J. Plasma Phys. **53**, 25 (1995)
Clarmann, T.V., Dudhia, A., Edwards, D.P., Funke, B., Höpfner, M., Kerridge, B.J.: J. Geophys. Res. **107**, 4631 (2002)
Dubouloz, N., Pottelette, R., Malingre, M., Treumann, R.A.: Geophys. Res. Lett. **18**, 155 (1991). doi:10.1029/90GL02677
El-Shewy, E.K.: Chaos Solitons Fractals **34**, 628 (2007)
Ergun, R.E., Calrson, C.W., McFadden, J.P., Mozer, F.S., Delory, G.T., et al.: Geophys. Res. Lett. **25**, 2041 (1998)
Feldman, W.C., Asbridge, J.R., Bame, S.J., Montgomery, M.D.: J. Geophys. Res. **78**, 2017 (1973)
Formisano, V., Moreno, G., Palmiotto, F.: J. Geophys. Res. **78**, 3714 (1973)
Gary, S.P., Tokar, R.L.: Phys. Fluids **28**, 2439 (1985)
Ghizzo, A., Johnston, T.W., Révéille, T., Bertrand, P., Albrecht-Marc, M.: Phys. Rev. E **74**, 046407 (2006)
Gill, T.S., Kaur, H., Saini, N.S.: Chaos Solitons Fractals **30**, 1020 (2006)
Gurnett, D.A., Frank, L.A., Lepping, R.P.: J. Geophys. Res. **81**, 6059–6071 (1976)
Hasegawa, A., Mima, K., Duong-Van, M.: Phys. Rev. Lett. **54**, 2608 (1985)
Hellberg, M.A., Mace, R.L.: Phys. Plasmas **9**, 1495 (2002)
Hellberg, M.A., Mace, R.L., Bakalu, T.K., Kourakis, I., Saini, N.S.: Phys. Plasmas **16**, 094701 (2009)
Kabantsev, A.A., Valentini, F., Driscoll, C.F.: AIP Conf. Proc. **862**, 13 (2006)
Leubner, M.P.: J. Geophys. Res. **87**, 6335 (1982). doi:10.1029/JA087iA08p06335
Mace, R.L., Hellberg, M.A.: Phys. Plasmas **2**, 2098 (1995)
Mace, R.L., Hellberg, M.A.: Phys. Plasmas **8**, 2649 (2001)
Mace, R.L., Baboolal, S., Bharuthram, R., Hellberg, M.A.: J. Plasma Phys. **45**, 323–338 (1991)
Magni, S., Roman, H.E., Barni, R., Riccardi, C., Pierre, Th., Guyomarc'h, D.: Phys. Rev. E **72**, 026403 (2005)
Mamun, A.A., Shukla, P.K.: J. Geophys. Res. **107**, 1135 (2002). doi:10.1029/2001JA009131
Mamun, A.A., Shukla, P.K., Stenflo, L.: Phys. Plasmas **9**, 1474 (2002)
Marsch, E., Muhlhauser, K.H., Schwenn, R., Rosenbauer, H., Pilipp, W., Neubauer, F.M.: J. Geophys. Res. **87**, 52 (1982)
Maxon, S., Viecelli, J.: Phys. Rev. Lett. **32**, 4 (1974a)
Maxon, S., Viecelli, J.: Phys. Fluids **17**, 1614 (1974b)
Olsson, A., Janhunen, P.: Ann. Geophys. **16**, 298 (1998)
Onsager, T.G., Thomsen, M.F., Elphic, R.C., Gosling, J.T., Anderson, R.R., et al.: J. Geophys. Res. **98**, 15509–15519 (1993)
Parks, G.K., McCarthy, M., Fitzenreiter, R.J., Etcheto, J., Anderson, K.A., et al.: J. Geophys. Res. **89**, 8885–8906 (1984)
Pickett, J.S., Menietti, J.D., Gurnett, D.A., Tsurutani, B., Kintner, P.M., et al.: Nonlinear Process. Geophys. **10**, 3 (2003)
Podesta, J.J.: Phys. Plasmas **12**, 052101 (2005)
Press, H.W., Teukolsky, A.S., Vetterling, T.W., Flannery, P.B.: Numerical Recipes in Fortran, 2nd edn. Cambridge University Press, Cambridge (1992)
Sahu, B.: Phys. Plasmas **17**, 122305 (2010)
Schamel, H.: Phys. Plasmas **7**, 4831 (2000)
Scudder, J.D., Sittler, E.C., Bridge, H.S.: J. Geophys. Res. **86**, 8157 (1981)
Shukla, P.K., Mamun, A.A., Eliasson, B.: Geophys. Res. Lett. **31**, L07803 (2004). doi:10.1029/2004GL019533
Singh, S.V., Lakhina, G.S.: Nonlinear Process. Geophys. **11**, 275 (2004)
Sircombe, N.J., Arber, T.D., Dendy, R.O.: Plasma Phys. Control. Fusion **48**, 1141 (2006)
Sultana, S., Kourakis, I., Saini, N.S., Hellberg, M.A.: Phys. Plasmas **17**, 032310 (2010)
Summers, D., Thorne, R.M.: Phys. Fluids B **3**, 1835 (1991)
Summers, D., Thorne, R.M.: Phys. Plasmas **1**, 2012 (1994)
Thorne, R.M., Summers, D.: Phys. Fluids B **3**, 2117 (1991)
Tribeche, M., Boubakour, N.: Phys. Plasmas **16**, 084502 (2009)
Tsurutani, B.T., Arballo, J.K., Lakhina, G.S., Ho, C.M., Buti, B., et al.: Geophys. Res. Lett. **25**, 4117 (1998)
Valentini, F., O'Neil, T.M., Dubin, D.H.E.: Phys. Plasmas **13**, 052303 (2006)
Vasyliunas, V.M.: J. Geophys. Res. **73**, 2839 (1968). doi:10.1029/JA073i009p02839
Watanabe, K., Taniuti, T.: J. Phys. Soc. Jpn. **43**, 1819 (1977)
Younsi, S., Tribeche, M.: Astrophys. Space Sci. **330**, 295 (2010)
Yu, M.Y., Shukla, P.K.: J. Plasma Phys. **29**, 409 (1983)