Capacity Region of Multiple Access Relay Channels with Orthogonal Components

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***Abstract*—In this paper, we introduce multiple access relay channel with orthogonal components from the senders to the relay receiver and from the senders and relay to the receiver. Our model is motivated by the practical constraint in wireless communications that a node cannot send and receive at the same time or in the same frequency band. For this model, we derive a capacity region. Superposition block Markov encoding and multiple access channel encoding and decoding strategies are used to prove the results and then we extend the results to the Gaussian case.**

***Index Terms*—multiple access relay channel; orthogonal components; decode and forward strategy; Gaussian channel.**

I. Introduction

The relay channel was first introduced by Van der Meulen [1]. In [2], the capacity of degraded and reversely degraded relay channels and the capacity of the relay channel with feedback as well as upper and lower bounds on the capacity of the general relay channel were established. In [3], the capacity of semi deterministic relay channel , in [4], the capacity of the relay channel with orthogonal components, in [5], the capacity of modulo-sum relay channel, in [6], the capacity of a class of deterministic relay channel and in [7], capacity of a more general class of relay channels have been determined.

Relaying has been proposed as a means to increase coverage area of wireless networks. Relay nodes in cooperation with the users, act as a distributed multi antenna system. Nowadays, there has been much research on a multi-user extension of the relay channel, e.g. multiple access relay channel (MARC). In [8], MARC is introduced, where some sources communicate with one single destination with the help of a relay node. An example of such a channel model is the cooperative uplink of some mobile stations to the base station with the help of the relay in a cellular based mobile communication system. Fig. 1 shows an N-source MARC.

Many recent results concerning coding strategies on the MARC can be found in [9]-[11]. An achievable rate region for the MAC with feedback was established in [12].

II. Preliminaries and Main Results

We Divide $X_k\,(k=1,\cdots,N)$, to orthogonal components (X_{Rk},X_{Dk}) and send these components from the senders to relay receiver (X_{Rk}) and from the sender and relay to the receiver (X_{Dk}). For this model (MARCO), we derive a capacity region and extend the result to the Gaussian case.

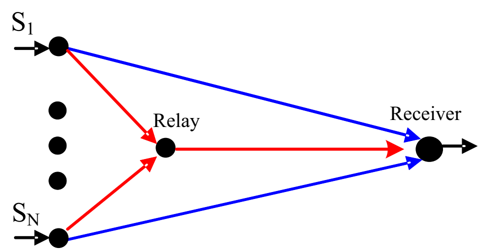

Fig. 1: An N-source multiple access relay channel

A discrete memoryless multiple access relay channel is said to have orthogonal components if the channel input-output distribution can be expressed as

$$P(y_D,y_R,x_{R1},\cdots,x_{RN},x_{D1},\cdots,x_{DN},x_R)= P(y_R|x_{R1},\cdots,x_{RN},x_R)P(y_D|x_{D1},\cdots,x_{DN},x_R) \times P(x_R)\prod_{k=1}^{N}P(x_{Rk}|x_R)P(x_{Dk}|x_R) \quad (1)$$

where X_R, X_{Rk} and $X_{Dk}\,(k=1,\cdots,N)$, are the inputs, Y_D and Y_R are the outputs, all with finite alphabets, and $P(y_D,y_R|x_{R1},\cdots,x_{RN},x_{D1},\cdots,x_{DN},x_R)$ is the channel probability function.

In Gaussian multiple access relay channel with orthogonal components the channel input sequences are subject to the following average power constraints

$$\frac{1}{n}\sum_{t=1}^{n}E[X_{Rk,t}^2+X_{Dk,t}^2]\le P_k,\ (k=1,\cdots,N) \quad (2)$$

$$\frac{1}{n}\sum_{t=1}^{n}E[X_{R,t}^2]\le P_R \quad (3)$$

At the t^{th} transmission $(t=1,\cdots,n)$, $X_{Rk,t},X_{Dk,t}$ $(k=1,...,N)$, and $X_{R,t}$ are sent and the channel outputs are

$$Y_{R,t}=\sum_{k=1}^{N}g_{Rk}X_{Rk,t}+Z_{1t} \quad (4)$$

$$Y_{D,t}=\sum_{k=1}^{N}g_{Dk}X_{Dk,t}+g_RX_{R,t}+Z_{2t} \quad (5)$$

where N is the number of senders, $Z_m=(Z_{m1},...,Z_{mn})$, $m=1,2$, is a sequence of independent identically distributed $(i.i.d)$ normal random variables with zero mean and variance N_m. g_{Rk}, $g_{Dk}\,(k=1,\cdots,N)$, and g_R are the channel gains; $S_{Rk}=g_{Rk}^2\frac{P_{Rk}}{N_1}$, $S_{Dk}=g_{Dk}^2\frac{P_{Dk}}{N_2}$, and $S_R=g_R^2\frac{P_R}{N_2}$ are the signal to noise ratio (SNR) of the channels, where $P_{Dk}=\alpha P_k$ and $P_{Rk}=\bar{\alpha}P_k$ $(0\le\alpha_k\le1,\ \bar{\alpha}_k=1-\alpha_k)$.

Now, we express main results as two following theorems.

Theorem 1. The capacity region $\mathcal{C}_{MARCO}$ of two-source multiple access relay channels with orthogonal components is given by

$$\mathcal{C}_{MARCO} = \bigcup \Big\{ (R_1, R_2)$$
$$0 \le R_1 \le \min\{ I(X_{D1}, X_R; Y_D | X_{D2}), \quad I(X_{R1}; Y_R | X_{R2}, X_R) + I(X_{D1}; Y_D | X_{D2}, X_R)\} \tag{6a}$$
$$0 \le R_2 \le \min\{ I(X_{D2}, X_R; Y_D | X_{D1}), \quad I(X_{R2}; Y_R | X_{R1}, X_R) + I(X_{D2}; Y_D | X_{D1}, X_R)\} \tag{6b}$$
$$0 \le R_1 + R_2 \le \min\{ I(X_{D1}, X_{D2}, X_R; Y_D), \quad I(X_{R1}, X_{R2}; Y_R | X_R) + I(X_{D1}, X_{D2}; Y_D | X_R)\} \Big\} \tag{6c}$$

where the union is taken over

$$P(x_R, x_{R1}, x_{R2}, x_{D1}, x_{D2}) = P(x_R)P(x_{R1}|x_R)P(x_{D1}|x_R)P(x_{R2}|x_R)P(x_{D2}|x_R) \tag{7}$$

Theorem 2. The capacity region $\mathcal{C}_{MARCO}$ of two-source Gaussian multiple access relay channels with orthogonal components is given by ($C(x) = \frac{1}{2}\log(1+x), \ \ x \ge 0$)

$$0 \le R_1 \le \min\{ C\left(S_{D1} + S_R + 2\rho_{D1}\sqrt{S_{D1}S_R}\right), \quad C\left((1-\rho_{R1}^2)S_{R1}\right) + C\left((1-\rho_{D1}^2)S_{D1}\right)\} \tag{8a}$$
$$0 \le R_2 \le \min\{ C\left(S_{D2} + S_R + 2\rho_{D2}\sqrt{S_{D2}S_R}\right), \quad C\left((1-\rho_{R2}^2)S_{R2}\right) + C\left((1-\rho_{D2}^2)S_{D2}\right)\} \tag{8b}$$
$$0 \le R_1 + R_2 \le \min\{ C\left((1-\rho_{D1}^2)S_{D1} + (1-\rho_{D2}^2)S_{D2}\right) + C\left((1-\rho_{R1}^2)S_{R1} + (1-\rho_{R2}^2)S_{R2}\right), \tag{8c}$$
$$C\left(S_{D1} + S_{D2} + S_R + 2\rho_{D1}\sqrt{S_{D1}S_R} + 2\rho_{D2}\sqrt{S_{D2}S_R}\right)\}$$

where ρ_{Dk} and ρ_{Rk} are the correlation coefficients between X_{Dk} and X_{Rk} with X_R, respectively.

The above two theorems can be easily generalized to N-source MARCO.

III. Proof of The Main Results

A. Proof of Theorem 1

1) Achievability part: The two-source multiple access relay channel with orthogonal components is shown in Fig. 2.

Definition: A $((2^{nR_1}, 2^{nR_2}, n)$ code, $(R_1 = R_{R1} + R_{D1}, R_2 = R_{R2} + R_{D2})$, for the multiple access relay channel with orthogonal components consists of two sets of integers $w_1 = (w_{R1}, w_{D1}) \in [1 : 2^{nR_{R1}}] \times [1 : 2^{nR_{D1}}]$ and $w_2 = (w_{R2}, w_{D2}) \in [1 : 2^{nR_{R2}}] \times [1 : 2^{nR_{D2}}]$, called the message sets; two encoding functions,

$$X_1 : \mathcal{W}_1 = (\mathcal{W}_{R1}, \mathcal{W}_{D1}) \to \mathcal{X}_1^n = \mathcal{X}_{R1}^n \times \mathcal{X}_{D1}^n \tag{9a}$$
$$X_2 : \mathcal{W}_2 = (\mathcal{W}_{R2}, \mathcal{W}_{D2}) \to \mathcal{X}_2^n = \mathcal{X}_{R2}^n \times \mathcal{X}_{D2}^n \tag{9b}$$

a set of relay function $\{f_t\}_{t=1}^n$ such that

$$X_{R,t} = f_t\{Y_R^{t-1}\}, \qquad 1 \le t \le n \tag{10}$$

and a decoding function,

$$g : \mathcal{Y}_D^n \to \mathcal{W}_1 \times \mathcal{W}_2 = (\mathcal{W}_{R1}, \mathcal{W}_{D1}) \times (\mathcal{W}_{R2}, \mathcal{W}_{D2}) \tag{11}$$

Sender 1 chooses an index $w_1 = (w_{R1}, w_{D1})$ uniformly distributed over $[1 : 2^{nR_{R1}}] \times [1 : 2^{nR_{D1}}]$ and sends the

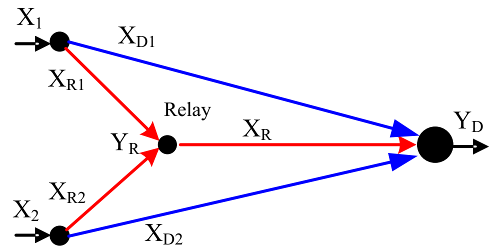

Fig. 2: A 2-source multiple access relay channel with orthogonal components

corresponding codeword over the channel. Sender 2 does likewise. Assuming that the distribution of messages over the product set $\mathcal{W}_1 \times \mathcal{W}_2$ is uniform, we define the average probability of error for the $((2^{nR_1}, 2^{nR_2}, n)$ code as follows:

$$P_e^{(n)} = \frac{1}{2^{n(R_1+R_2)}} \sum_{(w_1,w_2)\in\mathcal{W}_1\times\mathcal{W}_2} \Pr\{g(Y_D^n) \ne (w_1, w_2) | (w_1, w_2) \text{has been sent}\} \tag{12}$$

A rate pair (R_1, R_2) is said to be achievable for the multiple access relay channel with orthogonal components if there exists a sequence of $((2^{nR_1}, 2^{nR_2}), n)$ code with $P_e^{(n)} \to 0$. We consider B blocks, each of n symbols. We use superposition block Markov encoding. A sequence of B messages $w_{1,i} \times w_{2,i} = (w_{R1,i}, w_{D1,i}) \times (w_{R2,i}, w_{D2,i})$, $i = 1, 2, \cdots, B$ will be sent over the channel in nB transmissions. In each n-block, $b = 1, 2, \cdots, B+1$, we use the same set of codebooks:

$$\mathcal{C} = \{x_R^n(v), x_{R1}^n(v, j), x_{R2}^n(v, l), x_{D1}^n(v, q), x_{D2}^n(v, u)\}$$
$$v = (v_1, v_2) \in [1 : 2^{n(R_1+R_2)}], j \in [1 : 2^{nR_{R1}}],$$
$$l \in [1 : 2^{nR_{R2}}], q \in [1 : 2^{nR_{D1}}], u \in [1 : 2^{nR_{D2}}]$$

Now we proceed with the proof of achievability using a random coding technique.

Random codebook generation: First fix a choice of $P(x_R)P(x_{R1}|x_R)P(x_{R2}|x_R)P(x_{D1}|x_R)P(x_{D2}|x_R)$.

1) Generate $2^{n(R_1+R_2)}$ independent identically distributed n-sequences x_R^n, each drawn according to $P(x_R^n) = \prod_{t=1}^n P(x_{R,t})$. Index them as $x_R^n(v_1, v_2)$, $v_1 \in [1 : 2^{nR_1}]$, $v_2 \in [1 : 2^{nR_2}]$.
2) For each $x_R^n(v)$, generate $2^{nR_{R1}}$ conditionally independent n-sequence x_{R1}^n drawn according to $P(x_{R1}^n | x_R^n(v)) = \prod_{t=1}^n P(x_{R1,t} | x_{R,t}(v))$. Index them as $x_{R1}^n(v, j)$, $j \in [1 : 2^{nR_{R1}}]$.
3) For each $x_R^n(v)$, generate $2^{nR_{R2}}$ conditionally independent n-sequence x_{R2}^n drawn according to $P(x_{R2}^n | x_R^n(v)) = \prod_{t=1}^n P(x_{R2,t} | x_{R,t}(v))$. Index them as $x_{R2}^n(v, l)$, $l \in [1 : 2^{nR_{R2}}]$.
4) For each $x_R^n(v)$, generate $2^{nR_{D1}}$ conditionally independent n-sequence x_{D1}^n drawn according to $P(x_{D1}^n | x_R^n(v)) = \prod_{t=1}^n P(x_{D1,t} | x_{R,t}(v))$. Index them as $x_{D1}^n(v, q)$, $q \in [1 : 2^{nR_{D1}}]$.
5) For each $x_R^n(v)$, generate $2^{nR_{D2}}$ conditionally independent n-sequence x_{D2}^n drawn according to $P(x_{D2}^n | x_R^n(v)) = \prod_{t=1}^n P(x_{D2,t} | x_{R,t}(v))$. Index them as $x_{D2}^n(v, u)$, $u \in [1 : 2^{nR_{D2}}]$.

Encoding: Encoding is performed in $B + 1$ blocks, The

TABLE I: Encoding Strategy

Block 1	Block 2	$\cdots$	Block B+1
$x^n_{R,1}(1,1)$	$x^n_{R,2}(w_{R1,1},w_{R2,1})$	$\cdots$	$x^n_{R,B+1}(w_{R1,B},w_{R2,B})$
$x^n_{R1,1}(1,w_{R1,1})$	$x^n_{R1,2}(w_{R1,1},w_{R1,2})$	$\cdots$	$x^n_{R1,B+1}(w_{R1,B},1)$
$x^n_{R2,1}(1,w_{R2,1})$	$x^n_{R2,2}(w_{R2,1},w_{R2,2})$	$\cdots$	$x^n_{R2,B+1}(w_{R2,B},1)$
$x^n_{D1,1}(1,w_{D1,1})$	$x^n_{D1,2}(w_{D1,1},w_{D1,2})$	$\cdots$	$x^n_{D1,B+1}(w_{D1,B},1)$
$x^n_{D2,1}(1,w_{D2,1})$	$x^n_{D2,2}(w_{D2,1},w_{D2,2})$	$\cdots$	$x^n_{D2,B+1}(w_{D2,B},1)$

coding strategy is shown in Table I.

1) *Source terminals*: The messages are split into B equally sized blocks $w_{R1,b}$, $w_{R2,b}$, $w_{D1,b}$, $w_{D2,b}$, $b = 1,2,\cdots,B$. In block $b = 1,2,\cdots,B+1$, the sender 1 transmits $x^n_{R1,b}(w_{R1,b-1},w_{R1,b})$ and $x^n_{D1,b}(w_{D1,b-1},w_{D1,b})$, where $w_{R1,0} = w_{R1,B+1} = w_{D1,0} = w_{D1,B+1} = 1$. Sender 2 does likewise.

2) *Relay Terminal*: After the transmission of block b is completed, the relay has seen $y^n_{R,b}$. The relay tries to find a pair $(\widetilde{w}_{R1,b},\widetilde{w}_{R2,b})$ such that

$$\Big(x^n_{R1,b}(\widehat{\widehat{w}}_{R1,b-1},\widetilde{w}_{R1,b}),x^n_{R2,b}(\widehat{\widehat{w}}_{R2,b-1},\widetilde{w}_{R2,b}),\; x^n_{R,b}(\widehat{\widehat{w}}_{R1,b-1},\widehat{\widehat{w}}_{R2,b-1}),y^n_{R,b}\Big)\in A^n_\epsilon(X_{R1},X_{R2},X_R,Y_R)$$

where $\widehat{\widehat{w}}_{R1,b-1}$ and $\widehat{\widehat{w}}_{R2,b-1}$ are the relay terminal's estimate of $w_{R1,b-1}$ and $w_{R2,b-1}$, respectively. If one or more such $\widetilde{w}_{R1,b}$ and $\widetilde{w}_{R2,b}$ are found, then the relay chooses one of them, calls this choice $\widehat{\widehat{w}}_{R1,b}$ and $\widehat{\widehat{w}}_{R2,b}$ and then transmits $x^n_{R,b+1}(\widehat{\widehat{w}}_{R1,b},\widehat{\widehat{w}}_{R2,b})$ in block $b+1$. If no such $\widetilde{w}_{R1,b}$ and $\widetilde{w}_{R2,b}$ are found, the relay sets $\widehat{\widehat{w}}_{R1,b-1} = 1$ and $\widehat{\widehat{w}}_{R2,b-1} = 1$ and then transmits $x^n_{R,b+1}(1,1)$.

3) *Sink Terminal*: After block b, the receiver has seen $y^n_{D,b-1}$ and $y^n_{D,b}$ and try to find $\widetilde{w}_{R1,b-1}$, $\widetilde{w}_{R2,b-1}$, $\widetilde{w}_{D1,b-1}$, and $\widetilde{w}_{D2,b-1}$ such that

$$\Big(x^n_{R,b-1}(\widehat{\widehat{\widehat{w}}}_{R1,b-2},\widehat{\widehat{\widehat{w}}}_{R2,b-2}),x^n_{D1,b-1}(\widehat{\widehat{\widehat{w}}}_{D1,b-2},\widetilde{w}_{D1,b-1}),\; x^n_{D2,b-1}(\widehat{\widehat{\widehat{w}}}_{D2,b-2},\widetilde{w}_{D2,b-1}),y^n_{D,b-1}\Big)\in A^n_\epsilon(X_R,X_{D1},X_{D2},Y_D)$$

and

$$\Big(x^n_{R,b}(\widetilde{w}_{R1,b-1},\widetilde{w}_{R2,b-1}),y^n_{D,b}\Big)\in A^n_\epsilon(X_R,Y_D)$$

If one or more such $\widetilde{w}_{D1,b-1}$, $\widetilde{w}_{D2,b-1}$,$\widetilde{w}_{R1,b-1}$ and $\widetilde{w}_{R2,b-1}$ are found, then the sink chooses one of them and puts out these choices as $\widehat{\widehat{\widehat{w}}}_{D1,b-1}$, $\widehat{\widehat{\widehat{w}}}_{D2,b-1}$ and $\widehat{\widehat{\widehat{w}}}_{R1,b-1}$ and $\widehat{\widehat{\widehat{w}}}_{R2,b-1}$, respectively. If no such $\widetilde{w}_{D1,b-1}$, $\widetilde{w}_{D2,b-1}$, $\widetilde{w}_{R1,b-1}$ and $\widetilde{w}_{R2,b-1}$ are found, the sink sets $\widehat{\widehat{\widehat{w}}}_{D1,b-1} = \widehat{\widehat{\widehat{w}}}_{D2,b-1} = \widehat{\widehat{\widehat{w}}}_{R1,b-1} = \widehat{\widehat{\widehat{w}}}_{R2,b-1} = 1$.

Decoding and error Analysis: It can be shown with [2],[13, theorem 15.2.3] that the relay can decode reliably if

$$0 \le R_{R1} \le I(X_{R1};Y_R|X_{R2},X_R) \tag{13a}$$
$$0 \le R_{R2} \le I(X_{R2};Y_R|X_{R1},X_R) \tag{13b}$$
$$0 \le R_{R1}+R_{R2} \le I(X_{R1},X_{R2};Y_R|X_R) \tag{13c}$$

and the receiver can decode with arbitrarily small probability of error if

$$0 \le R_{D1} \le I(X_{D1};Y_D|X_{D2},X_R) \tag{14a}$$
$$0 \le R_{D2} \le I(X_{D2};Y_D|X_{D1},X_R) \tag{14b}$$
$$0 \le R_{D1}+R_{D2} \le I(X_{D1},X_{D2};Y_D|X_R) \tag{14c}$$
$$0 \le R_1 \le I(X_{D1},X_R;Y_D|X_{D2}) \tag{15a}$$
$$0 \le R_2 \le I(X_{D2},X_R;Y_D|X_{D1}) \tag{15b}$$
$$0 \le R_1+R_2 \le I(X_{D1},X_{D2},X_R;Y_D) \tag{15c}$$

Therefore,

$$0 \le R_1 \le \min\{I\left(X_{D1},X_R;Y_D|X_{D2}\right), \; I\left(X_{R1};Y_R|X_{R2},X_R\right)+I\left(X_{D1};Y_D|X_{D2},X_R\right)\} \tag{16a}$$
$$0 \le R_2 \le \min\{I\left(X_{D2},X_R;Y_D|X_{D1}\right), \; I\left(X_{R2};Y_R|X_{R1},X_R\right)+I\left(X_{D2};Y_D|X_{D1},X_R\right)\} \tag{16b}$$
$$0 \le R_1+R_2 \le \min\{I\left(X_{D1},X_{D2},X_R;Y_D\right), \; I\left(X_{R1},X_{R2};Y_R|X_R\right)+I\left(X_{D1},X_{D2};Y_D|X_R\right)\}\Big\} \tag{16c}$$

2) Converse part: It is possible to estimate $W_1,W_2\,((W_{R1},W_{D1}),(W_{R2},W_{D2}))$ from the received sequence Y^n_D with low probability of error. Hence, the conditional entropy of (W_1,W_2) given Y^n_D must be small. By Fano's inequality,

$$H(W_1|Y^n_D) \le n\epsilon_1, \quad H(W_1,W_2|Y^n_D) \le n\epsilon_{12} \tag{17}$$

To bound the R_1, we have

$$nR_1 = H\big(W_1\big) = I\big(W_1;Y^n_D\big)+H\big(W_1|Y^n_D\big) \tag{18}$$
$$\le^a I\big(W_1;Y^n_D\big)+n\epsilon_1 \tag{19a}$$
$$\le^b I\big(X^n_1;Y^n_D,Y^n_R,X^n_2\big)+n\epsilon_1 \tag{19b}$$
$$= I\big(X^n_1;X^n_2\big)+I\big(X^n_1;Y^n_R|X^n_2\big) +I\big(X^n_1;Y^n_D|Y^n_R,X^n_2\big)+n\epsilon_1 \tag{19c}$$
$$= I\big(X^n_1;Y^n_R|X^n_2\big)+I\big(X^n_1;Y^n_D|Y^n_R,X^n_2\big)+n\epsilon_1 \tag{19d}$$
$$= I_1+I_2+n\epsilon_1 \tag{19e}$$

where

$$I_1 = I(X^n_1;Y^n_R|X^n_2) = \textstyle\sum_{t=1}^n I(X^n_1;Y_{R,t}|Y^{t-1}_R,X^n_2) \tag{20}$$
$$= \textstyle\sum_{t=1}^n H(Y_{R,t}|Y^{t-1}_R,X^n_2) - H(Y_{R,t}|Y^{t-1}_R,X^n_2,X^n_1) \tag{21}$$
$$=^c \textstyle\sum_{t=1}^n H(Y_{R,t}|Y^{t-1}_R,X^n_{D2},X^n_{R2},X_{R,t}) -H(Y_{R,t}|Y^{t-1}_R,X^n_{D2},X^n_{R2},X^n_{D1},X^n_{R1},X_{R,t}) \tag{22}$$
$$\le^d \textstyle\sum_{t=1}^n H(Y_{R,t}|X_{R2,t},X_{R,t}) -H(Y_{R,t}|X_{R2,t},X_{R1,t},X_{R,t}) \tag{23}$$
$$= \textstyle\sum_{t=1}^n I(X_{R1,t};Y_{R,t}|X_{R2,t},X_{R,t}) \tag{24}$$

and

$$I_2 = I(X^n_1;Y^n_D|X^n_2,Y^n_R) = \textstyle\sum_{t=1}^n I(X^n_1;Y_{D,t}|Y^{t-1}_D,X^n_2,Y^n_R)$$
$$= \textstyle\sum_{t=1}^n H(Y_{D,t}|Y^{t-1}_D,X^n_2,Y^n_R) -H(Y_{D,t}|Y^{t-1}_D,X^n_2,X^n_1,Y^n_R) \tag{25}$$
$$=^e \textstyle\sum_{t=1}^n H(Y_{D,t}|Y^{t-1}_D,X^n_{D2},X^n_{R2},X_{R,t},Y_{R,t},Y^{t-1}_R) -H(Y_{D,t}|Y^{t-1}_D,X^n_{D1},X^n_{R1},X^n_{D2},X^n_{R2},X_{R,t},Y_{R,t},Y^{t-1}_R) \tag{26}$$
$$\le^f \textstyle\sum_{t=1}^n H(Y_{D,t}|X_{D2,t},X_{R,t}) -H(Y_{D,t}|X_{D2,t},X_{D1,t},X_{R,t}) \tag{27}$$
$$= \textstyle\sum_{t=1}^n I(X_{D1,t};Y_{D,t}|X_{D2,t},X_{R,t}) \tag{28}$$

Therefore,

$$nR_1 \le \sum_{t=1}^{n} I(X_{R1,t};Y_{R,t}|X_{R2,t},X_{R,t}) \quad (29)$$
$$+\sum_{t=1}^{n} I(X_{D1,t};Y_{D,t}|X_{D2,t},X_{R,t})+n\epsilon_1$$
$$\Longrightarrow R_1 \le I(X_{R1};Y_R|X_{R2},X_R)+I(X_{D1};Y_D|X_{D2},X_R)$$

Similarly,

$$R_2 \le I(X_{R2};Y_R|X_{R1},X_R)+I(X_{D2};Y_D|X_{D1},X_R)$$

where,

- 'a' follows from Fano's inequality.
- 'b' follows from the data-processing inequality.
- 'c' follows from the chain rule and definition of relay function.
- 'd' follows from the fact that $(Y_R^{t-1},X_{D2}^n,X_{R2}^{t-1},X_{D1}^n,X_{R1}^{t-1} \to X_{R2,t},X_{R1,t},X_{R,t} \to Y_{R,t})$ forms a Markov chain according to the distribution, the memoryless property of the channel, the chain rule and removing conditioning.
- 'e' follows from the fact that channel is memoryless.
- 'f' follows from the fact that $(Y_D^{t-1},X_{R2}^n,X_{D2}^{t-1},X_{R1}^n,X_{D1}^{t-1},Y_R^n \to X_{D2,t},X_{D1,t},X_{R,t} \to Y_{D,t})$ forms a Markov chain according to the distribution, the memoryless property of the channel, the chain rule and removing conditioning.

Again, we have

$$nR_1 = H\big(W_1\big) = I\big(W_1;Y_D^n\big)+H\big(W_1|Y_D^n\big) \quad (30)$$
$$\le I\big(W_1;Y_D^n\big)+n\epsilon_1 \quad (31)$$
$$\le I\big(X_1^n;Y_D^n,X_2^n\big)+n\epsilon_1 \quad (32)$$
$$= \sum_{t=1}^{n} I(X_1^n;Y_{D,t}|Y_D^{t-1},X_2^n)+n\epsilon_1 \quad (33)$$
$$= \sum_{t=1}^{n} H(Y_{D,t}|Y_D^{t-1},X_2^n) \quad (34)$$
$$-H(Y_{D,t}|Y_D^{t-1},X_2^n,X_1^n)+n\epsilon_1$$
$$\le \sum_{t=1}^{n} H(Y_{D,t}|Y_D^{t-1},X_{D2}^n,X_{R2}^n) \quad (35)$$
$$-H(Y_{D,t}|Y_D^{t-1},X_{D1}^n,X_{R1}^n,X_{D2}^n,X_{R2}^n,X_{R,t})+n\epsilon_1$$
$$\le \sum_{t=1}^{n} H(Y_{D,t}|X_{D2,t}) \quad (36)$$
$$-H(Y_{D,t}|X_{D2,t},X_{D1,t},X_{R,t})+n\epsilon_1$$
$$= \sum_{t=1}^{n} I(X_{D1,t},X_{R,t};Y_{D,t}|X_{D2,t})+n\epsilon_1 \quad (37)$$

Since,

$$nR_1 \le \sum_{t=1}^{n} I(X_{D1,t},X_{R,t};Y_{D,t}|X_{D2,t})+n\epsilon_1 \quad (38)$$
$$\Longrightarrow \quad R_1 \le I(X_{D1},X_R;Y_D|X_{D2}) \quad (39)$$

Similarly,

$$\Longrightarrow R_2 \le I(X_{D2},X_R;Y_D|X_{D1}) \quad (40)$$

To bound the sum rate, we have

$$n\big(R_1+R_2\big) = H\big(W_1,W_2\big) \quad (41)$$
$$\le I\big(W_1,W_2;Y_D^n\big)+n\epsilon_{12} \quad (42)$$
$$\le I\big(X_1^n,X_2^n;Y_D^n,Y_R^n\big)+n\epsilon_{12} \quad (43)$$
$$= I\big(X_1^n,X_2^n;Y_R^n\big)+I\big(X_1^n,X_2^n;Y_D^n|Y_R^n\big)+n\epsilon_{12} \quad (44)$$
$$= I_3+I_4+n\epsilon_{12} \quad (45)$$

where

$$I_3 = I(X_1^n,X_2^n;Y_R^n) \quad (46)$$
$$= \sum_{t=1}^{n} I(X_1^n,X_2^n;Y_{R,t}|Y_R^{t-1}) = \cdots \quad (47)$$

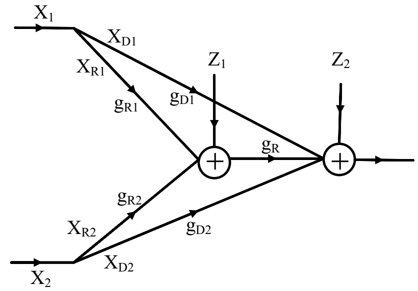

Fig. 3: A 2- source Gaussian multiple access relay channel with orthogonal components

$$= \sum_{t=1}^{n} I(X_{R1,t},X_{R2,t};Y_{R,t}|X_{R,t}) \quad (48)$$

and

$$I_4 = I(X_1^n,X_2^n;Y_D^n|Y_R^n) \quad (49)$$
$$= \sum_{t=1}^{n} I(X_1^n,X_2^n;Y_{D,t}|Y_D^{t-1},Y_R^n) = \cdots \quad (50)$$
$$= \sum_{t=1}^{n} I(X_{D1,t},X_{D2,t};Y_{D,t}|X_{R,t}) \quad (51)$$

Therefore,

$$n(R_1+R_2) \le \sum_{t=1}^{n} I(X_{R1,t},X_{R2,t};Y_{R,t}|X_{R,t}) \quad (52)$$
$$+\sum_{t=1}^{n} I(X_{D1,t},X_{D2,t};Y_{D,t}|X_{R,t})+n\epsilon_{12}$$
$$\Rightarrow R_1+R_2 \le I(X_{R1},X_{R2};Y_R|X_R)+I(X_{D1},X_{D2};Y_D|X_R)$$

Again, we have in a similar manner

$$n(R_1+R_2) \le \sum_{t=1}^{n} I(X_{D1,t},X_{D2,t},X_{R,t};Y_{D,t})+n\epsilon_{12} \quad (53)$$
$$\Longrightarrow \quad R_1+R_2 \le I(X_{D1},X_{D2},X_R;Y_D)$$

B. Proof of Theorem 2

The two-source Gaussian multiple access relay channel with orthogonal components is shown in Fig. 3 Achievability is established using the block Markov scheme just as the proof of achievability of theorem 1, where we let $X_{Dk} \sim N(0,\alpha_k P_k)$ and $X_{Rk} \sim N(0,\bar{\alpha}_k P_k)$ $(k=1,2)$, be independent and $X_R \sim N(0,P_R)$, is jointly Gaussian with X_{Dk} and X_{Rk} with correlation coefficients $\rho_{Dk} = \frac{E(X_R X_{Dk})}{\sqrt{E(X_R^2)E(X_{Dk}^2)}}$ and $\rho_{Rk} = \frac{E(X_R X_{Rk})}{\sqrt{E(X_R^2)E(X_{Rk}^2)}}$, respectively. The message is split into two parts, X_{Dk} and X_{Rk}; X_{Dk} is sent directly to the receiver and X_{Rk} is sent to the relay and decoded by it and sent cooperatively with the senders to receiver to remove the uncertainty of the receiver about the previous message.

To prove the converse, first note that from theorem 1, given any sequence of $((2^{nR_1},2^{nR_2}),n)$ code with $P_e^{(n)} \to 0$, the set (6) is capacity region $\mathcal{C}_{MARCO}$ of two-source multiple access relay channels with orthogonal components for some joint probability distribution

$$P(x_{R1},x_{R2},x_{D1},x_{D2},x_R) \quad (54)$$
$$= P(x_R)P(x_{R1}|x_R)P(x_{D1}|x_R)P(x_{R2}|x_R)P(x_{D2}|x_R)$$

The outputs of channel are

$$Y_R = g_{R1}X_{R1}+g_{R2}X_{R2}+Z_1 \quad (55)$$
$$Y_D = g_{D1}X_{D1}+g_{D2}X_{D2}+g_R X_R+Z_2 \quad (56)$$

We can now bound the rate R_{R1} as

$$I\big(X_{R1};Y_R|X_{R2},X_R\big) \quad (57)$$
$$= h(Y_R|X_{R2},X_R)-h(Y_R|X_{R2},X_{R1},X_R) \quad (58)$$

$$\leq h(g_{R1}X_{R1}+Z_1|X_R)-h(Z_1) \quad (59)$$

$$\leq \frac{1}{2}\log 2\pi e[E(Var(g_{R1}X_{R1}+Z_1|X_R))]-\frac{1}{2}\log 2\pi e N_1$$

Hence, we have

$$R_{R1}\leq \tfrac{1}{2}\log 2\pi e[N_1+g_{R1}^2\big(\tfrac{E(X_R^2)E(X_{R1}^2)-E^2(X_RX_{R1})}{E(X_R^2)}\big)]$$

$$-\frac{1}{2}\log 2\pi e N_1 \quad (60)$$

$$\leq \tfrac{1}{2}\log\big(1+\tfrac{g_{R1}^2\bar{\alpha}_1(1-\rho_{R1}^2)P_1}{N_1}\big) \quad (61)$$

$$= C\big((1-\rho_{R1}^2)S_{R1}\big) \quad (62)$$

Similarly, we have

$$R_{R2}\leq C\big((1-\rho_{R2}^2)S_{R2}\big)$$

To bound the sum rate, we have

$$I\big(X_{R1},X_{R2};Y_R|X_R\big) \quad (63)$$

$$= h(Y_R|X_R)-h(Y_R|X_{R2},X_{R1},X_R)=\cdots \quad (64)$$

$$= \tfrac{1}{2}\log\big(1+\tfrac{g_{R1}^2\bar{\alpha}_1(1-\rho_{R1}^2)P_1+g_{R2}^2\bar{\alpha}_2(1-\rho_{R2}^2)P_2}{N_1}\big) \quad (65)$$

Therefore,

$$R_{R1}+R_{R2}\leq C\big((1-\rho_{R1}^2)S_{R1}+(1-\rho_{R2}^2)S_{R2}\big)$$

Now, we want to bound the rate R_{D1} as

$$I\big(X_{D1};Y_D|X_{D2},X_R\big) \quad (66)$$

$$= h(Y_D|X_{D2},X_R)-h(Y_D|X_{D2},X_{D1},X_R)=\cdots \quad (67)$$

$$\leq \tfrac{1}{2}\log\big(1+\tfrac{g_{D1}^2\alpha_1(1-\rho_{D1}^2)P_1}{N_2}\big) \quad (68)$$

$$\Longrightarrow R_{D1}\leq C\big((1-\rho_{D1}^2)S_{D1}\big) \quad (69)$$

Similarly, we have

$$R_{D2}\leq C\big((1-\rho_{D2}^2)S_{D2}\big) \quad (70)$$

To bound the sum rate, we have similarly

$$R_{D1}+R_{D2}\leq C\big((1-\rho_{D1}^2)S_{D1}+(1-\rho_{D2}^2)S_{D2}\big)$$

and again

$$I\big(X_{D1},X_R;Y_D|X_{D2}\big) \quad (71)$$

$$= h(Y_D|X_{D2})-h(Y_D|X_{D2},X_{D1},X_R) \quad (72)$$

$$\leq h(g_{D1}X_{D1}+g_RX_R+Z_2|X_{D2})-h(Z_2) \quad (73)$$

$$\leq \tfrac{1}{2}\log\big(1+\tfrac{g_{D1}^2\alpha_1P_1+g_R^2P_R+2\rho_{D1}\sqrt{g_{D1}^2\alpha_1P_1P_R}}{N_2}\big)$$

$$\Longrightarrow R_1\leq C\big(S_{D1}+S_R+2\rho_{D1}\sqrt{S_{D1}S_R}\big) \quad (74)$$

similarly, we have

$$R_2\leq C\big(S_{D2}+S_R+2\rho_{D2}\sqrt{S_{D2}S_R}\big) \quad (75)$$

and for the sum rate

$$R_1+R_2\leq \quad (76)$$

$$C\big(S_{D1}+S_{D2}+S_R+2\rho_{D1}\sqrt{S_{D1}S_R}+2\rho_{D2}\sqrt{S_{D2}S_R}\big)$$

IV. Simulation

In this section with the help of computing, we show that the capacity region for a 2-source Gaussian multiple access relay channel with orthogonal components is larger than the achievable rate region of MARC with feedback. We fix SNR=0 dB for all channels and compute the capacity region of our model, multiple access channel and an inner bound for multiple access relay channels with relay-source feedback [14]. The results are shown in Fig. 4.

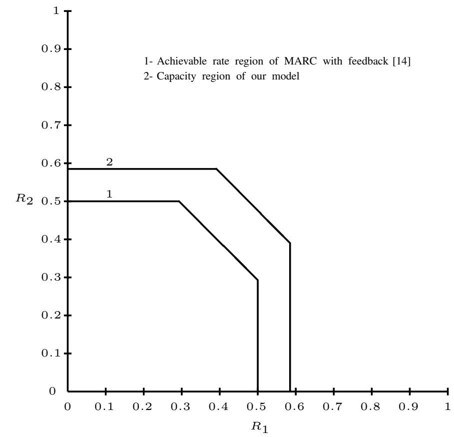

Fig. 4: Capacity Region for a 2-source Gaussian multiple access relay channel with orthogonal components

V. Conclusion

We have established capacity region for the multiple access relay channel with orthogonal components from the senders to the relay receiver and from the senders and relay to the receiver and extended the results to the Gaussian case. For the class of multiple access relay channels with orthogonal components discussed here, the optimal strategy is to split the messages into two parts, one is decoded by the relay and sent cooperatively with the senders to the receiver and the other is sent directly to the receiver.

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