APPLYING DYNAMIC RELAXATION METHOD IN BENDING ANALAYSIS OF MULTILAYER COMPOSITE PLATES WITH DIFFERENT GEOMETRICAL CIRCULAR SHAPES

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ABSTRACT

In this paper by applying a numerical method called "***Dynamic Relaxation***", the bending analysis of multi-layer composite plates with different circular shapes is studied. To illustrate the performance of this method, some multi-layer composite plates with various geometrical shapes are analyzed. These various geometrical shapes include circular plates with special geometries such as including a hole or variable sections. Moreover, to demonstrate the accuracy level of this method, its results are compared with the other numerical methods such as Finite Element or analytical ones by considering the both cases of small and large deflections. The analytical results of this study show that the DR method has a high ability in bending analysis of multi-layer composite plates with complicated shapes (including discontinuities & variable sections) and in some cases, it is the **only** possible method of analysis.

Keywords: Composite Plates, Multi-layer, Dynamic Relaxation (DR) Method, Small Deflection, Large Deflection, Discontinuity, Variable Section.

1. INTRODUCTION

Dynamic Relaxation Method (DR) is one of the ways of plates' analysis. Despite different methods of analyzing plates and shells, applying DR method make it possible to analyze plates with different complicated geometrical shapes and physical properties.

In recent years, applying of multi-layer composite plates in structure building has increased enormously. These plates, because of their low weight and high stiffness, play a special role in many industries such as Aviation Manufacturing, Rocket Building, Ship Building, Automotive Manufacturing and Construction. To use these multi-layer composite plates more efficiently, more analytical complicated methods should be applied to predict the elastic treatment of these structures against external loads, precisely. To this extent many researches on multi-layer plates (special thin plates) have been done to analyze nonlinear treatment of plates and their large deflections. Although Finite Element Numerical Analysis Methods are appeared, the analysis of large deflections in plates with complicated geometrical specifications is so difficult. Despite many applications of composite multi-layer plates, the complication of analyzing their structures makes it difficult to use them efficiently. Many types of these plates have different geometrical shapes including holes, cuts, variable sections, etc, but the lack of an appropriate analytical model to analyze their complicate treatment, make it difficult to apply them in engineering structures efficiently.

In this study, the multi-layer composite plates with different geometrical shapes are analyzed. The DR method is combined with the finite difference technique. So by deriving constitutive and compatibility equations in each node, the unknown forces and displacements are calculated. This job is done in both small and large deflections. Also the constitutive and compatibility equations are solved by applying DR method. This method provides the analysis possibility to composite multi-layer plates with different geometrical shapes and variable sections.

2. MODELING BY DYNAMIC RELAXATION METHOD AND ITS VALIDATION

Dynamic Relaxation Method is an explicit technique to solve simultaneous equations system. In this method, the solution is obtained by converting the static problem to a dynamic one. It should be mentioned that a system's response to a selected load include two transient and steady states responses. Moreover, each system has a damping ratio. Therefore, after duration of time, the transient response tends to zero. This time duration depends on the damp ratio. To this extent, if a static load exert to a dynamic system, the steady state solution would be obtained after time duration which makes the transient response equal to zero. DR method obeys the same treatment. In this method, the statically solution of a structural system is obtained by its dynamical response. Obviously, the nature of this method is dynamical. Generally, the Dynamic Relaxation Method can be figured out both physically and mathematically.

2.1. SIMPLIFIED DR METHOD'S STEPS

By applying Central Finite Difference Method, the DR Method's Calculation Steps are provided. It should be mentioned that many different factors are common to use to find the divergence of the response and terminating the steps. In this regard, the following factors can be applied:

1. Number of Iterations (N_{max}),
2. Zero amount of residual force by assuming an error amount e_R,
3. Zero amount of system's kinetic energy by assuming an error amount e_k.

The simplified steps of DR Method are as follows:

1. Determining the N_{max}, e_R and e_k factors by assuming n=0 and $\{\dot{D}\}^{\frac{1}{2}}$ =0.
2. Guess or calculate $\{D\}^0$ Vector,
3. Residual force is calculated according to equation (1),
4. If $\left\| R^n \right\| \le e_R$, then the process is terminated, else it will be continued,
5. The mass and attenuation matrices are calculated,
6. $\{\dot{D}\}^{n+\frac{1}{2}}$ is calculated by equation (2),
7. If $\sum_{j=1}^{q} \left(\dot{D}_j^{n+\frac{1}{2}} \right)^2 \le e_k$, then the process is terminated, else it will be continued,
8. $\{D\}^{n+1}$ vector is calculated from equation (3),
9. Boundary Conditions would be exerted on the equation,
10. If $n>N_{max}$, the process is terminated, else, n=n+1 and the analysis process comes back to step 3.

$$\{R\}^n = [M]\{\ddot{D}\}^n + [C]\{\dot{D}\}^n \quad (1)$$

$$\{\dot{D}\}^{n+\frac{1}{2}} = \frac{2-\tau^n c}{2+\tau^n c}\{\dot{D}\}^{n-\frac{1}{2}} + \frac{2\tau^n}{2+\tau^n c}[M]^{-1}\{R\}^n \quad (2)$$

$$\{D\}^{n+1} = \{D\}^n + \tau^{n+1}\{\dot{D}\}^{n+\frac{1}{2}} \quad (3)$$

By applying strain and stress equations in polar and Cartesian coordinates for small and large deflections, exerting boundary conditions, and displacement and force compatibility conditions, and according to Figure No.1, the following equations are obtained for both Small and Large Deflections:

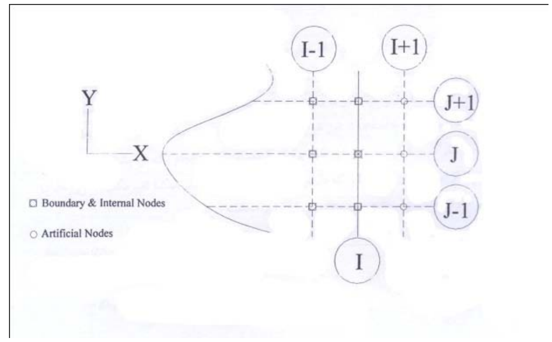

Figure No.1. Modeling for Simple Support

2.1.1. SMALL DEFLECTIONS

The following equation is derived after making assumptions of small deflections in plates:

$$\delta w_{I+1,J} = -\delta w_{I-1,J} + \frac{D_{16}}{D_{11}}\frac{\Delta x}{2\Delta y}\left(\delta w_{I+1,J+1} - \delta w_{I+1,J-1} - \delta w_{I-1,J+1} + \delta w_{I-1,J-1}\right) \quad (4)$$

2.1.2. LARGE DEFLECTIONS

In this case, the middle plate has a strain, so by making the assumptions of large deflections in plates, the following equations are derived:

$$a\left(\delta w_{I+1,J}\right)^2 + b\left(\delta w_{I+1,J}\right) + c = 0 \quad (5)$$

The constants in these equations are defined as follows:

$$a = B_{11}\left[\frac{1}{8\Delta x^2}\right]$$

$$b = B_{11}\left[\frac{1}{2}\left(\frac{\delta w_{I-1,J}}{2\Delta x^2}\right) + \frac{1}{2\Delta x}\frac{\partial w_{I,J}}{\partial x}\right] + B_{16}\left[\frac{1}{2\Delta x}\frac{\partial w_{I,J}}{\partial y} + \frac{1}{2\Delta x}\frac{\delta w_{I,J+1} - \delta w_{I,J-1}}{2\Delta y}\right] - D_{11}\frac{1}{\Delta x^2}$$

$$c = B_{11}\left[\frac{\delta u_{I,J} - \delta u_{I-1,J}}{\Delta x} + \frac{1}{2}\left(\frac{\delta w_{I-1,J}}{2\Delta x}\right)^2 - \frac{\partial w_{I,J}}{\partial x}\frac{\delta w_{I-1,J}}{2\Delta x}\right]$$
$$+ B_{12}\left[\frac{\delta v_{I,J} - \delta v_{I,J-1}}{\Delta y} + \frac{1}{2}\left(\frac{\delta w_{I,J+1} - \delta w_{I,J-1}}{2\Delta y}\right)^2 + \frac{\partial w_{I,J}}{\partial y}\frac{\delta w_{I,J+1} - \delta w_{I,J-1}}{2\Delta y}\right]$$
$$+ B_{16}\left[\frac{\delta u_{I,J} - \delta u_{I,J-1}}{\Delta y} + \frac{\delta v_{I,J} - \delta v_{I-1,J}}{\Delta x} - \frac{\partial w_{I,J}}{\partial y}\frac{\delta w_{I-1,J}}{2\Delta x}\right]$$
$$- D_{11}\frac{-2\delta w_{I,J} + \delta w_{I-1,J}}{\Delta x^2} - D_{12}\frac{\delta w_{I,J+1} - 2\delta w_{I,J} + \delta w_{I,J-1}}{\Delta y^2}$$
$$+ D_{16}\frac{\delta w_{I+1,J+1} - \delta w_{I+1,J-1} - \delta w_{I-1,J+1} + \delta w_{I-1,J-1}}{2\Delta x \Delta y}$$

2.2. VALIDATION OF DR METHOD

By applying DR Method on isotropic and composite multi-layer circular plates, the accuracy of Dynamic Relaxation is validated. In the following validations, "C" stands for fixed support and "S" stands for simple support. Moreover, for multi-layer composite circular plates, it is considered that the fiber properties in radius direction are constant. In all sections, the composite material is considered to be made of Kelvar/Epoxy with the following properties:

M_1	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
	54	5	2	0.4
Isotropic	E (GPa)	G (GPa)	ν	
	206.85	4.8265	0.3	

2.2.1. ISOTROPIC Circular SINGLE LAYER PLATE WITH CONSTANT SECTION

As the first step of DR Method validation, an isotropic circular single layer is considered in which r/h =20. “r” stands for radius and “h” stands for the thickness of the plate. “q” is the uniform load distribution on the plate. The result of this analysis is compared with the analytical results obtained from Timoshenko [T1] for an isotropic circular single layer with a constant section area.

The results of this comparison are shown in Table 1 for the amounts of $\frac{w_{max}}{h}$. S.D.A. and L.D.A. Stand for Small Deflections and Large Deflections, respectively.

Laminate	B.C.	$\bar{q}$	S.D.A.		L.D.A.	
			Timoshenko	*DR*	Timoshenko	*DR*
Isotropic Single Layer	*C*	*100*	*0.02125*	*0.0243*	*0.0208795*	*0.0239872*
	S		*0.0875*	*0.0823*	*0.085125*	*0.080132*

Table1. Comparing Results of DR Method with Timoshenko for Isotropic Circular plates

2.2.2. COMPOSITE CIRCULAR MULTILAYER PLATE WITH CONSTANT SECTION

As it is shown in Figure 2 and to make another validation for DR Method, the results of a considered three layers orthotropic circular plate (Figure 1) is compared with the results of Finite Element Method obtained in “Salehi” ’s paper [S1].

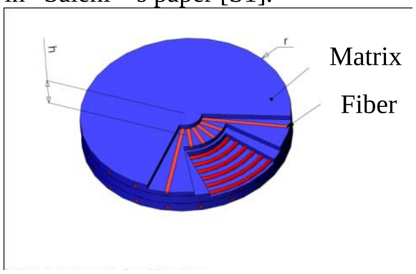

Figure No.2. General view of analyzed plate

The dimensions of the plate are. The results of this comparison are shown in Table 2.

Laminate	B.C.	$\bar{q}$	***S.D.A.***		***L.D.A.***	
			ANSYS	DR	ANSYS	DR
M1(0/90/0)	*C*	*1*	*0.00055*	*0.00051*	*0.00053*	*0.00050*
	S		*0.0014064*	*0.0014100*	*0.0014060*	*0.00139*
M2(60/-30/60)	*C*	*1*	*0.00081*	*0.00082*	*0.00079*	*0.0008*
	S		*0.00029*	*0.00032*	*0.00031*	*0.0003*

Table2. The comparison of DR results with “Salehi” for 3-layer orthotropic circular plates

2.2.3. COMPARISON OF DR METHOD RESULTS WITH THE OTHER METHODS

As it is shown in sections 2.2.1. and 2.2.2., different plates were analyzed with DR method and their results were compared with the other validated methods. The comparisons show a compatible adaptation between DR and the validated ones. Hence, it can be concluded that it is a compatible numerical method to analyze the composite multi-layer plates with different shapes.

3. Discussing DR Method’s Numerical Results

In this part, some other composite multi-layer circular plates with different geometrical properties are analyzed. The properties of the analyzed orthotropic plates are defined as Table 3.

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
M_1	54	5	2	0.4
M_2	177	108	7.6	0.27
M_3	216	5	4.5	0.25
M_4	294	6.4	4.9	0.23

Table3. The properties of the analyzed orthotropic plates

3.1. COMPOSITE MULTILAYER CIRCULAR PLATES WITH VARIABLE CROSS SECTION

3.1.1. LINEAR VARIABLE CROSS SECTION

We consider a circular plate with linear variable cross section as shown in Figure 3.

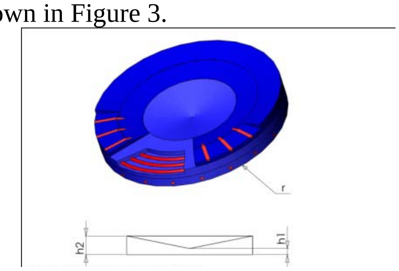

Figure No.3. Linear variable cross section composite circular plate

In this plate h is equal to $\frac{h_2}{h_1}=3$ and $\frac{w_{\max}}{h}$ is shown in Table 4.

Laminate	B.C.	$\bar{q}$	*S.D.A.*	*L.D.A.*
			DR	*DR*
M1(0/90/45)	*C*	*100*	*0.008765*	*0.008798*
	S		*0.06543*	*0.06545*
M2(-45/0/30/45)	*C*	*100*	*0.007345*	*0.01742*
	S		*0.05987*	*0.07857*
$(0_{M1}/0_{M2}/0_{M3})$	*C*	*100*	*0.00001*	*0.001032*
	S		*0.00341*	*0.007651*
$(0_{M1}/0_{M2}/0_{M3}/0_{M4})$	*C*	*100*	*0.000006*	*0.001531*
	S		*0.004391*	*0.007662*

Table4. Analysis of linear variable cross section circular plates with DR method

3.1.2. NON-LINEAR VARIABLE CROSS SECTION

The variable cross section is a function of e^r. The results of $\frac{w_{\max}}{h}$ are shown in Table 5.

Laminate	B.C.	$\bar{q}$	*S.D.A.*	*L.D.A.*
			DR	*DR*
M1(0/90/45)	*C*	*100*	*0.002922*	*0.002933*
	S		*0.02181*	*0.02182*
M2(-45/0/30/45)	*C*	*100*	*0.002448*	*0.005807*
	S		*0.01996*	*0.02619*
$(0_{M1}/0_{M2}/0_{M3})$	*C*	*100*	*0*	*0.000344*
	S		*0.001137*	*0.002550*
$(0_{M1}/0_{M2}/0_{M3}/0_{M4})$	*C*	*100*	*0*	*0.0005103*
	S		*0.001464*	*0.002554*

Table5. Analysis of non-linear variable cross section circular plates with DR method

3.2. COMPOSITE MULTILAYER ANNULAR PLATES

In this section, an annular multi-layer composite plate is analyzed. The geometrical property of the plate is assumed to be $\frac{a}{b}=3$.

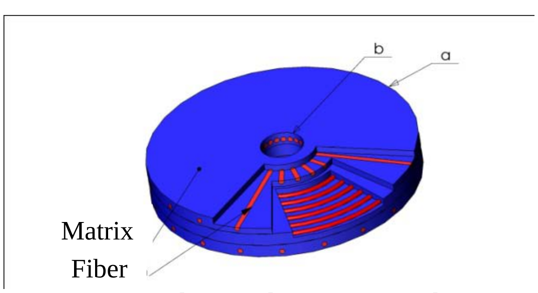

Figure No.4. Annular circular composite plates

The results shown in Table 6, figure out the obtained results for the maximum non-dimensional bending in an annular composite plate.

Laminate	B.C.	$\bar{q}$	*S.D.A.*	*L.D.A.*
			DR	*DR*
M1(0/90/45)	*C*	*100*	0.007382	0.007842
	S		0.024655	0.026561
M2(-45/0/30/45)	*C*	*100*	0.00536	0.006371
	S		0.024175	0.025552
$(0_{M1}/0_{M2}/0_{M3})$	*C*	*100*	0.04491	0.03941
	S		0.09713	0.07234
$(0_{M1}/0_{M2}/0_{M3}/0_{M4})$	*C*	*100*	0.06345	0.04139
	S		0.1471	0.01162

Table6. Analysis of composite annular plates with DR method

4. CONCLUSION AND DISCUSSION

As it was studied in this paper, DR is a very compatible method to analyze composite multi-layer plates with different complicated geometries. To this extent, applying this method is much easier than the other usual methods such as Finite Element and in some cases; it is the only possible method. DR method provides the new technique to analyze the plates which are not easy to solve by the other methods.

5. REFERENCES

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6. INDEX

Stress Elements σ_i, Stiffness Matrix Elements C_{ij}, Strain Elements ε_j, Displacements in X,Y&Z Directions: u, v, w, Stiffness Matrix Tension Elements A_{ij}, Stiffness Matrix Shearing Elements B_{ij}, Stiffness Matrix Torsion Elements D_{ij}, Nodal Velocity Vector in n Step $\{\dot{D}\}^n$, Nodal Acceleration Vector in n Step $\{\ddot{D}\}^n$, Residual Force in step n $\{R\}^n$, The Maximum Number of Iterations $N_{\max}$, The Acceptable Error for Kinetic Energy e_k, The Acceptable Error for Residual Force e_R.