A General Achievable Rate Region for Multiple-Access Relay Channels and Some Certain Capacity Theorems

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***Abstract*—In this paper, we obtain a general achievable rate region and some certain capacity theorems for multiple-access relay channel (MARC), using decode and forward (DAF) strategy at the relay, superposition coding at the transmitters. Our general rate region (i) generalizes the achievability part of Slepian-Wolf multiple-access capacity theorem to the MARC, (ii) extends the Cover-El Gamal best achievable rate for the relay channel with DAF strategy to the MARC, (iii) gives the Kramer-Wijengaarden anticipated rate region for the MARC, (iv) meets max-flow min-cut upper bound and leads to the capacity regions of some important classes of the MARC.**

***Index Terms*—Multiple-access relay channel; decode and forward strategy; superposition coding; degraded relay channel; orthogonal components.**

I. Introduction

THE relay channel was first introduced by Van der Meulen [1]. In the Cover-El Gamal seminal paper [2], the relay channel has been studied carefully. In [3], [4], the known capacity theorems for the relay channel have been unified into one capacity theorem. Relaying has been proposed as a means to increase coverage area and transmission rate of wireless networks. Relay nodes in cooperation with the users, act as a distributed multi-antenna system. In [5], MARC is introduced, where some sources communicate with one single destination with the help of a relay node. In [6], [7], some capacity regions were determined for the MARC.

A. Our Motivation and Work

In the literature, we had a general achievable rate for the relay channel [2] and Slepian-Wolf multiple-access capacity theorem [8]; also, extension of the general best rate for the relay channel to a relay network [9], and capacities of special relay channels [2], [10], [11].

In view of the above previous work and motivations, in this paper, we obtain a general achievable rate region for the multiple-access relay channel that might be considered as (i) generalization of Slepian-Wolf multiple-access capacity theorem to the MARC, (ii) extension of the Cover-El Gamal best achievable rate for the relay to the MARC. Also, our achievable rate, (iii) includes the Kramer-Wijengaarden anticipated achievable rate region for the MARC and, (iv) meets max-flow min-cut upper bound and leads to the capacity region for some important classes of the MARC.

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B. Paper Organization

The rest of the paper is organized as follows: In section II, we have preliminaries and some definitions . In the section III, we introduce and prove the main theorem. The results of the main theorem are studied in section IV. Finally, we conclude the paper in section V.

II. Preliminaries

A. Notation

In this paper, we use the following notations: random variables (r.v.) are denoted by uppercase letters and lowercase letters are used to show their realizations. The probability distribution function (p.d.f) of a r.v. X with alphabet set $\mathcal{X}$ is denoted by $P_X(x)$ where $x \in \mathcal{X}; P_{(X|Y)}(x|y)$ denotes the conditional p.d.f of X given Y, where $y \in \mathcal{Y}$. A sequence of r.v.'s $(X_{k,1}, \cdots, X_{k,n})$ with the same alphabet set $\mathcal{X}$ is denoted by X_k^n and its realization is denoted by $(x_{k,1}, \cdots, x_{k,n})$, where k is the index of the k^{th} sender. The set of all ϵ-typical n-sequences X^n with respect to the p.d.f $P_X(x)$, is denoted by $A_\epsilon^n(X)$.

B. Slepian-Wolf Multiple-Access Channel Capacity Region

For the discrete memoryless multiple-access channel (MAC) $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1, x_2), \mathcal{Y})$ with three independent uniformly distributed messages (Slepian and Wolf situation), the capacity region for this channel is given by the set of rate triples (R_0, R_1, R_2) such that

$$R_1 \leq I(X_1; Y|X_2, S) \tag{1a}$$

$$R_2 \leq I(X_2; Y|X_1, S) \tag{1b}$$

$$R_1 + R_2 \leq I(X_1, X_2; Y|S) \tag{1c}$$

$$R_0 + R_1 + R_2 \leq I(X_1, X_2; Y) \tag{1d}$$

for some $p(s)p(x_1|s)p(x_2|s)$.

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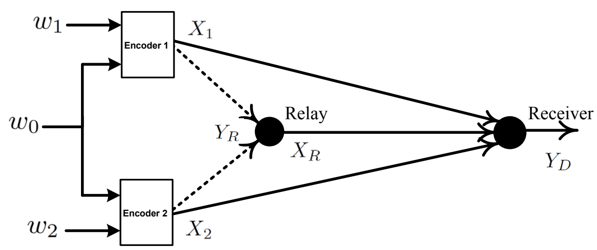

Fig. 1: A two-source multiple-access relay channel

C. *Multiple-Access Relay Channel*

In multiple-access relay channel, some sources communicate with one single destination with the help of a relay node. An example of such a channel model is the cooperative uplink of some mobile stations to the base station with the help of the relay in a cellular based mobile communication system. Fig. 1 shows a two-source discrete memoryless MARC which is defined by $(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_R, p(y_R, y_D|x_1, x_2, x_R), \mathcal{Y}_R \times \mathcal{Y}_D)$, where Y_D and Y_R are the channel outputs of the receiver and the relay, respectively; X_k, $(k = 1, 2)$ and X_R are the channel inputs which are sent by the transmitter and the relay, respectively.

D. *Kramer-Wijengaarden Multiple-Access Relay Channel Achievable Rate Region*

In [8], Kramer and Wijengaarden anticipated the following achievable rate region for multiple-access relay channel:

$$R_1 \leq \min\big(I(X_1; Y_R|X_2, X_R), I(X_1, X_R; Y_D|X_2)\big) \tag{2a}$$

$$R_2 \leq \min\big(I(X_2; Y_R|X_1, X_R), I(X_2, X_R; Y_D|X_1)\big) \tag{2b}$$

$$R_1 + R_2 \leq \min\big(I(X_1, X_2; Y_R|X_R), I(X_1, X_2, X_R; Y_D)\big) \tag{2c}$$

where

$$p(x_1, x_2, x_R) = p(x_1)p(x_2)p(x_R|x_1, x_2) \tag{3}$$

E. *Special Classes of Multiple-Access Relay Channel*

1) Multiple-Access Degraded Relay Channel (MADRC): In multiple-access degraded relay channel, all channels between senders and relay are better than direct channels, such that for MADRC, we have:

$$p(x_1, x_2, x_R, y_R, y_D) = \\ p(x_1, x_2, x_R)p(y_R|x_1, x_2, x_R)p(y_D|x_R, y_R) \tag{4}$$

2) Multiple-Access Reversely Degraded Relay Channel (MARDRC): In multiple-access reversely degraded relay channel, all channels between senders and receiver are better than channels between senders and relay, hence, we have:

$$p(x_1, x_2, x_R, y_R, y_D) = \\ p(x_1, x_2, x_R)p(y_D|x_1, x_2, x_R)p(y_R|y_D, x_R) \tag{5}$$

3) Multiple-Access Semi-Deterministc Relay Channel (MASDRC): If $Y_R = g(X_1, X_2, X_R)$ and the senders know the each other messages, then Y_R is known at the senders (assuming that the senders know the first symbol of X_R), hence, we have:

$$p(x_1, x_2, x_R, y_R, y_D) = \\ p(x_1, x_2, x_R, y_R)p(y_D|x_1, x_2, x_R, y_R) \tag{6}$$

4) Multiple-Access Relay Channel with Orthogonal Components (MARCO): $X_k\,(k = 1, 2)$ is divided to orthogonal components (X_{Rk}, X_{Dk}) and these components are sent from the senders to the relay (X_{Rk}) and from the senders and relay to the receiver (X_{Dk}, X_R). A discrete memoryless multiple-access relay channel is said to have orthogonal components if the channel input-output distribution can be expressed as

$$P(y_D, y_R, x_{R1}, x_{R2}, x_{D1}, x_{D2}, x_R) = P(y_R|x_{R1}, x_{R2}, x_R) \\ P(y_D|x_{D1}, x_{D2}, x_R)P(x_R)\prod_{k=1}^{2} P(x_{Rk}|x_R)P(x_{Dk}|x_R) \tag{7}$$

III. Main Theorem

Theorem. A general achievable rate region for two-source multiple-access relay channel is given by $\bigcup\{(R_0, R_1, R_2):$

$$R_1 \leq \min\Big(I(U_1, X_1; Y_D|U_0, U_2, X_2, X_R) + I(X_R; Y_D), \\ I(U_1; Y_R|U_0, U_2, X_R) + I(X_1; Y_D|U_0, U_1, U_2, X_2, X_R)\Big) \tag{8a}$$

$$R_2 \leq \min\Big(I(U_2, X_2; Y_D|U_0, U_1, X_1, X_R) + I(X_R; Y_D), \\ I(U_2; Y_R|U_0, U_1, X_R) + I(X_2; Y_D|U_0, U_1, U_2, X_1, X_R)\Big) \tag{8b}$$

$$R_1 + R_2 \leq \min\Big(I(U_1, U_2, X_1, X_2; Y_D|U_0, X_R) + I(X_R; Y_D), \\ I(U_1, U_2; Y_R|U_0, X_R) + I(X_1, X_2; Y_D|U_0, U_1, U_2, X_R)\Big) \tag{8c}$$

$$R_0 + R_1 + R_2 \leq \min\Big((I(X_1, X_2, X_R; Y_D), \\ I(U_0, U_1, U_2; Y_R|X_R) + I(X_1, X_2; Y_D|U_0, U_1, U_2, X_R)\Big)\} \tag{8d}$$

where the union is taken over all $p(x_1, x_2, x_R, u_0, u_1, u_2)$ for which

$$p(x_1, x_2, x_R, u_0, u_1, u_2) = \\ P(x_R)P(u_0)\prod_{k=1}^{2} P(u_k|u_0, x_R)P(x_k|u_0, u_k, x_R) \tag{9}$$

A. *Proof of the Theorem*

We split every message w_k into two parts w_k' and w_k'' with respective rates R_k' and R_k''. We consider B blocks, each of n symbols. We use superposition coding. In each block, $b = 1, 2, \cdots, B + 1$, we use the same set of codebooks:

$$\mathcal{C} = \{x_R^n(m), u_0^n(j), u_1(j, m, l_1), u_2(j, m, l_2), \\ x_1(j, m, l_1, q_1), x_2(j, m, l_2, q_2)\}$$

$$m = (m_1, m_2) \in [1 : 2^{nR_1'}] \times [1 : 2^{nR_2'}] \quad R = R_1' + R_2', \\ j \in [1 : 2^{nR_0}], l_k \in [1 : 2^{nR_k'}], \quad q_k \in [1 : 2^{nR_k''}],\ k = 1, 2.$$

Now, we proceed with proof of achievability using a random coding technique. *Random codebook generation:* First, fix a choice of $P(u_0, u_1, u_2, x_R, x_1, x_2) = P(x_R)P(u_0)\prod_{k=1}^{2} P(u_k|u_0, x_R)P(x_k|u_0, u_k, x_R)$

1) Generate 2^{nR} independent identically distributed n-sequence x_R^n, each drawn according to $P(x_R^n) = \prod_{t=1}^n P(x_{R,t})$ and index them as $x_R^n(m)$, $m \in [1:2^{nR}]$.
2) Generate 2^{nR_0} independent identically distributed n-sequence u_0^n, each drawn according to $P(u_0^n) = \prod_{t=1}^n P(u_{0,t})$. Index them as $u_0^n(j)$, $j \in [1:2^{nR_0}]$.
3) For each $\{x_R^n(m), u_0^n(j)\}$, generate $2^{nR'_k}$, $k = 1,2$, conditionally independent n-sequence u_k^n, each drawn according to $P(u_k^n|x_R^n(m), u_0^n(j)) = \prod_{t=1}^n P(u_{k,t}|x_{R,t}(m), u_{0,t}(j))$. Index them as $u_k^n(j,m,l_k)$, $l_k \in [1:2^{nR'_k}]$.
4) For each $\{x_R^n(m), u_0^n(j), u_k^n(j,m,l_k)\}$, generate $2^{nR''_k}$, $k = 1,2$, conditionally independent n-sequence x_k^n, each drawn according to $P(x_k^n|x_R^n(m), u_0^n(j), u_k^n(j,m,l_k)) = \prod_{t=1}^n P(x_{k,t}|x_{R,t}(m), u_{0,t}(j), u_{k,t}(j,m,l_k))$. Index them as $x_k^n(j,m,l_k,q_k)$, $q_k \in [1:2^{nR''_k}]$.
5) Partition the sequence (u_k^n, x_k^n), $k = 1,2$, into 2^{nR} bins, randomly.
6) Partition the sequence $(u_1^n, x_1^n, u_2^n, x_2^n)$ into 2^{nR} bins, randomly.
7) Partition the sequence $(u_0^n, u_1^n, x_1^n, u_2^n, x_2^n)$ into 2^{nR} bins, randomly.

Encoding: Encoding is performed in $B+1$ blocks. The encoding strategy is shown in table I.

1) *Source Terminals:* The message w_0 and w'_k are split into B equally sized blocks $w_{0,b}, w'_{k,b}$, $k = 1,2$, $b = 1,\dots,B$. Similarly, w''_k is split into B equally sized blocks $w''_{k,b}$, $k = 1,2$, $b = 1,\dots,B$. In block $b = 1,\cdots,B+1$, the k^{th} encoder sends $x_{k,b}^n(w_{0,b}, w'_{k,b-1}, w'_{k,b}, w''_{k,b})$ over the channel.

2) *Relay Terminal:* After the transmission of block b is completed, the relay has seen $y_{R,b}^n$. The relay tries to find $\widetilde{w}_{0,b}$, $\widetilde{w}'_{1,b}$ and $\widetilde{w}'_{2,b}$ such that

$$\Big(u_{1,b}^n(\widetilde{w}_{0,b}, \hat{w}'_{1,b-1}, \widetilde{w}'_{1,b}), u_{2,b}^n(\widetilde{w}_{0,b}, \hat{w}'_{2,b-1}, \widetilde{w}'_{2,b}), x_{R,b}^n(\hat{w}'_{1,b-1}, \hat{w}'_{2,b-1}), u_{0,b}^n(\widetilde{w}_{0,b}), y_{R,b}^n\Big) \in A_\epsilon^n(U_1, U_2, X_R, U_0, Y_R) \quad (10)$$

where $\hat{w}'_{1,b-1}$ and $\hat{w}'_{2,b-1}$ are the relay terminal's estimate of $w'_{1,b-1}$ and $w'_{2,b-1}$, respectively. If one or more such $w'_{1,b}$ and $w'_{2,b}$ are found, then the relay chooses one of them, and then transmits $x_{R,b+1}^n(\hat{w}'_{1,b}, \hat{w}'_{2,b})$ in block $b+1$.

3) *Sink Terminal:* After block b, the receiver has seen $y_{D,b-1}^n$ and $y_{D,b}^n$ and tries to find $\widetilde{w}_{0,b-1}$, $\widetilde{w}'_{1,b-1}$, $\widetilde{w}'_{2,b-1}$, $\widetilde{w}''_{1,b-1}$ and $\widetilde{w}''_{2,b-1}$ such that

$$\Big(x_{R,b}^n(\widetilde{w}'_{1,b-1}, \widetilde{w}'_{2,b-1}), y_{D,b}^n\Big) \in A_\epsilon^n(X_R, Y_D) \quad (11)$$

and

$$\Big(u_{1,b-1}^n(\widetilde{w}_{0,b-1}, \hat{w}'_{1,b-2}, \widetilde{w}'_{1,b-1}), u_{2,b-1}^n(\widetilde{w}_{0,b-1}, \hat{w}'_{2,b-1}, \widetilde{w}'_{2,b-1}), u_{0,b-1}^n(\widetilde{w}_{0,b-1}), x_{1,b-1}^n(\widetilde{w}_{0,b-1}, \hat{w}'_{1,b-2}, \widetilde{w}'_{1,b-1}, \widetilde{w}''_{1,b-1}), x_{2,b-1}^n(\widetilde{w}_{0,b-1}, \hat{w}'_{2,b-2}, \widetilde{w}'_{2,b-1}, \widetilde{w}''_{2,b-1}), x_{R,b-1}^n(\hat{w}'_{1,b-2}, \hat{w}'_{2,b-2}), y_{D,b-1}^n\Big) \in A_\epsilon^n(U_1, U_2, U_0, X_1, X_2, X_R, Y_D) \quad (12)$$

Decoding and error Analysis: It can be shown that the relay, after determining x_R^n from y_R^n, uses jointly decoding and can decode reliably if

$$R'_1 \le I(U_1; Y_R|X_R, U_2, U_0) \quad (13)$$
$$R'_2 \le I(U_2; Y_R|X_R, U_1, U_0) \quad (14)$$
$$R'_1 + R'_2 \le I(U_1, U_2; Y_R|X_R, U_0) \quad (15)$$
$$R_0 + R'_1 + R'_2 \le I(U_0, U_1, U_2; Y_R|X_R) \quad (16)$$

and the receiver decodes x_R^n, u_0^n and other messages with arbitrarily small probability of error if

$$R''_1 \le I(X_1; Y_D|U_0, U_1, U_2, X_2, X_R) \quad (17)$$
$$R''_2 \le I(X_2; Y_D|U_0, U_1, U_2, X_1, X_R) \quad (18)$$
$$R''_1 + R''_2 \le I(X_1, X_2; Y_D|U_0, U_1, U_2, X_R) \quad (19)$$
$$R'_1 + R''_1 - I(X_R; Y_D) \le I(U_1, X_1; Y_D|U_2, U_0, X_2, X_R) \quad (20)$$
$$R'_2 + R''_2 - I(X_R; Y_D) \le I(U_2, X_2; Y_D|U_1, U_0, X_1, X_R) \quad (21)$$
$$R'_1 + R''_1 + R'_2 + R''_2 - I(X_R; Y_D) \le I(U_1, U_2, X_1, X_2; Y_D|U_0, X_R) \quad (22)$$
$$R_0 + R'_1 + R''_1 + R'_2 + R''_2 - I(X_R; Y_D) \le I(U_0, U_1, U_2, X_1, X_2; Y_D|X_R) \quad (23)$$

Therefore, by fully considering (13)-(23), the theorem is proved.

IV. The Results of the Theorem

For the multiple-access relay channel, the achievable rate regions and for some special cases, the capacity regions have been found in the following:

A. Achievability Region of Slepian-Wolf Multiple-Access Channel

Suppose that X_k, $k = 1,2$, sees a source of rate R_k, and in addition, all X_k see a common source of rate R_0. All three sources are independent. To obtain the achievable rate region let $U_0 = S$ and $U_k = X_k$. With removing the relay, we obtain achievability of (R_0, R_1, R_2) according to the (1a)-(1d).

B. The anticipated Rate Region of Kramer-Wijengaarden Work

The Kramer-Wijengaarden anticipated achievable rate region for MARC in accordance with our definition, is multiple-access degraded relay channel which is discussed in the next section (*C*-1).

C. Capacity Region of Some Special Classes of Multiple-Access Relay Channel

1) The Capacity Region of Multiple-Access Degraded Relay Channel: With substitution $U_k = X_k$ or $U_k = f_k(X_k)$ (f_k is

TABLE I: Encoding Strategy

Block1	Block2	$\cdots$	Block B+1
$u_{0,1}^n(w_{0,1})$	$u_{0,2}^n(w_{0,2})$	$\cdots$	$u_{0,B+1}^n(1)$
$x_{R,1}^n(1,1)$	$x_{R,2}^n(w'_{1,1},w'_{2,1})$	$\cdots$	$x_{R,B+1}^n(w'_{1,B},w'_{2,B})$
$u_{1,1}^n(w_{0,1},1,w'_{1,1})$	$u_{1,2}^n(w_{0,2},w'_{1,1},w'_{1,2})$	$\cdots$	$u_{1,B+1}^n(1,w'_{1,B},1)$
$u_{2,1}^n(w_{0,1},1,w'_{2,1})$	$u_{2,2}^n(w_{0,2},w'_{2,1},w'_{2,2})$	$\cdots$	$u_{2,B+1}^n(1,w'_{2,B},1)$
$x_{1,1}^n(w_{0,1},1,w'_{1,1},w''_{1,1})$	$x_{1,2}^n(w_{0,2},w'_{1,1},w'_{1,2},w''_{1,2})$	$\cdots$	$x_{1,B+1}^n(1,w'_{1,B},1,1)$
$x_{2,1}^n(w_{0,1},1,w'_{2,1},w''_{2,1})$	$x_{2,2}^n(w_{0,2},w'_{2,1},w'_{2,2},w''_{2,2})$	$\cdots$	$x_{2,B+1}^n(1,w'_{2,B},1,1)$

reversible), $k=1,2$, and $U_0=\phi$ in (8a)-(8d) then MARC is an MADRC ($(X_1,X_2)\to(X_R,Y_R)\to Y_D$) and there exists $p(x_1,x_2,x_R,u_0,u_1,u_2)=p(x_1,x_2,x_R)$ such that for MADRC, we have:

$$p(x_1,x_2,x_R,y_R,y_D)= \qquad (24)$$
$$p(x_1,x_2,x_R)p(y_D|x_R,y_R)p(y_R|x_1,x_2,x_R)$$

$$R_1\le \min\big(I(X_1;Y_R|X_2,X_R),I(X_1,X_R;Y_D|X_2)\big) \qquad (25a)$$
$$R_2\le \min\big(I(X_2;Y_R|X_1,X_R),I(X_2,X_R;Y_D|X_1)\big) \qquad (25b)$$
$$R_1+R_2\le \min\big(I(X_1,X_2;Y_R|X_R),I(X_1,X_2,X_R;Y_D)\big) \qquad (25c)$$

It is shown in [7] that achievable rate in (25a)-(25c) meets its outer bound; therefore, the above achievable rate is the capacity region.

2) The Capacity Region of Multiple-Access Reversely Degraded Relay Channel: In multiple-access reversely degraded relay channel $U_k=X_R,\ k=1,2$, and $U_0=\phi$. We have $(X_1,X_2)\to(X_R,Y_D)\to Y_R$. Consequently, MARC is an MARDRC and there exists $p(x_1,x_2,x_R,u_0,u_1,u_2)=p(x_1,x_2,x_R)$ such that for MARDRC, we have:

$$p(x_1,x_2,x_R,y_R,y_D)= \qquad (26)$$
$$p(x_1,x_2,x_R)p(y_D|x_1,x_2,x_R)p(y_R|y_D,x_R)$$

It is easy to show that the capacity region for MARDRC is as follows

$$R_1\le I(X_1;Y_D|X_2,X_R) \qquad (27a)$$
$$R_2\le I(X_2;Y_D|,X_1,X_R) \qquad (27b)$$
$$R_1+R_2\le I(X_1,X_2;Y_D|X_R) \qquad (27c)$$

3) The Capacity Region of Multiple-Access Semi-Deterministic Relay Channel: If $Y_R=g(X_1,X_2,X_R)$ and the senders know the each other messages, then Y_R is known at the senders and according to the lemma in [3] and [4], $U_k,\ k=1,2$, is also a function of $X_k,\ k=1,2$, and X_R and we have $U_k=Y_R$, then there exists, $p(x_1,x_2,x_R,u_0,u_1,u_2)=p(x_1,x_2,x_R,y_R)$ such that for MASDRC, we have:

$$p(x_1,x_2,x_R,y_R,y_D)=p(x_1,x_2,x_R,y_R)p(y_D|x_1,x_2,x_R,y_R)$$

We obtain achievability of (R_1,R_2) as follows,

$$R_1\le I(X_1;Y_D|Y_R,X_2,X_R) \qquad (28a)$$
$$R_2\le I(X_2;Y_D|Y_R,X_1,X_R) \qquad (28b)$$
$$R_1+R_2\le \min\big(H(Y_R|X_R)+I(X_1,X_2;Y_D|Y_R,X_R)$$
$$I(Y_R,X_1,X_2;Y_D|X_R)+I(X_R;Y_D)\big) \qquad (28c)$$

It is easy to show that this rate is also an outer bound for MASDERC.

4) The Capacity Region of Multiple-Access Relay Channel with Orthogonal Components: In [6], the capacity region of MARCO was obtained. If $X_k=(X_{Rk},X_{Dk}),\ k=1,2$, and $U_k=X_{Rk},\ k=1,2$; therefore, the capacity region is obtained as following:

$$R_1\le \min\big(I(X_{D1},X_R;Y_D|X_{D2}), \qquad (29a)$$
$$I(X_{R1};Y_R|X_{R2},X_R)+I(X_{D1};Y_D|X_{D2},X_R)\big)$$
$$R_2\le \min\big(I(X_{D2},X_R;Y_D|X_{D1}), \qquad (29b)$$
$$I(X_{R2};Y_R|X_{R1},X_R)+I(X_{D2};Y_D|X_{D1},X_R)\big)$$
$$R_1+R_2\le \min\big(I(X_{D1},X_{D2},X_R;Y_D), \qquad (29c)$$
$$I(X_{R1},X_{R2};Y_R|X_R)+I(X_{D1},X_{D2};Y_D|X_R)\big)$$

where

$$P(x_{R1},x_{R2},x_{D1},x_{D2},x_R)$$
$$=P(x_R)\prod_{k=1}^{2}P(x_{Rk}|x_R)P(x_{Dk}|x_R) \qquad (30)$$

V. Conclusion

We obtain a general achievable rate region and some certain capacity theorems for the MARC. Our general rate region generalizes the achievability part of Slepian-Wolf multiple-access capacity theorem to the MARC, extends the Cover-El Gamal best achievable rate for the relay channel with DAF strategy to the MARC, gives the Kramer-Wijengaarden anticipated rate region for the MARC, meets max-flow min-cut upper bound and leads to the capacity regions of some important classes of the MARC such as MADRC, MARDRC, MASDRC and MARCO.

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