

# Application of time delay resonator to machine tools

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**Abstract** Undesirable vibration can negatively affect the performance of machineries. In case of a machine tool, it negatively affects machining accuracy and tool life. Performance may be improved at the design stage by increasing machine stiffness. However, this is difficult to accomplish with existing machines. This paper implements a delay resonator (DR) on model of a typical machine tool to minimize relative vibration between the cutting tool and workpiece during machining process. The DR is a tunable vibration absorber which converts a conventional passive absorber into a marginally stable resonator. This structure absorbs all the vibratory energy at its point of attachment. A strategy called “Stability charts” is used not only to resolve the stability question but also to find out gain and time delay of the absorption. Results show that relative motion between cutting tool and workpiece is reduced significantly.

**Keywords** Delay resonator · Machine tool · Cutting tool · Vibration · Stability charts · MATLAB Simulink

## 1 Introduction

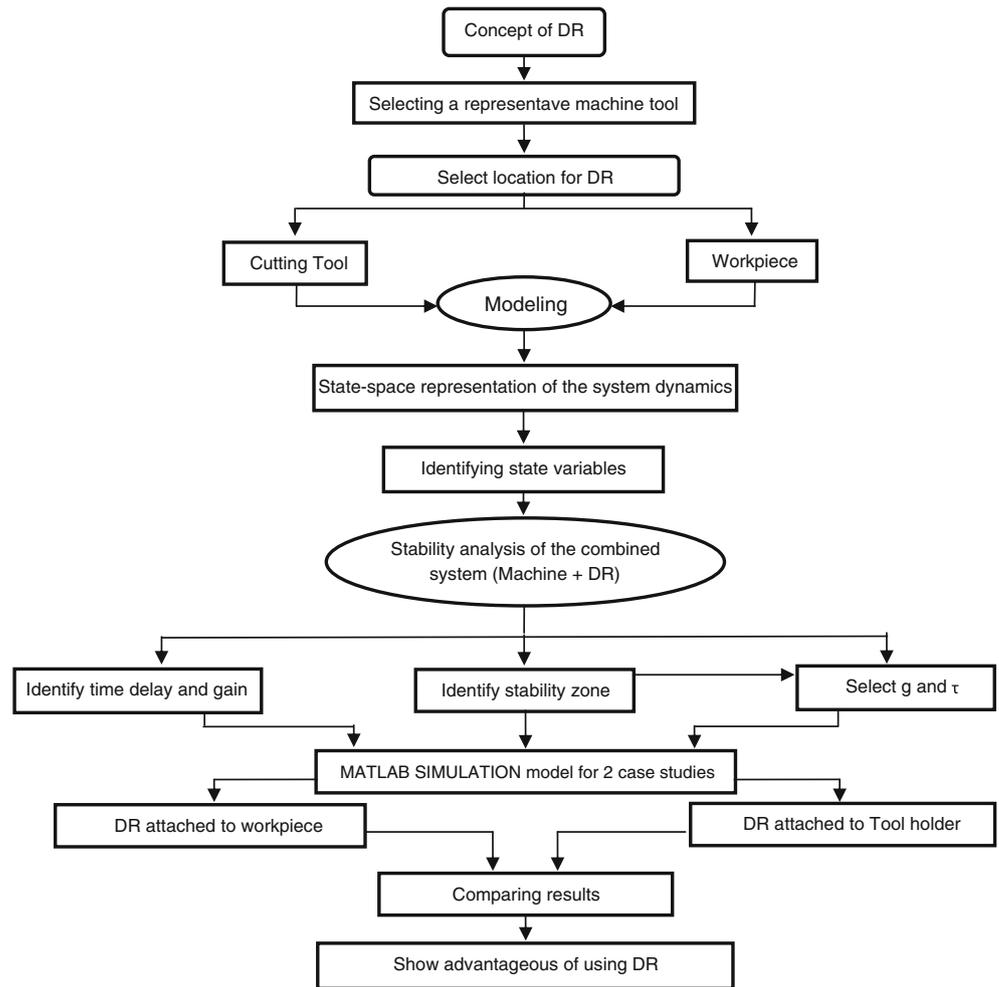
Today’s machines are required to take smaller foot print, weight less, more durable while having higher accuracy and operation speed. One way to achieve some of these requirements is by reducing undesirable vibrations. A great deal of engineering efforts is placed to minimize undesirable vibration

of machineries. There are many solutions that may be used such as changing electrical and mechanical stiffness. These parameters may be altered at the design stage for new machines that are not built yet. However, making any fundamental changes to existing machines is usually difficult to implement. Therefore, a method to improve the performance of an existing machine, with a given mechanical and electrical stiffness, is highly desirable. Among manufacturing equipment, machine tools and robots usually require higher accuracies. For example, in machining procedure, the cutting forces generated during the cutting process as well as external excitations will create undesirable vibratory motions of various structural components [1] that must be minimized. In particular, the relative motion between the cutting tool and the workpiece creates undulation on the machined surface, and hence, adversely affects the surface accuracy. Furthermore, if the amplitude of the vibration grows significantly, this undulation becomes the source of oscillatory force in the following pass, which again excites the cutting tool more and eventually leads to an unstable situation. This type of vibration is called “regenerative mechanism” and occurs due to the closed loop nature of machining operations [1]. One of the major concerns in designing a machine tool structure is, therefore, reducing the relative amplitude of vibration between the tool and the workpiece. If a machine tool is extremely rigid or the excitation frequency is below its first resonance frequency, the entire system will undergo a rigid body motion with no relative vibration between the tool and the workpiece. Thus, efforts are made to maximize the stiffness of a machine tool structure during design and construction. However, several functional requirements need to be satisfied within a limited space. The cost involved in building a very rigid system limits the achievable stiffness of the system, especially that of the cutting tool. Therefore, a certain amount of vibration is unavoidable during operation [2]. In the previous studies, optimization of passive vibration absorbers was effectively

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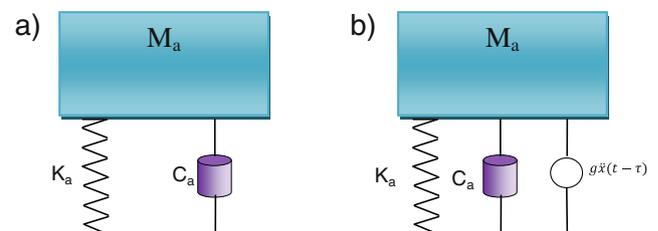
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**Fig. 1** A detailed flow chart depicting major steps carried out in this paper



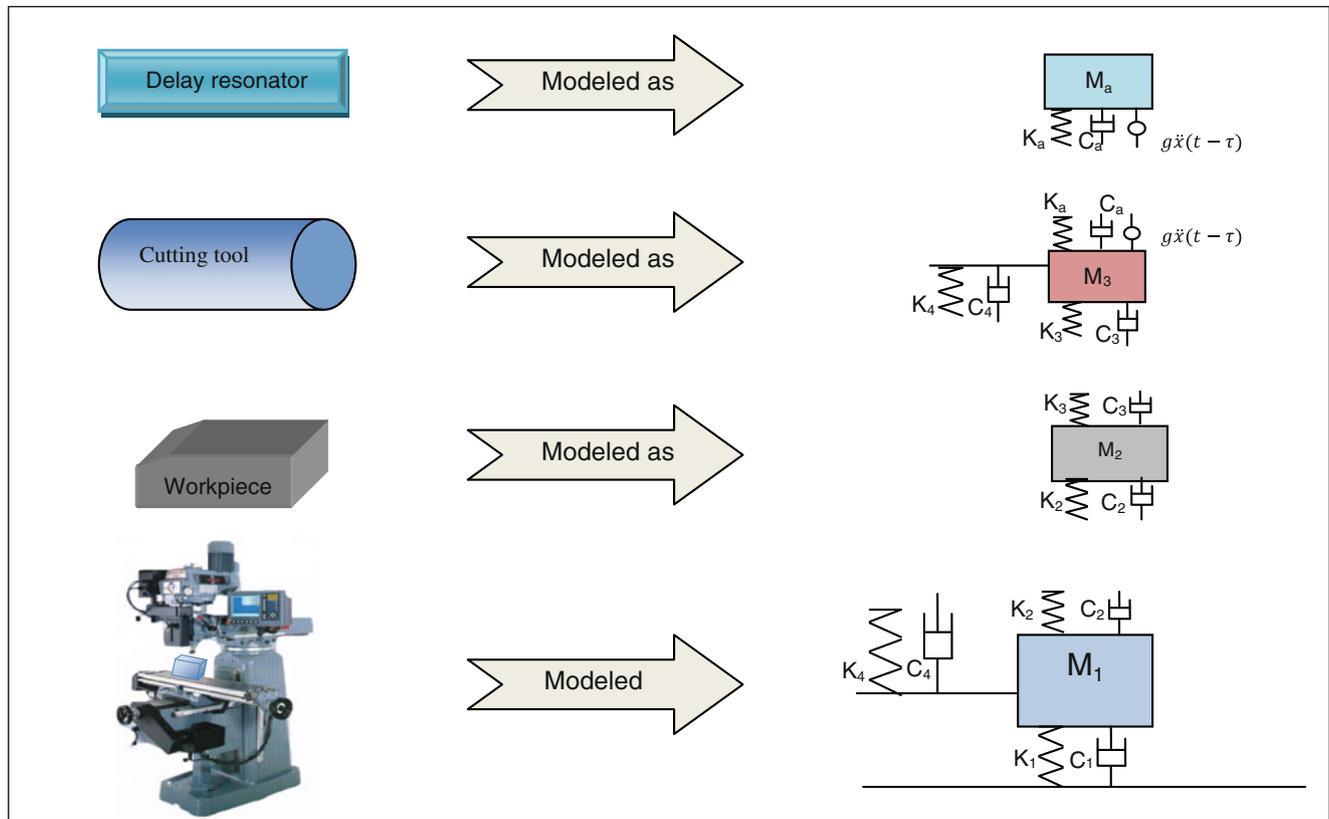
used to remove undesirable oscillations from mechanical structures by Y.C. Shin and K.W. Wang [2]. Other common passive approach, modification to the tool holder was considered by Rivin and Kang [3], and in other studies, adding a tuned mass vibration absorber was considered by Hopkins and Kosker [3]. In more recent studies, active vibration absorber was used by adding damping to a single axis of the structure. In this manner, Matsubara, Yamamoto, and Mizumoto used piezoelectric actuators to apply control moments to the boring bar. Their system uses time delay to phase shift an accelerometer signal and achieve narrowband velocity feedback control [3]. Subsequently, Tewani, Rouch, and Walcott mounted a piezoelectric reaction mass actuator in the bar itself and created what they termed an active dynamic absorber [3]. To eliminate undesirable torsional oscillations in rotating mechanical structures, an active vibration absorption device called centrifugal delayed resonator was introduced by Hosek et al. [4]. This device was forced to mimic an ideal real-time tunable absorber utilizing a control torque in the form of proportional angular position feedback with variable gain and time delay. In another study, a DR absorber was implemented on a flexible beam, and the dynamic features of

its structure were studied by Olgac and Jalili [5]. In particular, the stability features obtained through analytical and experimental studies were compared. Results obtained concur with the experimental findings better than the earlier dynamic models which use the finite difference method and ideal clamped-clamped BCs. In other researches, many experimental methods were developed for the optimization of the machine tool structural response. These approaches, however, were mainly dealt with the structural motion of a machine tool itself, not the relative vibration between a cutting tool and a workpiece. Since the cutting tool or workpiece often is the most flexible part



**Fig. 2** a Passive absorber. b DR absorber with acceleration feedback

**Table 1** Lumped spring-mass models

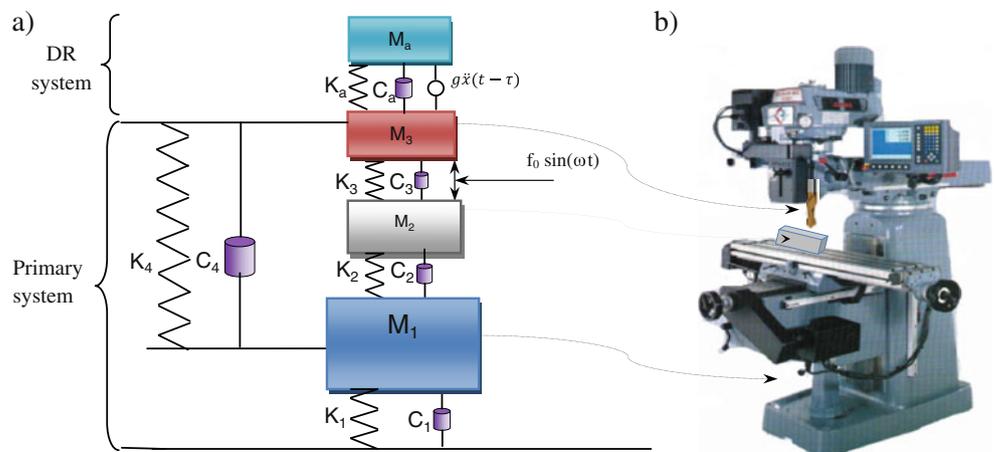


of an entire machine tool system, optimizing the rigidity of other structural components might have little effects on reducing the relative vibration at the cutting point [6, 7]. Robustness of the control strategy against fluctuations in the structural parameters of the controlled system was addressed by Martin Hosek and Olgac [8]. A new example was presented by Olgac and Sipahi for assessing the stability posture of a general class of linear time invariant–neutral time delayed systems. The ensuing method, which is named the direct method, offers several unique features: It returns the number of unstable characteristic roots of the system in

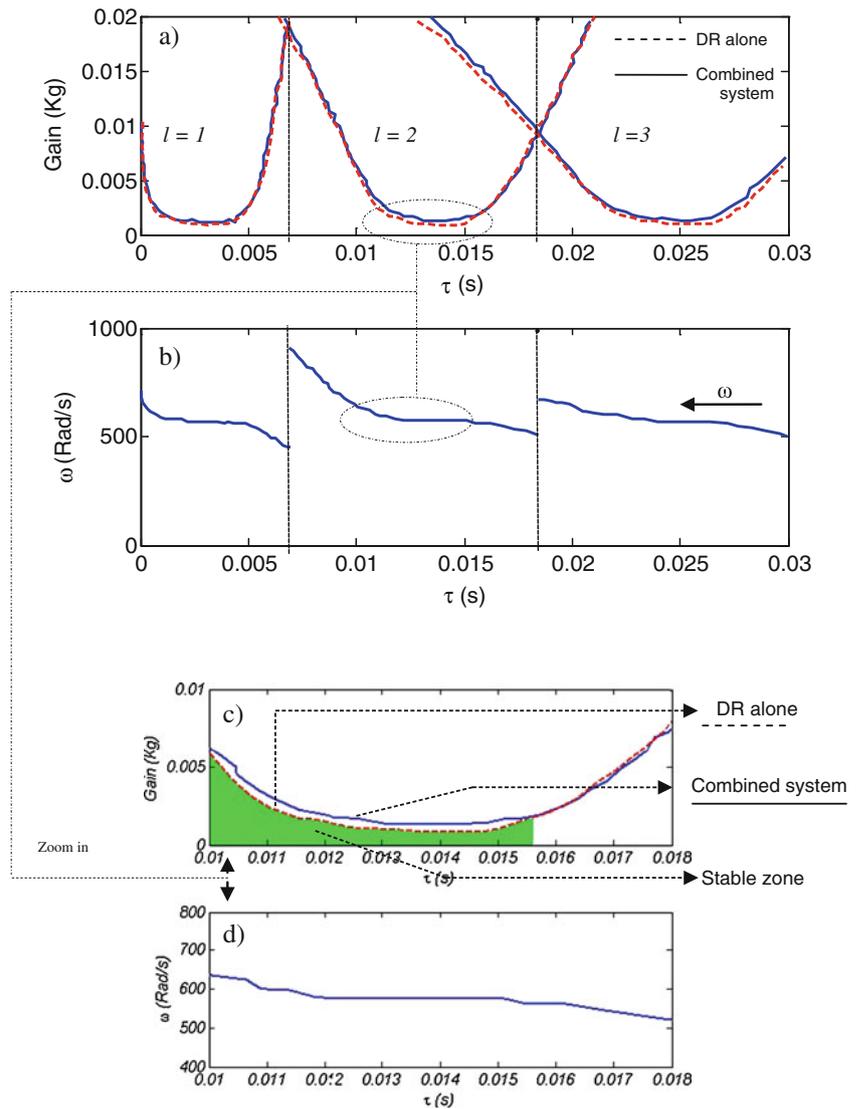
an explicit and non-sequentially evaluated function of time delay,  $\tau$ . Consequently, the direct method creates exclusively all possible stability intervals of  $\tau$  [9].

The DR vibration absorber has some attractive features in eliminating tonal vibrations from the system [10]. Some of them are ability of real-time tune, perfect tonal suppression, wide range of frequencies, simplicity of the control implementation, and robust design [11]. Additionally this single-degree-of-freedom absorber can also be tuned to handle multiple frequencies of vibration [12]. Z. H. Wang and H. Y. Hu presented a systematic

**Fig. 3** a Spring-mass model of the machine tool—DR attached to the  $m_3$ . b Machine tool, cutting tool, and workpiece



**Fig. 4** Stability charts. **a** Plot of gain versus time delay for combined system and DR alone. **b** Plot of excitation frequency versus time delay for DR alone. **c** A zoom of selected zone shown in **a**. **d** A zoom of selected zone shown in **b**



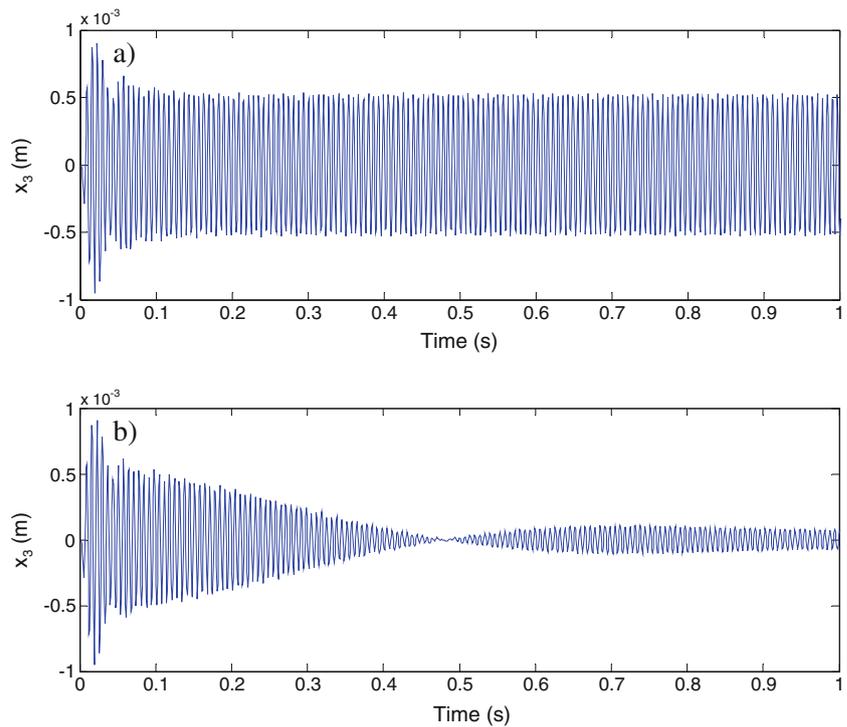
method of stability analysis for high-dimensional dynamic systems involving a time delay and some unknown parameters [13]. The term “unknown” means that the parameters are constants but yet to be determined. The analysis focuses on the stability switches of those systems with increase of the time delay from zero to infinity. On the basis of the generalized Sturm criterion, the parameter space of concern is divided into several regions determined by a discrimination sequence and the Routh–Hurwitz conditions. It is found that as the time delay increases, the system undergoes none, exactly one, or more than one stability switches when the parameters are chosen from different regions [13].

Takagi–Sugeno fuzzy model was extended by Chen-Yuan Chen to analyze the stability of interconnected systems with time delays in their subsystems [14]. They present a stability criterion in terms of Lyapunov’s theory for fuzzy interconnected models [14]. The stabilization problem was considered by F. H. Hsiao for a nonlinear multiple time-delay large-scale system [15]. They employed a neural-network (NN) model to approximate each subsystem of the large-scale system [15]. They established a linear differential inclusion state space representation for the dynamics of each NN model. Finally, a delay-dependent stability criterion was derived to guarantee the asymptotic stability of the nonlinear multiple time-delay large-scale systems.

**Table 2** System parameters value

$m_1$ (kg)	$m_2$	$m_3$	$m_4$	$k_1$ (N/m)	$k_2$	$k_3$	$k_4$	$k_a$	$C_1$ (Nm/s)	$C_2$	$C_3$	$C_4$	$C_a$
100	10	8	0.3	100e4	80e4	10e4	80e4	1e4	50	60	7	60	0.5

**Fig. 5** **a** Displacement of  $m_3$  while  $g=0$ . **b** Displacement of  $m_3$  while  $g \neq 0$



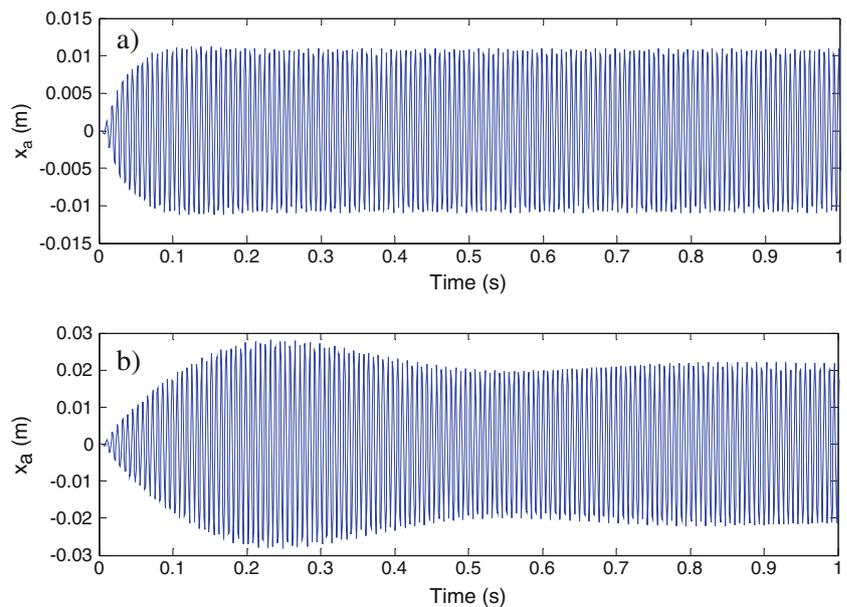
In this study, our goal is to demonstrate that the undesirable vibration of a machine tool with specific working frequency can be minimized by utilizing a DR and without altering the existing control system or mechanical structure of the machine. Therefore, a machine tool and a delay resonator are chosen to minimize the relative vibration between the cutting tool and workpiece. The DR is first attached to the cutting tool and next to the workpiece. In both cases, the relative vibration during

machining process are studied and reported in this paper. A detailed flow chart depicting major steps carried out in this paper is shown in Fig. 1.

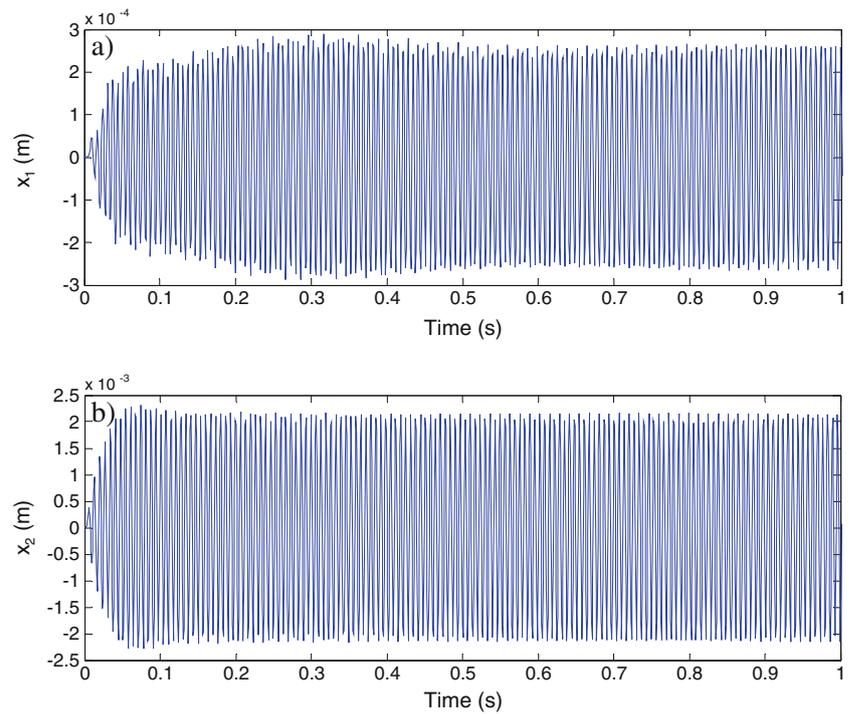
**2 The delay resonator concept**

A brief overview of DR is presented here. DR has an unconventional control logic which is implemented on a

**Fig. 6** **a** Displacement absorber while  $g=0$ . **b** Displacement of absorber while  $g \neq 0$



**Fig. 7** **a** Displacement of  $m_1$ , while  $g \neq 0$ . **b** Displacement of  $m_2$ , while  $g \neq 0$

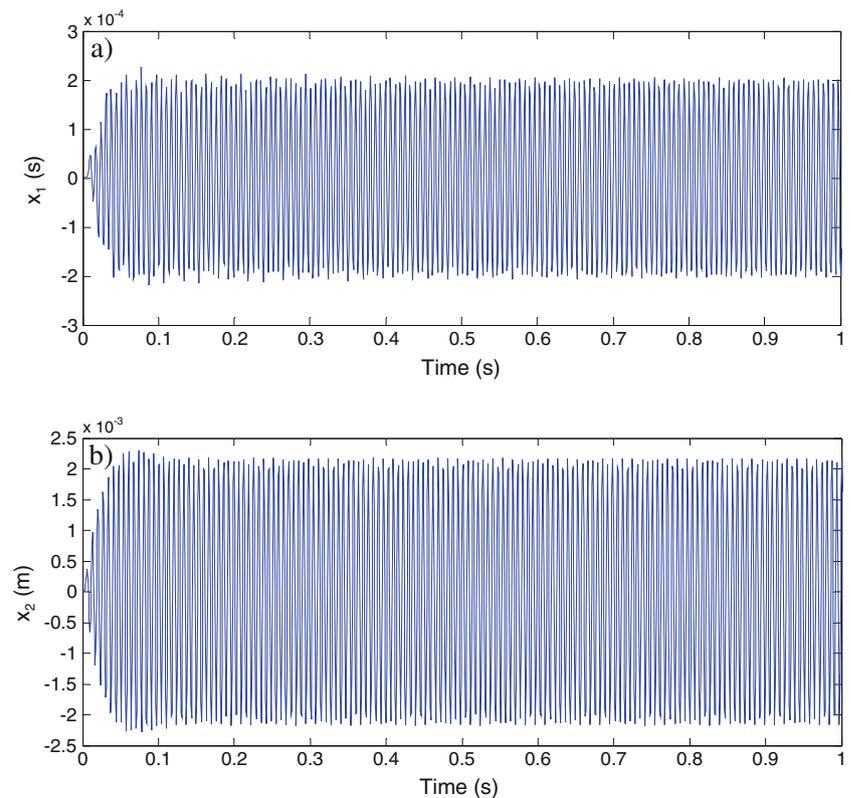


passive absorber. See Fig. 2a. It consists of a proportional position feedback with time delay. The position feedback can be based on absolute or relative displacements of the absorber. These displacement measurements are prohibitively difficult

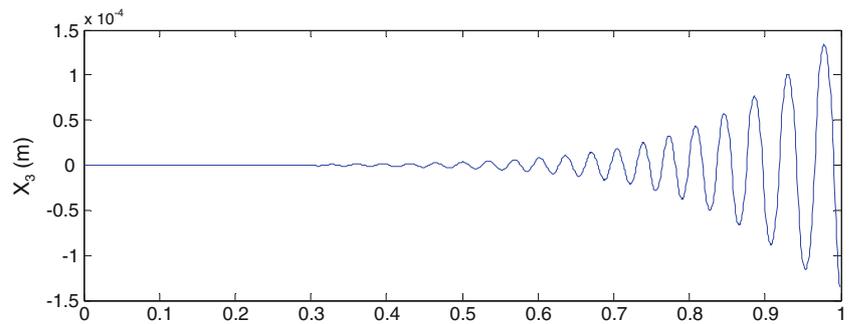
for high frequency-low amplitude applications. Acceleration feedback is much more feasible for such cases. See Fig. 2b.

Naturally, the effects of substituting displacement measurements with acceleration should be carefully analyzed.

**Fig. 8** **a** Displacement of  $m_1$ , while  $g=0$ . **b** Displacement of  $m_2$ , while  $g=0$



**Fig. 9** Control parameters are selected from unstable zone



Using this feedback control converts the dissipative passive absorber structure into a marginally stable one, i.e., a resonator with a specified resonance frequency,  $\omega_c$  [12].

Control goal is to place the dominant poles at  $\pm\omega_j$ , where  $j = \sqrt{-1}$ . Recommended force to achieve pole position is

$$g \times \ddot{x}_a(t - \tau) \tag{1}$$

where  $g$  is gain and  $\tau$  is time delay. The corresponding new system dynamics is

$$m_a \ddot{x}_a + c_a \dot{x}_a + k_a x_a - g \ddot{x}_a(t - \tau) = 0 \tag{2}$$

The Laplace domain representation leads to the transcendental characteristic equation

$$m_a s^2 + c_a s + k_a - g s^2 e^{-\tau s} = 0 \tag{3}$$

This equation possesses infinitely many finite roots for  $g \neq 0$  and  $\tau \neq 0$ . Their distribution can be sketched following the

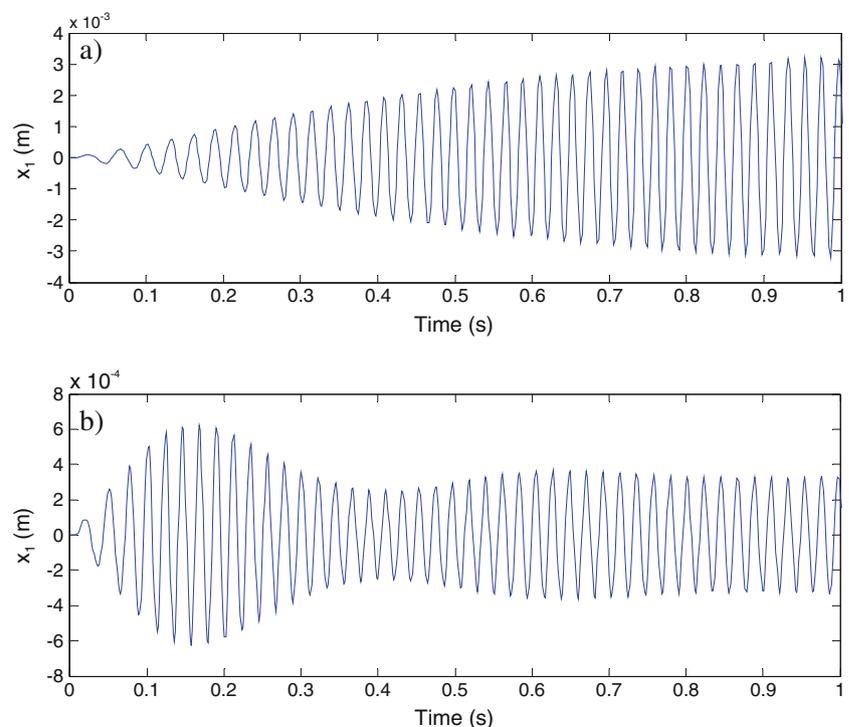
root locus analysis [13]. To achieve ideal resonator behavior, two dominant roots of Eq. 2 should be placed on the imaginary axis at the desired crossing frequency  $\omega_c$  while the others remain in the left half of the complex plane. Substituting  $S = \pm\omega_j$  into Eq. 3 and solving for the control parameters  $g_c$  and  $\tau_c$ , one obtains

$$g_c = \left(\frac{1}{\omega_c^2}\right) \sqrt{(c_a \omega_c)^2 + (m_a \omega_c^2 - k_a)^2} \tag{4}$$

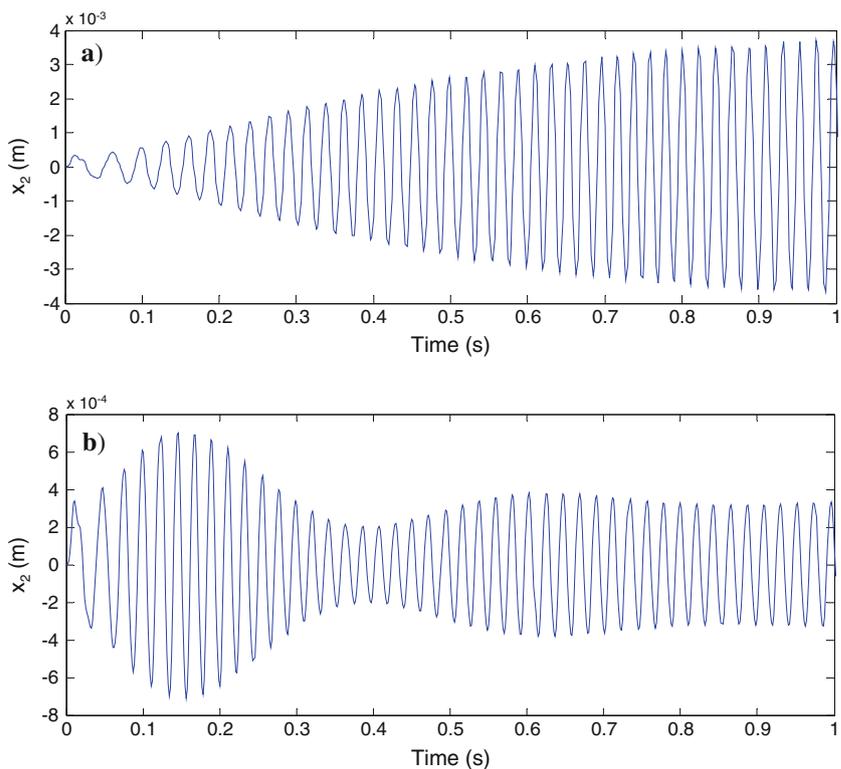
$$\tau_c = \left(\frac{1}{\omega_c}\right) \left\{ \tan^{-1} \left[ \frac{c_a \omega_c}{m_a \omega_c^2 - k_a} \right] + 2(L - 1)\pi \right\}, L = 1, 2, \dots \tag{5}$$

The variable parameter  $L$  refers to the branch of root loci that happens to cross the imaginary axis at  $\omega_c$ . Extensive studies on stability analysis have been done previously [7–10].

**Fig. 10** **a** Displacement of  $m_1$  without DR. **b** Displacement of  $m_1$  with DR and gain  $\neq 0$



**Fig. 11** **a** Displacement of  $m_2$  without DR. **b** Displacement of  $m_2$  with DR and gain

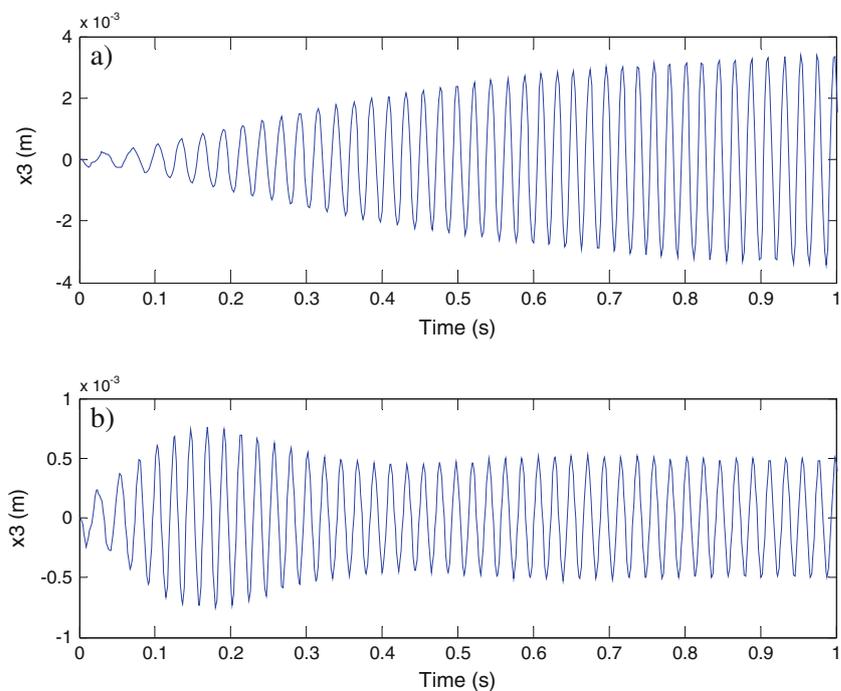


**3 DR application on machine tool**

A mechanical system can be described by a lumped spring-mass model. Since an actual machine tool structure cannot be regarded as a system with proportional damping, general viscous damping must be used to describe the behavior of the

system. A typical machine tool, workpiece, cutting tool, and time delay resonator with their lumped spring-mass models are explained in Table 1. Equivalent masses of the machine tool, workpiece, and the cutting tool are represented by  $m_1$ ,  $m_2$ , and  $m_3$ , respectively. The delay resonator mass,  $m_a$ , is attached to the cutting tool through a spring and a damper.

**Fig. 12** **a** Displacement of  $m_3$  without DR. **b** Displacement of  $m_3$  with DR and gain



The stiffness and damping of the machine tool, workpiece, cutting tool, and delay resonator are represented by  $k_1, C_1, k_2, C_2, k_3, C_3$ , and  $k_a, C_a$ , respectively. Additional relation between the cutting tool and the machine tool can be modeled by using an additional set of spring and damper, as  $k_4, C_4$ . Four basic coordinates,  $x_1, x_2, x_3$ , and  $x_a$  are defined to represent the 4-degree-of-freedom of the system.

In milling process, an exciting force is generated between cutting tool and workpiece. This force leads to additional vibration in cutting tool, workpiece, and machine tool. The goal of this study is to improve the quality of the milling process by reducing the vibration between cutting tool and the workpiece.

Assembled model is shown in Fig. 3.

The state space representation of the system dynamics is written in the form:

$$\dot{y}(t) = A_0y(t) + gA_\tau\dot{y}(t - \tau) + f(t) \tag{6}$$

Where,  $y(t) = \{x_1, \dot{x}_1, x_2, \dot{x}_2, x_3, \dot{x}_3, x_a, \dot{x}_a\}$  and  $f(t) = \{0, 0, 0 - f, 0, f, 0, 0\}$  are state space variables and excitation force, respectively. Furthermore, the amplitude deflection of the primary structure and absorber are denoted by  $x_i$ , where  $i=1, 2, 3, a$ . The parameters  $A_0$  and  $A_\tau$  represent corresponding system matrices. These matrices are shown in Appendix. Finally,  $g$  and  $\tau$  are gain and time delay, respectively. The Laplace transform of Eq. 6 is

$$(sI - A_0 - gse^{-\tau s}A_\tau)Y(s) = F(s) \tag{7}$$

The characteristic equation can be written as

$$Q(s, g, \tau) = |sI - A_0 - gse^{-\tau s}A_\tau| = 0 \tag{8}$$

Poles of the system are those (complex) values of  $s$  for which  $|sI - A_0 - gse^{-\tau s}A_\tau|$  is zero. For non-delay systems, these poles are the eigenvalues of the system matrix,  $A_0$ .

#### 4 Stability analysis of the combined system

The stability is an important property of any feedback control system. The sufficient and necessary condition for asymptotic stability is that the roots of the transcendental characteristic Eq. 8 have negative real parts. The verification of the root locations, however, is not a trivial task. The characteristic Eq. 8 can be written in a simplified form as

$$\sum_{k=0}^1 H_k(s)g^k e^{-k\tau s} = 0 \tag{9}$$

One can obtain its most general form as

$$M(g, s) + N(g, s)ge^{-\tau s} = 0, \tag{10}$$

where  $M$  and  $N$  are polynomials of  $s$ . When the combined (machine tools with DR on cutting tool) system is marginally

stable, there are at least two roots on the imaginary axis. Enforcing  $S = \pm\omega j$  into Eq. 10 and solving for the control parameters  $g_{cs}$  and  $\tau_{cs}$  yields the delay and feedback values that make the combined system marginally stable as

$$g_{cs} = \left\| \frac{M(g, s)}{N(g, s)} \right\| \tag{11}$$

$$\tau_{cs} = \left( \frac{1}{\omega_{cs}} \right) \left\{ 2(l-1)\pi + \tan^{-1} \left( \frac{M(g, s)}{N(g, s)} \right) \right\} \tag{12}$$

As shown in Fig. 4a and b, plots of  $g_c$  versus  $\tau_c$  and  $g_{cs}$  versus  $\tau_{cs}$  illustrate the stability points. Zoomed sections for an interval containing operating frequency are shown in Fig. 4c and d. The ratio of the combined system gain and DR gain ( $g_{cs}/g_c$ ) can be defined as the “stability margin” of the control system, at the particular delay value  $\tau$ . That is, for a given  $\tau_{cs}$ , the system is stable if  $g_c < g_{cs}$  or the stability margin is greater than one. This criterion is shown in Fig. 4c. This figure depicts the frequency range in which the combined system remains stable. An example of such treatment is also presented in the simulations section.

#### 5 Dynamic simulations and result

Determining the stiffness and damping coefficients of a real machine tool is very difficult and possible only by experimental means. Analytical techniques run into difficulty because the behavior of the most significant source of flexibility such as structural joints is hard to predict; however, methods to construct a lumped parameter system from the experiment have been proposed by others [13]. In this paper, system parameters of a machine tool, shown in Table 1, are used [1].

The  $g_{cs}$  versus  $\tau_{cs}$  plots for the given system are generated using Eqs. 11 and 12. The following procedure is used to identify the values of  $g_{cs}$  and  $\tau_{cs}$ :

1. —An interval of  $\omega_{cs}$  is selected.
2. — $g_{cs}$  values which satisfy Eq. 11 for the  $\omega_{cs}$  are numerically calculated.
3. —Using Eq. 12, the corresponding values for  $\tau_{cs}$  are calculated. Notice that  $\tau_{cs}$  is also multi-value due to the crossing root loci identifier,  $L=1, 2, 3, \dots$

The lower envelop of the operating points,  $\{g_{cs}, \tau_{cs}\}$   $L=1, 2, 3$  form the marginal stability boundaries for the combined system. Note that, the  $g=0$  (no feedback) line lies always in the stable zone. These stability boundaries (which are alternatively known as the “stability chart”) are depicted in Fig. 4.

The comparison of stability charts of the combined system and that of the DR (dotted lines) reveals a stable frequency range to absorb and reduce system vibration. It should be noted

that for ideal suppression, the control must maintain the DR within the stability zone. As shown in Fig. 4c, this zone is below dotted lines and where gain of the DR is less than gain of the combined system (meshed zone).

Clearly, the desirable values of the combined system gain and time delay should be in the stable operating zones. Figure 4b shows the relation between the delay,  $\tau_c$ , and the frequency,  $\omega_c$ . For instance, the stable interval of  $\tau$  given in example above corresponds to  $575 < \omega_c < 585$  rad/s.

The procedure outlined can be used as a design tool for selecting the DR optimum parameters  $m_a$ ,  $k_a$ , and  $c_a$  to minimize the cutting tool vibration. If, for instance, the excitation frequency falls outside a stable frequency range,  $m_a$ ,  $k_a$ , and  $c_a$  can be altered until the satisfactory stability picture is reached. As shown in Fig. 4c, the majority of the operating points fall in the stable zone (meshed zone). However, the stability margin is not large, and therefore, the absorption transients are long. It should be noted that the stability margin is, indeed, an indication of the location of the dominant roots of the combined system.

For single DR systems, one can conclude, decreasing the stability margin causes increasing the settling time since the dominant roots are much closer to the imaginary axis [7, 8]. This feature is shown next using simulation examples.

## 6 Case study—I

The machine tool structure depicted in Fig. 3 is considered. The objective is to reduce the vibration of the  $m_3$  (cutting tool). This is achieved by applying DR attached to this mass. The system is excited by a simple harmonic force at  $m_2$  and  $m_3$ , located on the primary structure. The numerical values for the primary structure are taken as shown in Table 2. The characteristics Eq. 8, with the uncontrolled ( $g=0$ ), is used to obtain the eigenvalues and natural frequencies of the combined system. The eigenvalues of the combined system are

$$S_{1,2} = -2.1 \pm 288.2i \quad S_{3,4} = -8.3 \pm 574.7i \\ S_{5,6} = -36.7 \pm 993i \quad S_{7,8} = -45.4 \pm 1, 108.2i$$

This means the real part of the dominant roots is at “-2.1” and the system natural frequencies are roughly

$$288.2(\text{rad/s}), \quad 574.75(\text{rad/s}), \\ 993.67(\text{rad/s}), \quad 1109.1(\text{rad/s}).$$

In order to verify the validity of the stability zones found in Fig. 4, a hypothetical operating frequency close to the natural frequency of the system is selected. To do this, excitation frequency  $\omega_{cs}=581$  rad/s corresponding to stable operation of system is selected. Using this frequency and Fig. 4, the DR tuning parameters for the second branch of root loci,  $L=2$ , are determined to be  $g_c=0.000947$  kg and  $\tau_c=0.01277$  s.

The displacements of the primary structure at the points of attachment of the DR without and with gain are shown in Fig. 5a and b, respectively. As shown, the displacement of the cutting tool is reduced by more than 77.4%.

The DR system, absorber, and responses are shown in Fig. 6. As shown in this figure, the displacement of the DR increases. This shows the DR absorbs the vibration energy.

The behavior of  $m_1$  and  $m_2$  with and without gain are shown in Figs. 7 and 8, respectively.

As shown in these figures, with or without DR attached to  $m_3$ , the behavior of both  $m_1$  and  $m_2$  do not significantly change. Furthermore, it was shown in Fig. 5 that displacement of  $m_3$  is reduced. Therefore, we can conclude that the relative displacement between  $m_2$  and  $m_3$  is reduced.

Finally, to further confirm the stability chart, an operating point in unstable zone is selected. Figure 9 shows results of selecting an unstable operation point, same frequency as before, with  $g_c=0.000985$  kg and  $\tau_c=0.01418$  s. Results indicate that displacement of the  $m_3$  increases.

## 6.1 Case study—II

In this section, the DR is attached to  $m_2$ , workpiece. The same as the previous case study, the system is excited by a simple harmonic force between  $m_2$  and  $m_3$ . The objective is to study the vibration of the workpiece. The numerical values for the primary structure are taken from Table 2.

Excitation frequency is  $\omega_{cs}=254$  rad/s, and other conditions are same as previous case. Displacements of  $m_1$  in two conditions, system without DR and system with DR, are shown in Fig. 10. The displacement of the primary structure is reduced considerably.

Displacements of  $m_2$  in two conditions, with  $g=0$  and  $g \neq 0$ , are shown in Fig. 11. Results indicate that displacement of  $m_2$  at the points of attachment of the DR is reduced significantly.

Displacement of  $m_3$  in two conditions: systems with DR and zero gain as well as system with DR and gain are shown in Fig. 12a and b, respectively. The Appendix section includes the MATLAB Simulink models, Figs. 13 and 14, used for the case studies I and II.

## 7 Conclusion

The number of machining tools on factory floors is very large. These machines are usually expensive, and their replacement with newer machineries is not always cost-effective. An approach to improve the performance, reduced unwanted vibration of existing machines, without making changes to its existing mechanical or electrical components, is highly desirable. Therefore, a general method is developed and successfully demonstrated through simulation using a 4-degree-of-freedom machine tool model. The presented method

is based on the time-delay resonator, which is a tonal vibration absorber. Unlike previous studies which try to reduce the total machine tool structure vibration, this research is concentrated on minimizing the cutting tool and the workpiece vibrations under simple sinusoidal excitations. This approach is thus more realistic and addresses the metal cutting excitation problem directly with minimal computation efforts. What makes this method more advantages is its additive nature. This means that its control logic is independent of the controller used in the machine tool. Furthermore, the

additional mechanical components, DR, are added to the system without altering the existing mechanical structure. Simulink model of the system is also presented. Results of this research indicate that the use of DR technique for minimizing the vibration system is highly efficient and effective. Results also show that optimum location of DR is on the cutting tool. This method can be expanded to more complex systems with large DOF and multiple DRs. The procedure outlined can also be used as a design tool for selecting the DR optimum parameters.

Appendix

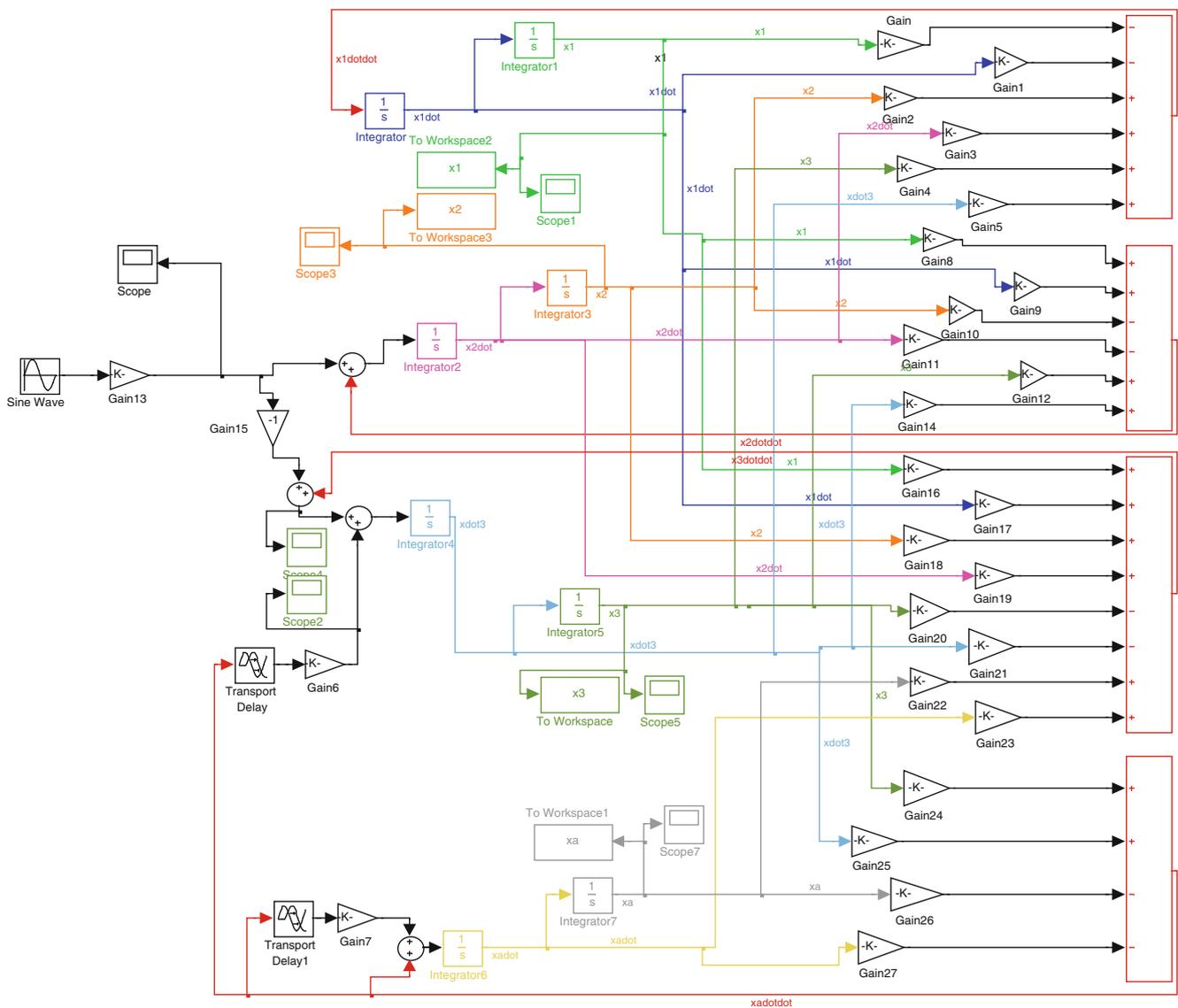


Fig. 13 MATLAB Simulink model—DR attached on  $m_3$

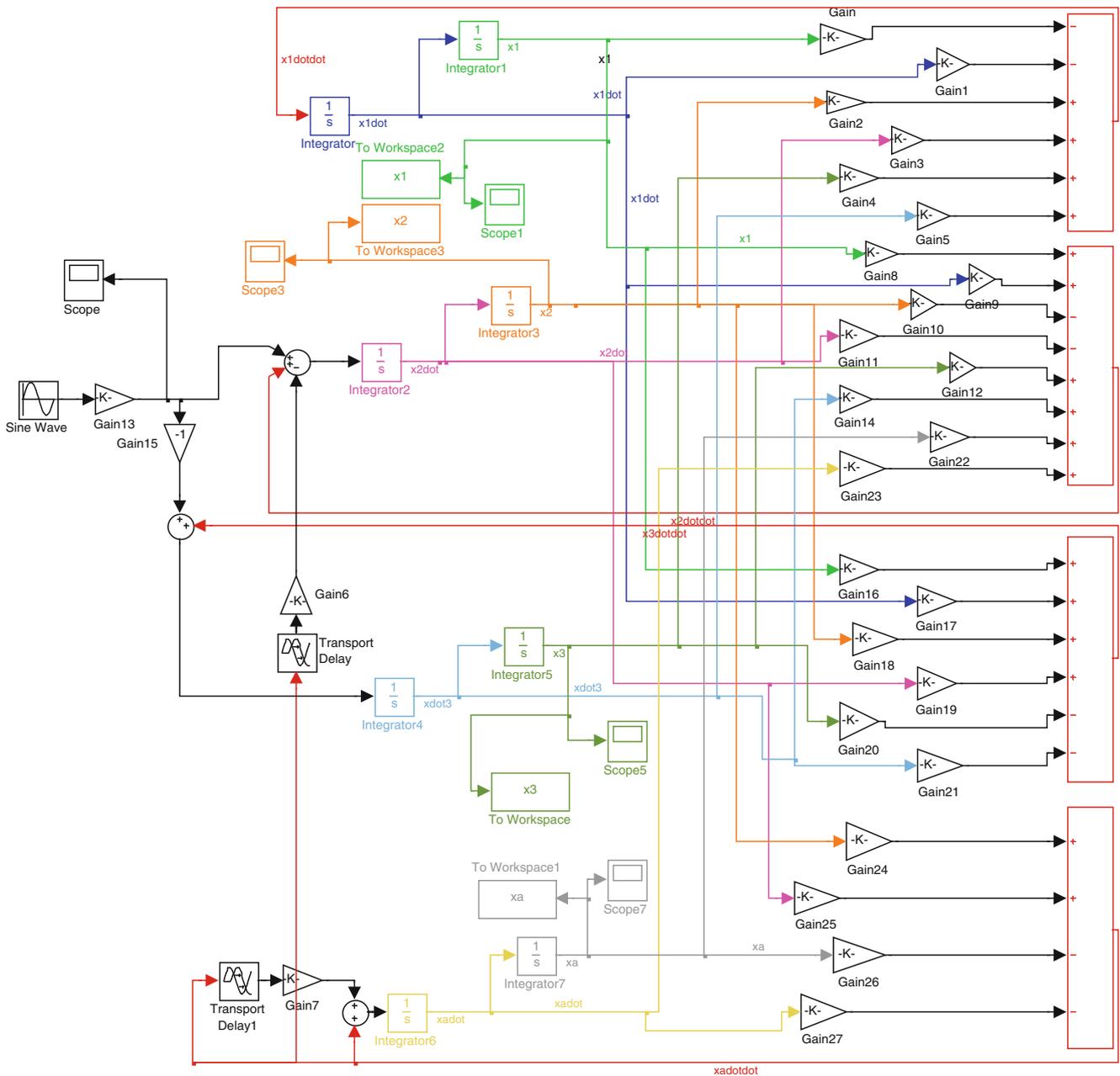


Fig. 14 MATLAB Simulink model—DR attached on  $m_2$

$$A_0 = \begin{pmatrix} -(k_1 + k_2 + k_4)/m_1 & -(c_1 + c_2 + c_4)/m_1 & k_2/m_1 & c_2/m_1 & k_4/m_1 & c_4/m_1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_2/m_2 & c_2/m_2 & -(k_2 + k_3)/m_2 & -(c_2 + c_3) & k_3/m_2 & c_3/m_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ k_4/m_3 & c_4/m_3 & k_3/m_3 & c_3/m_3 & -(k_3 + k_4 + k_a)/m_3 & -(c_3 + c_4 + c_a)/m_3 & k_a/m_3 & c_a/m_3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_a/m_a & C_a/m_a & -k_a/m_a & -c_a/m_a \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_r = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g/m_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & g/m_a \end{pmatrix}$$

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