A novel Vector Perturbation Based On Joint Transceiver Algorithm In Cooperative Mu-MIMO

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Abstract: In this paper we attend to introduce a novel nonlinear vector perturbation approach to improve qualification of communications in a downlink multi user Multi-Input Multi-Output systems where base stations work together cooperatively. In this algorithm, nonlinear vector perturbation based on lattice reduction and VBLAST algorithms is applied at the base stations, whereas linear receiver processing and modulo operation are applied at each mobile. It is shown that the proposed algorithm effectively decomposes the multiuser MIMO channel into parallel independent single-user MIMO channels, and then, the performance of each mobile can be separately optimized. When used on the downlink of multiuser MIMO systems with multiple-antenna mobiles, this algorithm achieves significantly better performance than the ZFcriterion-based nonlinear preprocessing algorithm designed for the multiuser MIMO systems with single-antenna mobiles, because it effectively utilizes the processing capabilities of the mobiles. This method is comparable to nonlinear THP approach and also has outperformance rather than linear types such as block diagonalization because of conservation of the energy in a specify constraint. Moreover, the proposed algorithm achieves a much higher sum capacity at a high signal-to-noise ratio (SNR) than the known block diagonalization technique due to the effective application of the nonlinear preprocessing at the transmitter. Also we propose some methods to achieve better system performance by suitably ordering the channel matrices of different mobiles.

Keywords: Joint transmitter-receiver processing (joint Tx-Rx processing),Vector perturbation (VP), Lattice Reduction(LR), Multiple-Input Multiple-Output MIMO) antenna systems, Tomlinson-Harashima precoding (THP), Zero-forcing (ZF), Vertical Bell Labs layered space time (V-BLAST).

1. Introduction

In wireless local area networks (WLANs), multiple access points (APs) run by different operators may co-exist in the same frequency band and in the same geographical location. Ideally, to achieve maximum capacity, the operators would like their APs to transmit simultaneously to their respective station without interfering with each other. In reality, however, these co-working APs do interfere with each other [1]. One way to mitigate CI in WLANs is extra resources such as additional frequencies or time slots. But It is more efficient for APs to ransmit cooperatively without interfering with each other. In this paper, we consider a cooperative base station system to eliminate the coworking interference in WLAN or other wireless networks. In the proposed cooperative transmission scheme, multiple BSs share information about the transmitted messages to their respective users and wireless channels via a backbone network. It means that there is a centeral unit where gather all of informations from the local BSs and after preprocessing, send to them necessary data. Each base station (BS) can either transmit a single symbol stream or multiple symbol streams to its respective mobile station (MS). Individual BSs and MSs are equipped with multiple transmit and receive antennas, respectively. Each BS transmitter uses the transmitted signal information from other BSs under wireless channel conditions which comes from the center unit to precode its own signal.

Also, in cellular mobile systems interference comes from the adjacent cells is strong [2] especially when the users are located near the cell edges. Most of the published papers in this area such as [3,4] considered only a multi-user multiple-input single-output (MISO) system with a single receive antenna. In the multiuser MIMO systems with multiple-antenna mobiles, through spatial multiplexing, each mobile user can receive multiple data streams, while different data streams can also be transmitted at the same time to different users. Thus, a very high throughput can be achieved. However, each stream of data will encounter spatial interference from the other data streams, including the data streams of other users and the other data streams of the same user. Therefore, the cancellation of the interstream interference is an important problem. In [5], the authors studied a ZF method for a multi-stream multi-user MIMO system, where transmit–receive antenna weights are jointly optimized by a ZF diagonalization technique. The water-filling power allocation method is then applied to allocate power to each user. The method in [5] is then significantly improved in [6] by iteratively finding the transmit–receive weights. Moreover, the zero-forcing (ZF)-criterion-based nonlinear preprocessing algorithm with single-antenna mobiles [7], [8], have been applied to preeliminate the interstream interference at the transmitter for a multiuser MIMO downlink. It is noted that the nonlinear preprocessing algorithm proposed in [7] and [8] can achieve significantly better performance than the linear preprocessing algorithm, since it uses Tomlinson-Harashima precoding (THP) to successfully limit the transmitted power increase while preeliminating the interstream interference. On the other hand, nonlinear joint transmitter-receiver inside of the linear processing (joint Tx-Rx processing) algorithm based on THP, has been proposed in [9]. The vector perturbation technique proposed in [10] is based on the fixed-complexity sphere encoder (FSE) and the QR-decomposition. In [11]VP is derived that minimizes the MSE but is restricted to maximize the mutual information of the MIMO channel and in [12] Approximate vector perturbation techniques assisted by LLL lattice reduction (LR) can exploit all the diversity that is available in multi-user multi-antenna broadcast systems.

The rest of this paper is organized as follows. System model and assumptions are first described in Section 2. The nonlinear preprocessing algorithm developed in [7] and [8] and the basic idea of the THP are briefly described in Section 3. In Section 4, our novel nonlinear joint Tx-Rx processing algorithm based on VP is introduced, the diversity advantage of this algorithm over the algorithm in [7] and [8] is shown, and the sum capacity of a multiuser MIMO system when the proposed algorithm is applied is analyzed. The ordering problem of the proposed algorithm is solved in this section. Simulation results are given in Section 5. Conclusions follow in Section 6.

2. System model and assumptions

In this paper, we consider a multi-user MIMO system, where K BSs transmit to K MSs. Each BS and MS are equipped with $\mathrm{M_T}^{k}$ and $\mathrm{N_R}^{k}$ antennas, for k = 1,…,K, respectively. All BSs cooperate with each other to transmit N symbol streams to their respective MSs via $\bar{\mathrm{M}}_\mathrm{T} = \sum_{k=1}^{K} \mathrm{M_T}^{k}$ antennas.

Transmitter structure based on Cooperative THP and vector perturbation (VP) precoding is shown in Figure1 and figure2 respectively. Let a_k shows the symbol for user k*th* before the preprocessing. It is assumed that all the data symbols are independent and have unit power, i.e., $\boldsymbol{R_a} \equiv E[\boldsymbol{aa}^H] = \boldsymbol{I}$, where *E*[.] denotes expectation, and $(.)^H$ denotes conjugate transpose. Let $\boldsymbol{x}_\mathrm{k} = [x_{k,1} \dots x_{k,s} \dots x_{k,S}]^T$represents the precoded and modulated signal vector, consisting of *K* M-QAM (M-ary Quadrature Amplitude Modulation) modulated symbols, where $x_{k,s}$ is the *s*th modulated symbol stream from BS *k* intended for MS. Thus, we have a multi-stream transmission where *S* symbol streams are transmitted from BS *k* to MS *k* simultaneously. The constellation points for M-QAM are drawn from the signal set A=$\{\frac{1}{2} \pm \mathrm{j}\frac{1}{2}, \dots, \frac{\sqrt{\mathrm{M}}-1}{2} \pm \mathrm{j}\frac{\sqrt{\mathrm{M}}-1}{2}\}$. The received signal vector $y_k \equiv ({y_k}^1, {y_k}^2, \dots, {y_k}^{N_r})^T$ at Ms_k is

$$\boldsymbol{y}_k = \boldsymbol{H}_k\boldsymbol{x} + \boldsymbol{n}_k \qquad (1)$$

Where $\boldsymbol{n}_k \equiv ({n_k}^1, {n_k}^2, \dots, {n_k}^{N_r})^T$ is i.i.d. zero-mean complex Gaussian random variables with the covariance matrix $\boldsymbol{R}_{\boldsymbol{n}_k} \equiv E[\boldsymbol{n}_k\boldsymbol{n}_k^H] = \sigma_n^2\boldsymbol{I}$**.** Another way to represent the received signals is

$$\boldsymbol{y} = \boldsymbol{Hx} + \boldsymbol{n} \qquad (2)$$

Where

$$\boldsymbol{H} = \left({\boldsymbol{H}_1}^T, {\boldsymbol{H}_2}^T, \dots, {\boldsymbol{H}_K}^T\right)^T,$$
$$\boldsymbol{x} = ({\boldsymbol{x}_1}^T, {\boldsymbol{x}_2}^T, \dots, {\boldsymbol{x}_K}^T)^T \equiv [x_{1,1} \dots x_{k,s} \dots x_{K,S}]^T,$$
$$\boldsymbol{n} = ({\boldsymbol{n}_1}^T, {\boldsymbol{n}_2}^T, \dots, {\boldsymbol{n}_K}^T)^T \quad ,$$
$$\boldsymbol{y} = ({\boldsymbol{y}_1}^T, {\boldsymbol{y}_2}^T, \dots, {\boldsymbol{y}_K}^T)^T.$$

3. Nonlinear THP Preprocessing For The Downlink Of Multiuser MIMO

In this section we are interested to investigate the proposed algorithm in [9]. Moreover. we briefly review the ZF-criterion-based nonlinear preprocessing algorithm in [7] and the basic idea of the THP. Fig. 1 shows the structure of the nonlinear preprocessing algorithm in [9] when it is used on the downlink of a multiuser MIMO system with multiple-antenna mobiles. In this algorithm, matrix **F** $(N_R \times M_T)$ at the transmitter works as a feedforward filter, and matrix (**B** − **I**) works as a feedback filter, where **B** $(M_T \times M_T)$ should be a unit triangular matrix (a triangular matrix with 1s on the main diagonal) [12]. In Fig. 1, the block outlined by the dashed line at the transmitter is the Tomlinson-Harashima precoder. Without considering of the modular block, closed loop of the (**B** − **I**) construct an inverse lower triangular of the matrix B, thus it converts vector a to large values. It means that the inverse block make power to increase in transmitter side. Therefore using of the nonlinear THP block comes benefit because of constraining the value of symbols in a specific range. Equivalently, nonlinear THP can be seen as linear model with additive vector **d** where generates an effective data vector $\boldsymbol{v} \equiv ({\boldsymbol{v}_1}^T, {\boldsymbol{v}_2}^T, \dots, {\boldsymbol{v}_K}^T)^T = \boldsymbol{a} + \boldsymbol{d}$ to be the input to

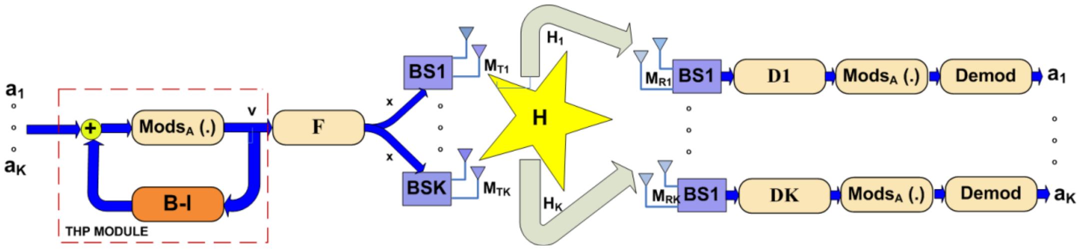

Fig.1. Cooperative Nonlinear Joint Transceiver THP Module for Multi User MIMO

$\boldsymbol{B}^{-1}$, where $\boldsymbol{d} = \left(\boldsymbol{d}_1{}^T, \dots, \boldsymbol{d}_K{}^T\right)^T$ is the precoding vector. This equivalent structure of the precoder is also shown in Fig. 1. It is noted that $\boldsymbol{d}_k$ can be removed from the received effective data vector $\boldsymbol{v}_k$ using a modulo device at MS_k. It should be noted that using the THP makes slightly power increase in precoder output $\boldsymbol{v}$ because, it is not taken from the constellation points. This transmitted power increase is quantified by the *precoding loss factor* $E[\|\boldsymbol{v}^2\|]/E[\|\boldsymbol{a}^2\|]$ [7]. It is noted that the transmitted power increase of the THP is significantly smaller compared with that of the linear preprocessing algorithms, and it is negligible for moderate and high modulation sizes [7]. In the receiver side, the signal streams collected from the multiple antennas are first processed by a linear receiver processing matrix $\boldsymbol{G}_k$ (k=1,..,K) before going into the modulo and decision devices for each of MSs. Howevere, the processing matrices $\boldsymbol{D}_1, \dots,$ $\boldsymbol{D}_K$ are determined at the transmitter, and hence, $\boldsymbol{D}_K$ needs to be transmitted to MS_k. In against to approaches in [1,7,8] where exploit the duality between downlink and uplink transmission, auther in [9] uses another way to ontain the transmitter and receiver matrixs.

If we use the ZF criterion for eliminating the interstream interference, therefore

$$\boldsymbol{DHFB}^{-1} = \boldsymbol{I} \tag{3}$$

is required. The processing matrix **B** and **F** are obtained by QR factorization on **H**. (az anjaeeke) QR gives a unitary and upper triangle matrixes, we use $\boldsymbol{H}^H$ instead of **H**. by QR factorization we can find the null space of the channel matrix. Thus, that is a good way to place precoding matrix in orthogonal space of $\boldsymbol{H}, \overline{\boldsymbol{H}}$ (where is interference channel) for canceling of inter user interference (IUI). Thus, we remove a part of interference with upper triangle matrix $\boldsymbol{B}$ and another one will be eliminated by preprocessing matrix $\boldsymbol{F}$. So, for nullifying of the effect of part of interference we have,

$$\boldsymbol{H}_j\boldsymbol{F}_k = \boldsymbol{0}_{M_{R_j}M_{R_k}} \qquad \text{j>k} \tag{4}$$

Where $\boldsymbol{F}_k$ should be in the null space of $\overline{\boldsymbol{H}}_k$, and

$$\overline{\boldsymbol{H}}_k = [\boldsymbol{H}_{k+1}^T, \boldsymbol{H}_{k+2}^T, \dots, \boldsymbol{H}_K^T]^T \tag{5}$$

tead of the SVD on $\overline{\boldsymbol{H}}_k$, one can apply QR factorization to a new matrix

$$\widehat{\boldsymbol{H}} = [\boldsymbol{H}_K^T, \boldsymbol{H}_{K-1}^T, \dots, \boldsymbol{H}_1^T]^T \tag{6}$$

Then QR is performed on $\widehat{\boldsymbol{H}}^H$,ie,

$$\widehat{\boldsymbol{H}}^H = [\boldsymbol{Q}_K \;\; \boldsymbol{Q}_{K-1} \; \dots \; \boldsymbol{Q}_1 \quad \boldsymbol{Q}^{\perp}]\left[\begin{smallmatrix}\boldsymbol{R}\\ \boldsymbol{0}\end{smallmatrix}\right] \tag{7}$$

Where $\boldsymbol{Q}_k$ is $N_T \times M_{R_k}$ and $\boldsymbol{Q}^{\perp}$ is $N_T \times (N_T - M_R)$If we wirte $\boldsymbol{F}_k = \boldsymbol{N}_k\boldsymbol{A}_k$ then $\boldsymbol{N}_k =$ $[\boldsymbol{Q}_k \;\; \boldsymbol{Q}_{k-1} \; \dots \; \boldsymbol{Q}_1 \quad \boldsymbol{Q}^{\perp}]$. Authers in [9] has shown that for satisfying the (3), it needs to apply GMD algorithm [14] to $\boldsymbol{H}_k\boldsymbol{N}_k$. Hence,

$$\boldsymbol{H}_k\boldsymbol{N}_k = \boldsymbol{Q}_k\boldsymbol{R}_k\boldsymbol{P}_k^H \tag{8}$$

$$\boldsymbol{A}_k = \boldsymbol{P}_k \tag{9}$$

$$\boldsymbol{G}_k = diag\left[\boldsymbol{R}_k^{11}, \dots., \boldsymbol{R}_k^{M_{Rk}M_{Rk}}\right] \tag{10}$$

$$\boldsymbol{D}_k = \boldsymbol{G}_k\boldsymbol{Q}_k^H \tag{11}$$

$$\boldsymbol{B}_k = \boldsymbol{G}_k\boldsymbol{R}_k \tag{12}$$

Since $\boldsymbol{N}_k$ with size of $N_T \times \left(N_T - \sum_{j=k+1}^{K} M_{R_j}\right)$ has orthogonal columns, so we can choose it for preprocessing matrix $\boldsymbol{F}$, where $\boldsymbol{F}^H\boldsymbol{F} = \boldsymbol{I}$, and

$$\text{Trace}\{\boldsymbol{FF}^H\} = M_R \tag{13}$$

Therefore, the linear preprocessing matrix $\boldsymbol{F}$ will not change the power of the precoder output vector. One can also see that with this matrix $\boldsymbol{F}$, the power allocated to each mobile is proportional to the number of data streams transmitted to the mobile. We represent matrix $\boldsymbol{F}$ as $\boldsymbol{F} = [\boldsymbol{F}_1, \boldsymbol{F}_2, \dots, \boldsymbol{F}_K]$ where $\boldsymbol{F}_k$ is a $\left(M_R \times N_{T_k}\right)$.

4. Nonlinear Vector Perturbation For The Downlink Of Multiuser MIMO

The goal of the vector perturbation techniques is to generate the vector $\widetilde{\boldsymbol{a}}_k$ from the data vector $\boldsymbol{a}_k$, such that the norm of $\boldsymbol{S}_{eff_k}\widetilde{\boldsymbol{a}}_k$ becomes smaller than that of $\boldsymbol{S}_{eff_k}\boldsymbol{a}_k$. Here, $\boldsymbol{S}_{eff_k}$ is the precoding matrix for user *k*.

The vector perturbation technique is employed to reduce the required transmission power. The perturbed vector $\widetilde{\boldsymbol{a}}_k$ is then derived from the THP technique at the transmitter side as follows:

$$\widetilde{\boldsymbol{a}}_k = \boldsymbol{a}_k + \tau\boldsymbol{l} \tag{14}$$

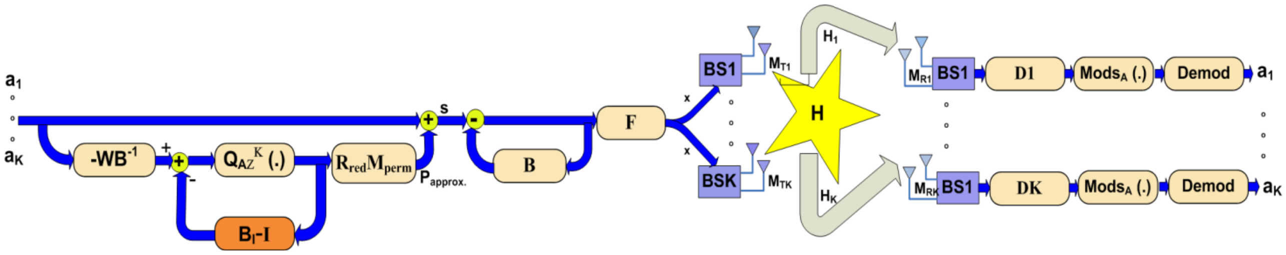

Fig.2. Cooperative Nonlinear Joint Transceiver Vector Perturbation Module for Multi User MIMO

integer vector. In [15], τ is given by:

$$\tau = 2(|C_{max}| + \tfrac{\Delta}{2}) \tag{15}$$

where $|C_{max}|$ is the absolute value of the symbol with the largest magnitude, and Δ is the spacing between any two neighbor symbols.

While we use precoding matrixes **F** and $\boldsymbol{B}^{-1}$ which have obtained from section III, the received vector at user k is given by:

$$\boldsymbol{y}_k = \widetilde{\boldsymbol{a}}_k + \widehat{\boldsymbol{n}}_k \tag{16}$$

Where $\widehat{\boldsymbol{n}}_k = \boldsymbol{D}_k \boldsymbol{n}_k$.

At the receiver, the original data vector $\boldsymbol{a}_i$ is recovered, without knowledge of vector $\boldsymbol{l}$, using the nonlinear

modulo operation as follows:

$$\widehat{\boldsymbol{a}}_k = mods_A(\boldsymbol{y}_k) \tag{17}$$

where $mods_A(.)$ is the symmetric modulo operation as

$$mods_A(x) = x - A\left\lfloor\frac{x+\frac{A}{2}}{A}\right\rfloor, \qquad A = \sqrt{M} \tag{18}$$

that $\lfloor x \rfloor$ is the greatest integer smaller than x, and reduces the range of the received signal to the interval $[-M, M)$, where M depends on the used modulation scheme [7].

The vector $\boldsymbol{l}$, introduced in (14), is found by solving the following N-dimensional integer lattice problem:

$$\boldsymbol{l} = \arg\min_{l \in \mathbb{Z}^N} \left\|\boldsymbol{S}_{eff_k}(\boldsymbol{a} + \tau \boldsymbol{l})\right\|^2 \tag{19}$$

One way to solve the (19) is using of the sphere encoding which was introduced in [15]. Its complexity is polynomial and it is comparable to the *brute-force search* which was introduced in [16] with exponential complexity. Another method was introduced in [10] where author has exploited a fixed sphere encoding (FSE) with the BD precoding for small complexity in multiuser MIMO system.

But here, we propose a method based on the closest point approximation [17] in order to obtain a simple but efficient vector precoding scheme instead of prohibitive search on lattice. Figure 2 shows the details. Here the basis for the approximate solutions is the LLL reduction of the matrix $\boldsymbol{B}^{-1}$, and similar to (3) we write

with the reduced matrix $\boldsymbol{H}_{red}$ and unimodular $\boldsymbol{R}$.

If we take the "rounding-off" approximation from [17] the solution of (19) is given by

$$\boldsymbol{p}_{approx.} = A\boldsymbol{R}Q_{\mathbb{Z}^K}\left\{\frac{1}{A}\boldsymbol{H}_{red}^{-1}\boldsymbol{H}^{-1}\boldsymbol{a}\right\} = \\ -\boldsymbol{R}Q_{A\mathbb{Z}^K}\{\boldsymbol{R}^{-1}\boldsymbol{a}\} \quad , \; A = \sqrt{M} \tag{21}$$

where we have used $Q_{A\mathbb{Z}^K}\{.\}$ to denote componentwise rounding of a K-dimensional vector to the scaled integer lattice $A\mathbb{Z}^K$.(Note that $Q_{A\mathbb{Z}^K}\{x\} = A\left\lfloor\frac{1}{A}x\right\rceil$).

Consequently, the transmit signal is given as

$$\boldsymbol{x} = \boldsymbol{F}\boldsymbol{B}^{-1}(-\boldsymbol{R}Q_{A\mathbb{Z}^K}\{\boldsymbol{R}^{-1}\boldsymbol{a}\}) \tag{22}$$

We can also consider the nearest plane approximation vector precoding [18] in combination with the V-BLAST algorithm for the solution of the closest point problem. From the V-BLAST algorithm applied to $\mathbf{H}_{\mathrm{red}}^{-1}$ obtained from the LLL algorithm ,we get, as in (5),

$$\boldsymbol{B}_{vblast} = \boldsymbol{W}\boldsymbol{H}_{red}\boldsymbol{M}_{perm} \tag{23}$$

Here $\boldsymbol{B}_{vblast} \in \mathbb{R}^{MR \times NT}$ is lower triangular matrix with unit diagonal, $\boldsymbol{W} \in \mathbb{R}^{MR \times NT}$ the matrix with orthonormal rows and $\boldsymbol{M}_{perm}$ is a $N_T \times N_T$ permutation matrix corresponding to the optimized decision order.

As we know, symbols are prepared without any ordering at transmitter at first. When the interference matrix has only a few very large eigenvalues, the strength of interference is mainly concentrated in the direction of eigenvectors corresponding to large eigenvalues. Since the interference comes only from a few directions, we could easily avoid the interference by transmitting the signals in the directions with minimum interference level. Thus, it seems to have improved performance while it is found the path with the least interference. Here we use the approach which was introduced in [1].

5. Simulation results

In this section, we compare the performance of three models of cooperative transmission schemes, 1) Cooperative-THP with scaling factor at receiver which calculated at the transmitter side and is sent to receiver, 2)Cooperative nonlinear THP with joint design of the precoding and postcoding, and 3) Cooperative nonlinear vector perturbation based on lattice reduction and VBLAST algorithms under

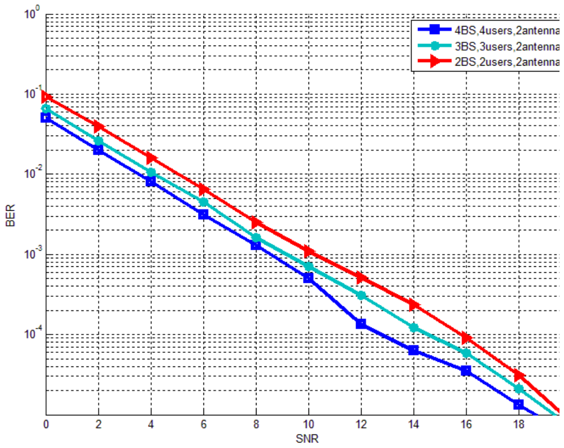

Fig.3. Cooperative THP Multi-User MIMO

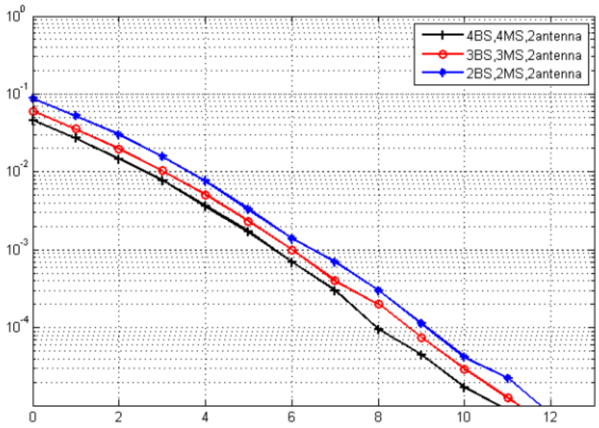

Fig.4. Cooperative Joint THP Multi-User MIMO

consideration of BER and capacity. Channel supposes to be flat Rayleigh fading Gaussian and there is no any delay at receivers. It is assumed that there are base stations in different cells and each of them has several transmit antennas and there exist mobiles in boundary of cells , each with several antennas(for a spatial case, M_{T_j}= 2, N_{R_j}= 2, K=4), in that multiuser MIMO system. Also it is supposed that all of base stations are connected to a center with a backbone networks, thus center has full CSI because each BS can share its CSI.

Rectangular 4-QAM (M=4) modulation is used. Figure 3 shows performance of the proposed Model I (Cooperative-THP with scaling factor) for different deployments. Simulation shows that the better performance while the number of BSs increase. Figure 4 illustrates performance of the proposed Model II (Cooperative nonlinear THP with joint design of the precoding and decoding) . It is obvious the better behavior rather than the model I because of considering matrix D. figure 5 shows the result of proposed algorithm (vector perturbation under the nearest plane consideration). the that VP-LR-VBLAST method outperforms the BD algorithm and slightly better than the its THP counterpart. Also users growth makes a relative improvement in BER because of better cooperation between transmitter antennas and using the full diversity in the nonlinear methods while in linear BD precoding is reverse. Finally, figure 6 compares capacity status explained in [9] with different antenna arrangements.

6. Conclusion

This paper compares three practical cooperative transmission schemes employing linear (BD) and nonlinear precoding (THP,VP), where VP is based on

V-BLAST approach for Mu-MIMO in different cells. The proposed designs eliminates ISI and MUI center on transmitter side and no have need to joint design, thus Nearest plane lattice reduction. the modulated data to be sent to all of the base stations with a backbone network. In the first design, the cooperative scheme among BSs, combines THP with beamforming and tries to minimize the total transmitted power.

The second, exploits vector perturbation concept and combines nonlinear rounding process to attain nearest point in lattice at receiver. both of designs propose a simple receiving structure and they just need to process at the transmitted side. In contrast to the BD they don't need to have a normalization factor because of their nonlinear structures.

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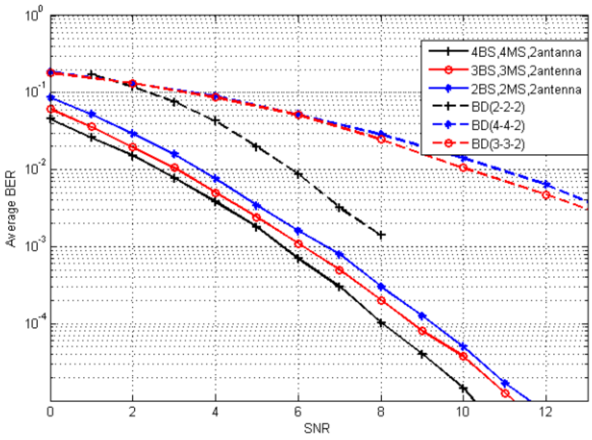

Fig.5. Cooperative Joint VP Multi-User

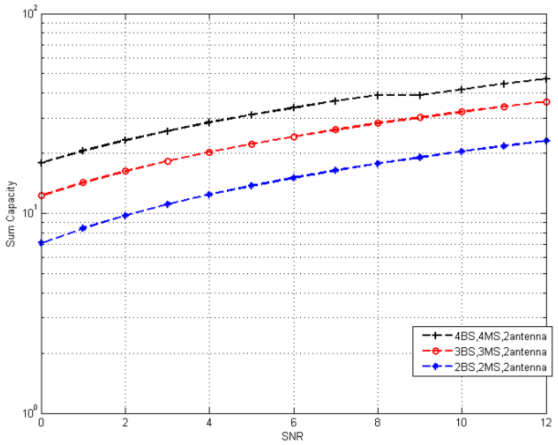

Fig.6. Sum Capacity Of VP(THP)Multi-User

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