

A Novel DS-CDMA Direction of Arrival Estimator for Frequency- Selective Fading Channel with Correlated Multipath by using Beam Forming Filter

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Abstract: In this paper a new method for estimating the direction of arrival of Asynchronous DS-CDMA signals is proposed. In this method we have assumed to have a frequency-selective fading channel with correlated multipath. In the proposed method first for removing effects of undesired paths we apply the coherent signals of the undesired paths perpendicularly to the under process signal, next, signal is passed through a filter bank and then using the “beam space” idea direction of signal is discovered. Since beam forming filters are used, effects from other users on the desired users signal is also decreased which in turn increase the efficiency of the algorithm and search area decrease almost to one tenth. In this algorithm it is not required to search all angles as in conventional methods like MUSIC. Besides, this algorithm does not require information criteria like minimum description length minimum description length (MDL) and Akaike’s information criterion (AIC) for estimating the number of sources. Another advantage of this method is that number of users can exceed the number of antenna arrays contrary to many of the conventional methods. At the end, some simulation results are illustrated to confirm the efficiency of the method.

Keywords: Direction of arrival estimation, MUSIC algorithm, MDL, orthogonal projection, beam forming, frequency-selective fading channel.

1. Introduction

In recent years, estimating the direction of arrival has been an attractive area of research because of its important application in radar and wireless location finding. Among the methods proposed, the signal subspace algorithms have attracted a lot of interest due to their high resolution. However, in a highly correlated or coherent environment due to multipath propagation, the direction of coherent signals cannot be detected via conventional subspace methods like the MUSIC algorithm [1], [3] and ESPRIT [2] since the spatial signatures cannot be resolved in the signal subspace.

Furthermore, in order to estimate DOA by conventional methods, the number of array elements must be greater than the number of users; this is impractical in CDMA systems with a large number of active users. A DOA estimator employing code matched filters and parallel MUSIC is proposed in [4]. In this method other user except of active users models to Gaussian noise as a interference.

In this paper, we propose a method for DOA estimation with an array of antennas at the receiver. First, we decorrelate the received signal of each path by orthogonal projection and then apply a bank of $2M$ beam forming filters to the received 2-D signal which results in $2M$ time domain sequences. The total energy of interfering signals is reduced after each beam forming filter. The energy of desired signal is also reduced, since the DOA is still unknown at this stage. However, the proposed algorithm, together with this receiver structure is able to control the loss of energy while maintaining the SNR level.

The paper is organized as follows. In Section 2, we describe the mathematical model of the system, set the underlying assumptions and define the problem objectives. In Section 3, the proposed model is developed for synchronous single path and asynchronous multipath cases. In Section 4, performance of the proposed method is evaluated by simulation, and Section 5 concludes the paper.

2. Mathematical model

Consider a DS-CDMA system with K active users transmitting binary information sequences of b_1, b_2, \dots, b_K with normalized spreading waveforms s_1, s_2, \dots, s_K that are randomly distributed in space. A Q bit transmitted baseband signal from the k -th user is:

$$x_k(t) = A_k \sum_{i=0}^{Q-1} b_k(i) s_k(t - iT_b) \quad k = 1, 2, \dots, K \quad (1)$$

Where T_b is the bit interval, $b_k(i) \in \{-1, +1\}$ is the i -th bit of a sequence of i.i.d random variables transmitted by the k -th user and A_k denotes the amplitude of the k -th user. $s_k(t)$ is defined as follows and its energy is limited to $[0, T_b]$:

$$s_k(t) = \sum_{j=0}^{N_c-1} c_k(j) \psi(t - jT_c) \quad 0 \leq t \leq T_b \quad (2)$$

where $N_c = \frac{T_b}{T_c}$ is the processing gain; and $\psi(t)$ is a chip

waveform of duration T_c and $\{c_k(n)\}_{n=1}^{N_c-1}$ is a signature code sequence of ± 1 s assigned to the k -th user that can be represented as $\underline{c}_k = [c_k(0) \ c_k(1) \ \dots \ c_k(N_c - 1)]^T$ where T denotes the transposition operator.

At the receiver an antenna array of M elements is employed and the baseband multipath channel of the k -th user can be modeled as a single-input multiple-output channel with $M \times 1$ vector impulse response $\underline{h}_k(t)$ given as [5]

$$\underline{h}_k(t) = \sum_{l=1}^L g_{kl} \delta(t - \tau_{kl}) \underline{a}_{\theta_{kl}} \quad (3)$$

where L is the number of paths in each user's channel, g_{kl} and τ_{kl} are gain and delay of the l -th path of the k -th user's signal respectively, $\delta(\cdot)$ is the Dirac delta function and $\underline{a}_{\theta_{kl}} = [a_{\theta_{kl}}^1, \dots, a_{\theta_{kl}}^M]^T$ is the array response vector corresponding to the l -th path of the k -th user's signal with DOA of θ_{kl} .

The total received baseband signal at the i -th antenna denoted by $r_i(t)$ is the superposition of the signals from all users plus the additive ambient noise and the $M \times 1$ vector $\underline{r}(t) = [r_1(t), \dots, r_M(t)]^T$ can be expressed as:

$$\underline{r}(t) = \sum_{k=1}^K x_k(t) * \underline{h}_k(t) + \sigma \underline{n}(t) \quad (4)$$

$$= \sum_{i=0}^{Q-1} \sum_{k=1}^K A_k b_k(i) \sum_{l=1}^L \underline{a}(\theta_{kl}) g_{kl} \times s_k(t - iT_b - \tau_{kl}) + \sigma \underline{n}(t)$$

where $*$ denotes convolution, σ^2 is the variance of the ambient noise at each antenna element and $\underline{n}(t) = [n_1(t), \dots, n_M(t)]^T$ is a vector of independent zero mean complex white Gaussian noise processes with unit variance, i.e.

$$E\{\underline{n}(t) \underline{n}(t')^H\} = I_M \delta(t - t') \quad (5)$$

Where E is the expectation operator, H denotes the conjugate transpose and I_M is the $M \times M$ identity matrix. We also assume that the noise processes and transmitted sequences of users are statistically independent.

To find the directions of the received signals from the l -th path of the k -th user, the receiver's chip matched filter is synchronized with the delay of τ_{kl} . The receiver works at the discrete chip rate. The sample of m -th antenna is:

$$y_{kl,m}(i, j) = \int_{iT_b + \tau_{kl} + jT_c}^{iT_b + \tau_{kl} + (j+1)T_c} r_m(t) \psi^*(t - iT_b - \tau_{kl} - jT_c) dt \quad (6)$$

M samples of antenna output can be represented by a $M \times 1$ vector as:

$$\underline{y}_{kl}(i, j) = [y_{kl,1}(i, j), y_{kl,2}(i, j), \dots, y_{kl,M}(i, j)]^T \quad (7)$$

s Or:

$$\underline{y}_{kl}(i, j) = \begin{bmatrix} y_{kl,1}(i, j) \\ y_{kl,2}(i, j) \\ \vdots \\ y_{kl,M}(i, j) \end{bmatrix} = \int_{iT_b + \tau_{kl} + jT_c}^{iT_b + \tau_{kl} + (j+1)T_c} \underline{r}(t) \psi^*(t - iT_b - \tau_{kl} - jT_c) dt \quad (8)$$

By combining the vectors \underline{y}_{kl} , we define the $M \times N_c$ matrix

$BL_{kl}(i)$ as follows:

$$BL_{kl}(i) = [\underline{y}_{kl}(i, 0), \underline{y}_{kl}(i, 1), \dots, \underline{y}_{kl}(i, N_c - 1)] \quad (9)$$

The contribution of k -th user at m -th antenna ($r_m(t)$) is:

$$r_m^k(t) = A_k \sum_{q=0}^{Q-1} b_k(i) \sum_{l=1}^L g_{kl} a_m(\theta_{kl}) s_k(t - qT_b - \tau_{kl}) \quad (10)$$

Therefore the contribution of k -th user at $y_{kl,m}(i, j)$ is:

$$BL_{kl,m}^k(i) = y_{kl,m}^k(i, j) \quad 0 \leq j \leq N_c - 1 \quad (11)$$

$$= A_k \sum_{q=0}^{Q-1} b_k(q) \sum_{p=1}^L g_{kp} a_m(\theta_{kp})$$

$$\times \int_{iT_b + jT_c + \tau_{kl}}^{iT_b + (j+1)T_c + \tau_{kl}} s_k(t - qT_b - \tau_{kp}) \psi^*(t - iT_b - jT_c - \tau_{kl}) dt$$

Due to $\psi(t)$ waveform at $[0, T_c]$ and $s_k(t)$ at $[0, T_b]$, product of $s_k(t - qT_b - \tau_{kp})$ and $\psi(t - iT_b - jT_c - \tau_{kl})$ is always zero except in two cases:

$$iT_b + jT_c + \tau_{kl} + T_c \geq qT_b + \tau_{kp} \quad (12)$$

$$iT_b + jT_c + \tau_{kl} < qT_b + \tau_{kp} + T_b \quad (13)$$

Since the effect of $i-1$, i , $i+1$ -th bits in integral, $y_{kl,m}^k(i, j)$ simplifies to:

$$y_{kl,m}^k(i, j) = A_k b_k(i-1) \sum_{p=1}^L g_{kp} a_m(\theta_{kp}) \underline{c}_{kp}^{i,-1}$$

$$+ A_k b_k(i) \sum_{p=1}^L g_{kp} a_m(\theta_{kp}) \underline{c}_{kp}^{i,0}$$

$$+ A_k b_k(i+1) \sum_{p=1}^L g_{kp} a_m(\theta_{kp}) \underline{c}_{kp}^{i,+1}$$

Where $\underline{c}_{kp}^{l,s}$ is a $1 \times N_c$.

$$\underline{c}_{kp}^{l,s} = [c_{kp}^{l,s}(0), c_{kp}^{l,s}(1), \dots, c_{kp}^{l,s}(N_c - 1)] \quad (15)$$

$$c_{kp}^{l,-1} = \int_{jT_c + \tau_{kl}}^{(j+1)T_c + \tau_{kl}} s_k(t + T_b - \tau_{kp}) \psi^*(t - jT_c - \tau_{kl}) dt \quad (16)$$

$$c_{kp}^{l,0} = \int_{jT_c + \tau_{kl}}^{(j+1)T_c + \tau_{kl}} s_k(t - \tau_{kp}) \psi^*(t - jT_c - \tau_{kl}) dt \quad (17)$$

$$c_{kp}^{l,+1} = \int_{jT_c + \tau_{kl}}^{(j+1)T_c + \tau_{kl}} s_k(t - T_b - \tau_{kp}) \psi^*(t - jT_c - \tau_{kl}) dt \quad (18)$$

As a result, the contribution of n -th user at $BL(i)$ matrix is:

$$\begin{aligned} BL_{kl}^k(i) &= A_k b(i-1) \sum_{p=1}^L g \underline{a}(\theta_{kp}) \otimes \underline{c}_{kp}^{l,-1} \\ &+ A_k b(i) \sum_{p=1}^L g \underline{a}(\theta_{kp}) \otimes \underline{c}_{kp}^{l,0} \\ &+ A_k b(i+1) \sum_{p=1}^L g \underline{a}(\theta_{kp}) \otimes \underline{c}_{kp}^{l,+1} \end{aligned} \quad (19)$$

Where \otimes is Kronecker matrix product.

3. The proposed method

3.1 Proposed method for synchronous single path case

First we consider the case where all users are transmitting synchronously in a single path fading system, i.e. $L=1$. Without loss of generality, assume $\tau_1 = \tau_2 = \dots = \tau_K$. In this case BL matrix is independent of indices k and l and simplifies to:

$$BL_{kl}^k(i) = A_k b_k(i) g_k \underline{a}(\theta_k) \otimes \underline{c}_k \quad (20)$$

A bank of $2M$ beam forming filters W_s is applied to BL . The beam forming filter bank W_s is of the form $[w_{s,1}, w_{s,2}, \dots, w_{s,2M}]$, where it has $M \times 2M$ dimension. Each beam forming filter steers at a different direction. The normalized m -th beam forming is set up as:

$$\underline{w}_{s,m} = \frac{1}{M} [1, e^{-j\pi \frac{(m-1)}{M}}, \dots, e^{-j\pi \frac{(M-1)(m-1)}{M}}]^T \quad (21)$$

The filter output has $2M \times M$ dimension is:

$$X = W_s^H BL(i) = \begin{bmatrix} \underline{w}_{s,1}^H BL(i) \\ \vdots \\ \underline{w}_{s,2M}^H BL(i) \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ x_{2M} \end{bmatrix} \quad (22)$$

The filter response for direction of k -th user is:

$$\underline{\Gamma}_k = W_s^H \underline{a}(\theta_k) \quad (23)$$

And if we use ULA antenna, then:

$$\underline{\Gamma}_k = \frac{1}{M^2} \begin{bmatrix} \sum_{v=0}^{M-1} e^{-jv\pi \sin \theta} \\ \vdots \\ \sum_{v=0}^{M-1} e^{-jv\pi(\sin \theta - \frac{2M-1}{M})} \end{bmatrix} \quad (24)$$

The signal in i -th block is:

$$SW = \sum_{k=1}^K A_k g_k \underline{\Gamma}_k d_k(i) \underline{c}_k^T \quad (25)$$

If the direction of k -th user is desired and assuming:

$$\begin{cases} E[d_k(j_1)d_k(j_2)] = 0 & j_1 \neq j_2 \\ E[d_k(j_1)d_m(j_2)] = 0 & k \neq m \end{cases} \quad (26)$$

The correlation matrix for the m -th row (i.e. the m -th beam forming filter output) is:

$$R_m = E\{\underline{x}\underline{x}^H\} = E\{\underline{x}_m^T \underline{x}_m^*\} \quad (27)$$

Where:

$$\underline{x}_m = \sum_{k=1}^K A_k g_k d_k(i) \sum_{v=0}^{M-1} e^{-j\pi v(\sin \theta_k - \frac{m-1}{M})} \underline{c}_k^T \quad (28)$$

Thus:

$$R_m = \left(\sum_{k=1}^K A_k g_k d_k(i) \underline{\Gamma}_{k,m} \underline{c}_k \right) \left(\sum_{k=1}^K A_k g_k d_k(i) \underline{\Gamma}_{k,m}^* \underline{c}_k^T \right) \quad (29)$$

Where:

$$\underline{\Gamma}_{k,m} = \sum_{v=0}^{M-1} e^{-j\pi v(\sin \theta_k - \frac{m-1}{M})} \quad (30)$$

(e,f)-th element of this matrix is:

$$\{A_1 g_1 d_1(i) \underline{\Gamma}_{1,m} c_1(e) + A_2 g_2 d_2(i) \underline{\Gamma}_{2,m} c_2(e) + \dots + A_K g_K d_K(i) \underline{\Gamma}_{K,m} c_K(e)\} \times$$

$$\{A_1 g_1 d_1(i) \underline{\Gamma}_{1,m}^* c_1(f) + A_2 g_2 d_2(i) \underline{\Gamma}_{2,m}^* c_2(f) + \dots + A_K g_K d_K(i) \underline{\Gamma}_{K,m}^* c_K(f)\}$$

R_m stands for the MAI plus noise covariance matrix:

$$R_m = R_{N,m} + |A_k g_k \underline{\Gamma}_{k,m}|^2 \underline{c}_k \underline{c}_k^T \quad (32)$$

The direction finding algorithm first identifies a section that desired signal may fall in. for determine the desired section we use the largest responses that belonging to the beam forming filter. To receive this goal we use following function that is constructed for each row [6]:

$$\varphi_m = \frac{1}{\underline{c}_k^T R_m^{-1} \underline{c}_k} \quad (33)$$

Now we can compute equation 33 for each row then select tow largest value of them witch named $M1$ and $M2$. For a given $\sin \theta$ value in the desired section, we can compare two response values into a vector represented as:

$$\underline{b}_s(\sin(\theta)) = \frac{1}{M^2} \begin{bmatrix} \left| \sum_{v=0}^{M-1} e^{-j\pi v \sin(\theta)} \right|^2 \\ \left| \sum_{v=0}^{M-1} e^{-j\pi v(\sin(\theta) - \frac{1}{M})} \right|^2 \end{bmatrix} \quad (34)$$

Where $\sin \theta$ ranges from 0 to $\frac{1}{2M}$. After identification of

target section, fine search is performed. $\underline{b}_s(\sin(\theta))$ can be treated as a steering vector to search in the range $\sin \theta = 0 \sim \frac{1}{2M}$. By collecting the two largest soft values from (33) to form a vector and constructing an orthogonal projector, we obtain:

$$\hat{\underline{\phi}} = \begin{bmatrix} \hat{\phi}_{M1} \\ \hat{\phi}_{M2} \end{bmatrix} \quad (35)$$

$$P_{\hat{\underline{\phi}}} = \frac{\hat{\underline{\phi}} \hat{\underline{\phi}}^T}{\|\hat{\underline{\phi}}\|^2} \quad (36)$$

$$\sin(\hat{\theta}) = \arg \max_{\sin(\theta)} \frac{\|\underline{b}_s(\sin \theta)\|^2}{\underline{b}_s(\sin \theta)^T (I - P_{\hat{\underline{\phi}}}) \underline{b}_s(\sin \theta)} \quad (37)$$

Let \sec stands for the estimated section, then the angle θ_k can be obtained by :

$$\hat{\theta}_k = \begin{cases} \sin^{-1}(\sin(\hat{\theta}) + (\sec - 2M - 1) \frac{1}{2M}) & \text{sec is odd} \\ \sin^{-1}(\frac{1}{2M} - \sin(\hat{\theta}) + (\sec - 2M - 1) \frac{1}{2M}) & \text{sec is even} \end{cases} \quad (38)$$

3.2 Proposed method for asynchronous multipath case

The algorithm for asynchronous multipath fading is an extension of the method for the single-path synchronous case. In this case, we collect the signals of the desired user from all L different paths. The method of DOA estimation for the single-path case (relation (38)) is not applicable directly since the correlation between signals from different paths of the k -th user will hinder its applicability. We should try to decorrelate the received signal of each path from other paths of the k -th user. Without loss of generality we assume that the different paths of users are numbered in increasing order of path delays, i.e. $\tau_{k1} \leq \tau_{k2} \leq \dots \leq \tau_{kL}$ for $k \in \{1, \dots, K\}$. The received vector of N_c samples of chip matched filter synchronised with τ_{kl} (the delay of the l -th path of the k -th user) at any antenna will contain interference of the same symbol from the m -th path of the k -th user, which can be represented by a $N_c \times 1$ vector as:

$$\underline{t}_m = [t_m(0), \dots, t_m(N_c - 1)]^T \quad (39)$$

$$t_m(j) = \int_{jT_c + \tau_{kl}}^{(j+1)T_c + \tau_{kl}} s_k(t - \tau_{km}) \psi^*(t - jT_c - \tau_{kl}) dt \quad (40)$$

In fact \underline{t}_m is the interference of m -th path of the k -th user on the l -th path of the k -th. We define the $N_c \times (L-1)$ matrix C_{kl} with columns \underline{t}_m for $\{m=1, \dots, L, m \neq l\}$

$$C_{kl} = [\underline{t}_1 : \underline{t}_2 : \dots : \underline{t}_{l-1} : \underline{t}_{l+1} : \dots : \underline{t}_L] \quad (41)$$

The column space of matrix C_{kl} (space spanned by columns of C_{kl}) is in fact the interference space caused by the different paths of the k -th user on the l -th path of the k -th user. To decorrelate the received signal from the l -th path of the k -th user and the other paths of the k -th user we just need to project the received vector of N_c samples of the chip matched filter into a space orthogonal to the column space of matrix C_{kl} .

The projection operator into the orthogonal space of C_{kl} is [7]:

$$P_{kl}^\perp = I_N - C_{kl} (C_{kl}^H C_{kl})^{-1} C_{kl}^H \quad (42)$$

In other words, the vector $BL_{kl}^{k\perp}(i) = P_{kl}^\perp BL_{kl}^k(i)$ is not effected by other paths and is totally decorrelated from that user's other paths signals, and then we can apply the previous method to $BL_{kl}^{k\perp}$ and estimate direction of arrival.

4. Simulation

In this section, the performance of the proposed method is evaluated in a multipath DS-CDMA system using gold code sequences of length 31. In all simulations, the receiver has a uniform linear array with half wavelength spacing between adjacent antennas and the antenna elements are assumed to be omnidirectional. There are eight active users ($K=8$) and three paths for each user. The real and imaginary parts of the complex channel gains have been generated randomly by a zero mean Gaussian distribution with unit variance.

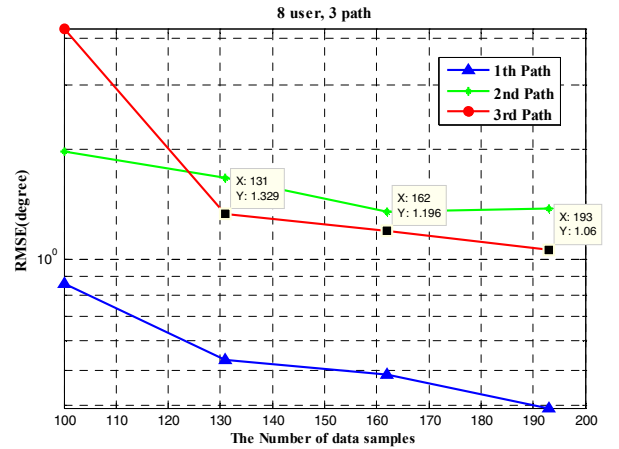


Fig 1. RMSE versus the Number of data samples ($M=3$)

The root mean square error (RMSE) and bias versus number of data samples, SNR and number of array elements are evaluated for the first, second and third path of 1-th user and plotted in Fig 1-6. To compare the performance of the algorithm, we compare this method with algorithms proposed in [8] [9] and [10].

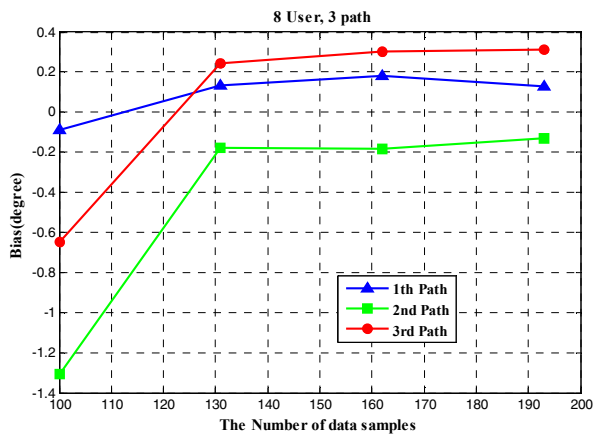


Fig 2. Bias versus the Number of data samples (M=3)

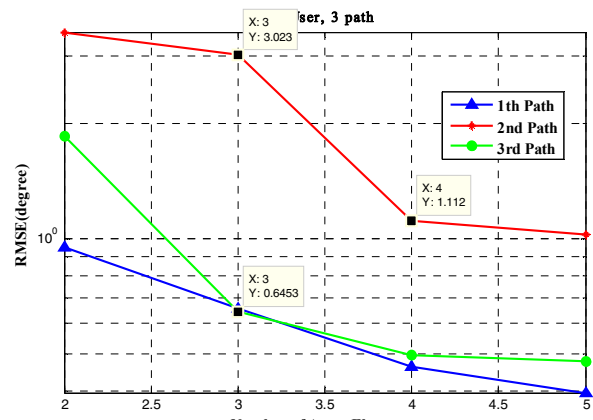


Fig 5. RMSE versus Number of Array Elements

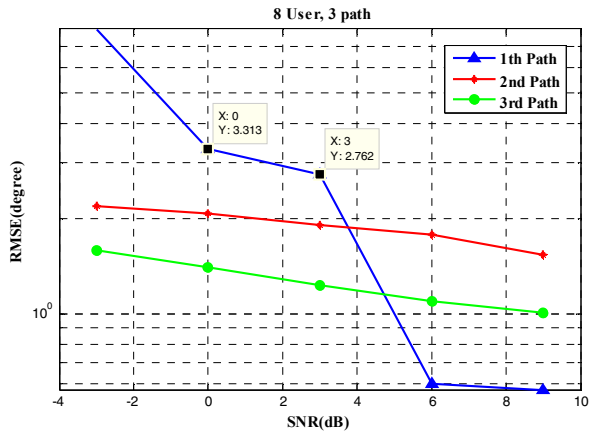


Fig 3. RMSE versus SNR (M=3)

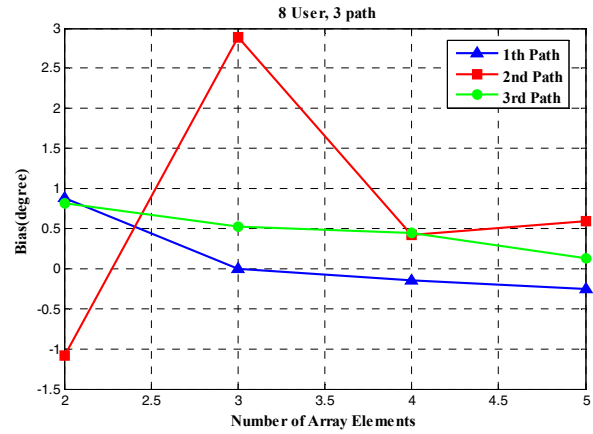


Fig 6. Bias versus Number of Array Elements

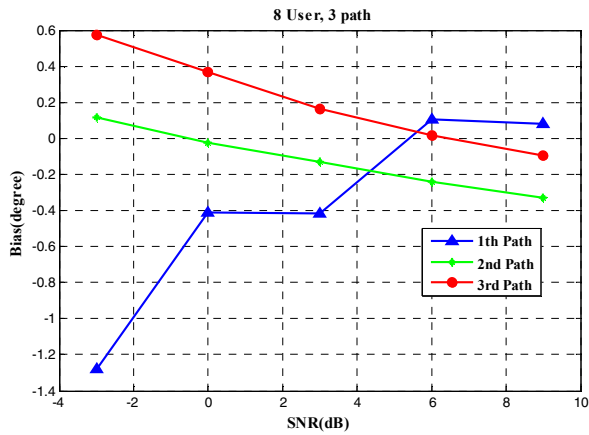


Fig 4. Bias versus SNR (M=3)

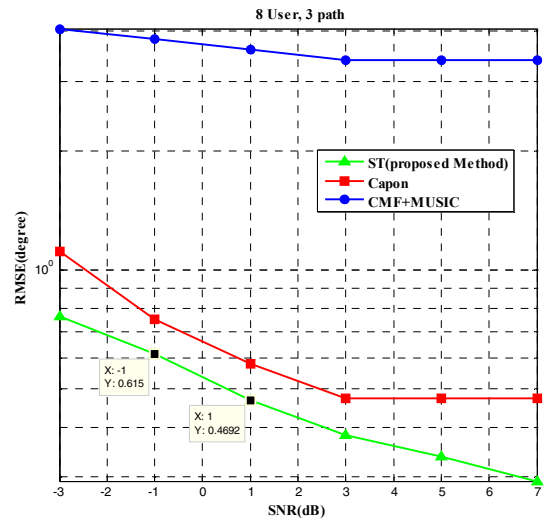


Fig 8. Comparison of RMSE against SNR for three methods

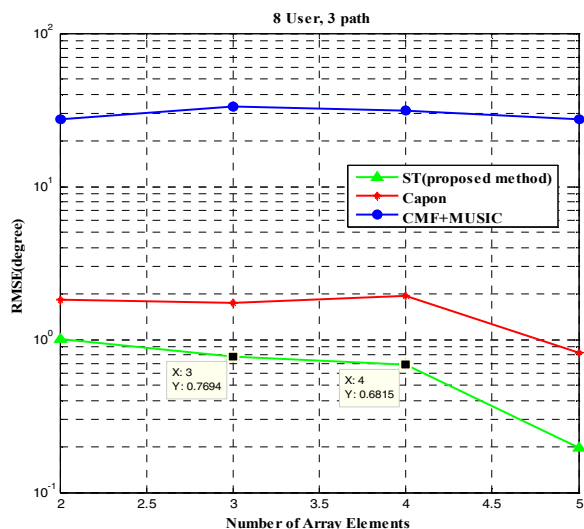


Fig 9. Comparison of RMSE against Number of Array Elements for three methods

5. Conclusion

In this paper a new method for estimating the direction of arrival of signals in frequency-selective fading channel with correlated multipaths is proposed. In this method first, we decorrelate the received signal of each path from other paths, and then pass it from a beam forming filter. In this approach contrary to other DOA estimation approaches, it is not required to search all angles, and it does not require EVD. Because the scenario is equivalent to a DOA estimation of single source in the noise environment, AIC and MDL criterions are not required for estimating the number of sources. Since beam forming filters are used, effects from other users on the desired users signal is also decreased which in turn increase the efficiency of the algorithm and search area decrease almost to one tenth some of the other advantageous of the proposed method are, forming of correlation matrix including vectors with smaller dimension which result to lower complexity of calculation. Also, number of users in this method can exceed the number of antenna arrays. Simulations have shown that suggested approach works properly in the case that the number of sources exceeds the number of array elements

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