A Survey on Topological Observability of Power Systems

Mohammad Shahraeini
Dept. of Electrical Engineering
Ferdowsi University of Mashhad
Mashhad, Iran
shahr@ieee.org

M. Hossein Javidi
Dept. of Electrical Engineering
Ferdowsi University of Mashhad
Mashhad, Iran
h-javidi@ferdowsi.um.ac.ir

Abstract—**Monitoring of system operation conditions is essential for secure operation of power systems. This is traditionally accomplished by state estimators which provide creditable data from raw data acquired from measurement devices. Measurement devices of state estimators may vary from power flow/injection and voltage/current magnitude to phasor measurement unit (PMU). On the other hand, observability is fundamental part of state estimators. Four kinds of observability algorithms are used in state estimators; algebraic, numerical, topological and hybrid. This paper aims to provide a survey of topological observability algorithms. To do this, such algorithms are firstly classified into conventional and PMU based. Then, related studies are investigated and summarized. It is observed that PMU based algorithms are computationally more simple, systematic, and efficient than conventional ones.**

State Estimation; Power System Observability Algorithms; Topological Observability; Phasor Measurement Unit

I. INTRODUCTION

Monitoring of system operation conditions is essential for secure operation of power systems [1]. Historically, in monitoring process, system data are acquired from measurement devices, which are distributed in the entire system, and they are transmitted to the control system through communication systems. After that, received data are processed by some computer aided tools called energy management systems (EMS) [2]. State estimation (SE) is one of the EMS functions which has been known as basis of EMS since it provides creditable data from raw data supplied by measurement devices. Indeed, due to the fact that other EMS functions utilize obtained creditable data, state estimation should be considered as kernel of EMS [2].

Operational data supplied by measurement systems can be classified into the two main groups: First, conventional data e.g. real/reactive power flows-injections, and magnitudes of bus voltages and branch currents which are provided by remote terminal units (RTU) and supervisory control and data acquisition (SCADA). The second one is phasor data which are provided by phasor measurement units (PMU). PMUs use synchronization signals from global positioning system (GPS) satellites, consequently; they provide the positive sequence phasor voltages and currents measured at a given system bus. The second type of system data will in turn improve the performance of EMS functions especially state estimators [1].

State estimation tries to assign values to unknown system state variables based on obtained measurements from system according some criteria [3]. Typically, the state estimators consist of the following functions [3]: Topology processor, observability analysis, state estimation solution, bad data processing, and parameter and structural error processing.

State estimators; based on types of measurement devices, can be classified into conventional, PMU based, and hybrid state estimators. Traditional SEs, using conventional measurements i.e. power flow-injections and voltage magnitudes, are known as conventional SEs. In cases, when state estimations utilize phasor data, they are referred as PMU based SEs. Hybrid SEs use data obtained from both conventional and phasor measurements [2].

In state estimations, the observability of the system is of major concern. A system can be observable if the number of measured independent variables is equal with or more than the number of variables that should be estimated. However, this condition may not necessarily be adequate. The observability of a system is significantly related to the location of data measuring devices and the way they are distributed [4]. Four main types of algorithms are used for observability analysis: algebraic, numerical, topological, and hybrid [5].

The aim of this paper is to provide a survey of power system topological observability. The rest of this paper is organized as follows: In Section II, classical formulation of state estimation will be investigated, and algebraic and topological observability of a power system will be defined. A new classification of topological observability will be introduced as well. Conventional and PMU based observability algorithms based on network topology will be surveyed in Sections III and IV, respectively. Finally, this paper will end with some concluding remarks and future perspective in Section V.

II. POWER SYSTEM OBSERVABILITY

In this section, we first review the classical formulation of the state estimation. Consider an N-bus system provided m

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measurement contained in vector z [3]. The equations relating the measurement z to the state vector x are:

$$z = h(x) + e \qquad (1)$$

where z is m dimensional measurement vector, x is n dimensional state vector and e is measurement error vector. State vector includes $|V|$ and δ at all buses except δ_1, which serves as a reference (i.e. $n = 2N - 1$).

h(x) is a vector function relating measurement vector to the state vector. In conventional and hybrid SE cases, equation (1) may include non-linear equations [3], while in PMU based SE cases, equation (1) is a vector of linear equations [6]. In former cases, because the measurement is non-linear, the state estimation problem is a non-linear estimation problem for which no direct solution is generally available. As a result, iterative method based on successive linearization of measurement equations should be used to calculate the estimate. Consequently, the linearized equations in former cases or equations in latter cases can be rewritten as follows:

$$z = H.x + e \qquad (2)$$

where H is $m \times n$ Jacobian matrix of *h(x)*.

As explained before, four different types of algorithms are used for system observability; algebraic, numerical, topological and hybrid.

- Algebraic observability is defined as the ability of system model to be solved for a state estimate [6]. According to the linearized equation (2), if the matrix H is of full rank and well-conditioned, then the system is algebraically observable.
- The topological algorithms use graph theory and determine network observability strictly based on the type and location of the measurements in the entire system [7].

Remarks: Algebraic observability implies topological observability, but the converse is not always true [5].

A. Topological Observability

In this paper, we aim to provide a survey on topological observability of power systems and its different algorithms. As discussed before, topological algorithms utilize graph theory and determine system observability based on the type and the location of the measurements in the system [7]. As previously mentioned, state estimators; based on measurements; have been classified into conventional, PMU based, and hybrid state estimators [2]. On the other hand, topological algorithms of system observability are not investigated separately based on their measurement types [5]. As a result, for the first time, we generalize classification of estimators into topological observability algorithms. Thus, topological observability algorithms are classified into conventional and PMU based observability algorithms.

According to the Kirchhoff and ohm laws, three kinds of virtual measures can be applied in power systems as follows [8]:

- When the voltage of one node of a branch and the branch current are directly measured by measurements, then the voltage of the other node is virtually measured.
- If the voltages of two nodes of a branch are directly measured by measurements, the current of this branch can be virtually measured.
- If all branch currents of a node are known except one, the unknown branch current is virtually measured.

By using graph theory together with above rules, observability of the power systems can be topologically estimated. In next two sections, the two classes of topological observability algorithms; conventional and PMU based; will be reviewed.

III. Conventional Observability Algorithms

Conventional observability algorithms use real/reactive power flows, power injections, and magnitudes of bus voltages and branch currents for system observability. The idea of topological observability by conventional measurements was introduced for the first time in [9]. Consider an N-bus power system network. Such a system can be represented by non-oriented graph $G = (V, E)$, where V is a set of graph vertices (including all system nodes), and E is a set of graph edges (including all system branches). The sub-graph $T = (V_T, E)$ is defined as a full rank spanning tree of graph G if T contains all of the graph vertices (i.e. V) and exactly N-1 edges (i.e. $E_T \subseteq E$). It is noted that a spanning tree is a loop-free graph. An N-bus system is topological observable if its measurements are placed in the way that at least one full rank spanning tree of measurements is existed [9], [10]. As a result, the sub-graph created by the measurement set should be investigated in order to examine topological observability.

For a power network containing power flow/injection measurements and at least one voltage measurement at reference bus, measurement sub-graph can be generated as follows [3]: measurement sub-graph creation starts out by assigning flow measurements to their respective branches. These branches are assumed as edges of measurement sub-graph and the pair of nodes of each branch are assigned as vertices of measurement sub-graph. If generated sub-graph is a full rank spanning tree, then entire system is observable. If flow measurements do not form a spanning tree, then a forest is yielded in which it consists of several smaller size trees. In the case of establishing a forest by flow measurements, remaining injection measurements are used for merging trees of forest and therefore, the size of the forest is reduced. It is noted that reduction process uses pre-mentioned virtual measurement rules. If at least one spanning tree is caused by this reduction process, the entire system is observed by this set of flow/injection measurements.

This method is known as spanning tree method. The essential steps of pre-mentioned observability algorithm can be summarized as follows [3]:

- Assign all the flow measurements to their respective branches;
- Using virtual measurement rules, the injection measurements are assigned in order to reduce the size of the forest by merging trees of forest.

IV. PMU Based Observability Algorithms

Phasor measurement units are different from other conventional metering devices. By using synchronization signals received from global position system (GPS), the voltage and the current phasors of entire system can be obtained by PMUs, all at once. Due to the fact that PMUs can offer real-time synchronized phasor measurements, PMU applications such as state estimation have been widely used for system security, monitoring and control [2].

According to the pre-mentioned rules for virtual measurement (described in subsection A of Section II), if all system PMUs have enough voltage and current channels, then three main topological observability rules can be concluded as follows [8]:

Rule 1: If a PMU is placed at a bus, this bus and all of its neighbor buses can be observed (Fig 1.a);

Rule 2: For a zero injection node which is observed, if all of its connected nodes are observable except one, then the unobserved node can be observed (Fig 1.b);

Rule 3: If all the nodes connected a zero injection node are observable, then the zero injection node can be observed too, as depicted in Fig 1.c.

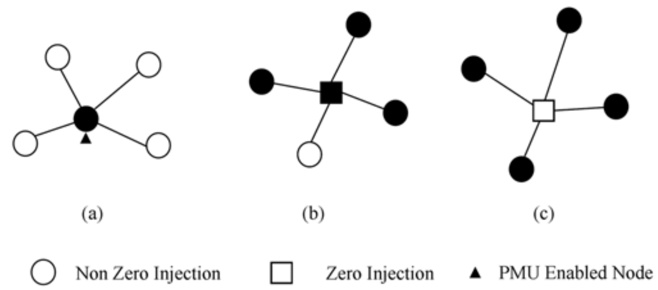

Figure 1. Topological Observability Rules based on PMU.

For systems whose some of their phasors are directly measured by PMUs, above rules together with some topological characteristics of network introduce some topological observability algorithms [1], [6], [11]. In this paper, we review *spanning tree*, *topology transformation* and *augmented incidence matrix* methods.

A. Spanning Tree Method

This method is similar to the spanning tree method described in previous section. The main difference is the way in which measurement sub-graph is generated. This difference results from the difference between PMUs and conventional measurements. The sub-graph is created based on the following criterions [6]:

Criteria 1: Assign a current phasor measurement all branches in which they connect to a PMU enabled bus;

Criteria 2: Assign a pseudo-current measurement to each branch connecting two buses with known voltages;

Criteria 3: Assign a pseudo-current measurement to a branch whose current can be concluded by using Kirchhoff's current law.

Using above criterions, measurement sub-graph can be generated. As same as previous case, if measurement sub-graph have at least one full rank spanning tree (including all system buses and be a loop free), then entire system is topological observable [6].

B. Topology Transformation Method

This method has been introduced by Xu in [1]. At the first of this section, three observability rules were introduced. Considering the rules 2 and 3, if a PMU is placed at a zero injection bus or one of its neighbors, such a node and all of its neighbors are observed by this single PMU. Consequently, the first step of this method is merging zero injection buses to one of their neighbors. This process called topology transformation [1]. As a result of topology transformation, a new network is created. The next step is examination of new network for observability based on rule 1. For this step, the information of new network is represented by adjacency matrix (*A*) as follows:

$$a_{ij} = \begin{cases} 1 & f\ i^{th}\ node\, connected\, to\ j^{th}\ node \\ 0 & otherwise \end{cases} \qquad (3)$$
$$a_{ii} = 1$$

where, the size of *A* is $N - N_Z$, *N* is number of system buses and N_Z is number of zero injection buses.

The locations of PMUs at the system buses are presented by *X* vector. The size of this vector is also $N - N_Z$. The typical array of X (x_i) is one if a PMU is installed at i^{th} bus and zero if not. The observability of entire system can be calculated as the following way:

$$F(X) = A.X > \hat{1} \qquad (4)$$

where, $\hat{1}$ is one vector whose it's all arrays are equal to one. In new network, each row of *F(X)* is pointed to the observability of one of nodes. Zero value of f_i means that corresponding node is not observed; one means such a node is observed by a PMU and if value of f_i is more than one, it can be concluded that corresponding node is observed by more than one PMU.

C. Augmented Incidence Matrix Method

This observability method has been introduced in [11] for the first time. In this approach, the rule 1 described at the first of this section is expanded into two sub-rules as follows:

Rule 1-1: For a branch in which voltage phasor of one of its nodes is known and its current phasor is also known, then the voltage phasor at the other end of the branch can be calculated;

Rule 1-2: For a branch in which voltage phasor of its end nodes are known, then the current phasor of this branch can be calculated.

In this method, three measurement layers; extended-measurement, pseudo-measurement and measurement; are defined. These layers of measurement are represented by *W*, *V*, *U* symbols, respectively. For a power network with *N* nodes and *B* branches, W_V, V_V and U_V are extended-measurement, pseudo-measurement and measurement for node voltage, while W_I, V_I and U_I are extended-measurement, pseudo-measurement and measurement for branch current, respectively. Finally, S_V and S_I are state vectors of bus voltages and branch currents, respectively.

When a set of PMU is installed at system buses, the corresponding extended measurements, pseudo-measurements and measurement can be found from augmented incidence matrix according to the following steps:

1- Measurement Assigning: The directly measurements can be assigned through the following search: first, for a PMU enabled bus, set the node voltage observability $U_V = 1$; then assign a current phasor measurement to each branch incident to this bus ($U_I = 1$).

2- Pseudo-measurement Assigning: Apply rule 1-1 and search every observable current ($U_I = 1$) branch incidence. Assign a voltage pseudo-measurement to their other node ($V_V = 1$). Similarly, apply rule 1-2 and search all the pseudo-voltage nodes ($V_V = 1$); Assign the current pseudo-measurement to the unobservable branch current incident to these nodes ($V_I = 1$). Then, the system observability indices can be calculated as follows:

$$\begin{aligned} S_V(i) &= U_V(i) + V_V(i) \quad for\ i = 1,2,...,N \\ S_I(j) &= U_I(j) + V_I(j) \quad for\ j = 1,2,...,B \end{aligned} \tag{5}$$

3- Extended-measurement Assigning: For all the unmeasured ($U_V = 0$) and observable ($S_V \neq 0$) zero-injection node, check whether observability rule 2 is satisfied; In this case, assign the branch current as current extended-measurement $W_I = 1$; When the new current extended-measurement is searched, repeat applying observability rules 1-2 and 2 to find new extended-measurement, all are assigned as extended-measurement W_V, W_I ; observability rule 3 is checked and if satisfied; assign the corresponding voltage and current as extended-measurements.

At last, by applying above assigning steps, the performance indices of network observability of the PMU set can be calculated as follows:

$$\begin{aligned} S_V(i) &= U_V(i) + V_V(i) + W_V(i) \quad for\ i = 1,2,...,N \\ S_I(j) &= U_I(j) + V_I(j) + W_I(j) \quad for\ j = 1,2,...,B \end{aligned} \tag{6}$$

V. CONCLUSION

Online data and information are essential for power system operation and control. This is traditionally accomplished by state estimators which provide creditable data from raw data acquired from measurement devices. In state estimation applications, system observability is of major concern. Four kinds of algorithms are used for checking observability of a system; algebraic, numerical, topological and hybrid. Algebraic observability is achieved by checking rank of Jacobian matrix (*H*), while topological observability is obtained by using network topology and the locations of measurements in the system. Consequently, topological algorithms are strictly faster than algebraic ones.

This paper is mainly focus on topological observability algorithms. First, topological observability algorithms are classified into conventional and PMU based which are used conventional and phasor measurements, respectively. Second, these classes of algorithms are surveyed and summarized. Summarization of related works indicates that PMU based algorithms are computationally more simple, systematic, and efficient than algebraic algorithms. These resulted from these facts that 1) PMUs directly measure phasor information, and 2) an installed PMU at a system bus can measure phasor of voltage and phasor currents of all incidence branches, all at once. Further investigation should be performed which compare topological observability algorithms in terms of speed and accuracy.

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