Computers, Environment and Urban Systems 33 (2009) 472–481

Contents lists available at ScienceDirect

Computers, Environment and Urban Systems

journal homepage: www.elsevier.com/locate/compenvurbsys

Predicting air pollution using fuzzy genetic linear membership kriging in GIS

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ARTICLE INFO

Article history:
Received 3 January 2009
Received in revised form 13 October 2009
Accepted 13 October 2009

Keywords:
Indicator kriging
Genetic algorithms
Fuzzy
Air pollution

ABSTRACT

Predicting air pollution is an important prerequisite for estimating, monitoring and mapping unknown pollution values. We can use fuzzy spatial prediction techniques to determine pollution concentration areas in practical situations where our observations are imprecise and vague. Fuzzy membership kriging with a semi-statistical membership function is an example of this type of technique. The implementation of fuzzy membership kriging extracts semi-statistical membership functions from data, and applies these functions to an indicator kriging model. Such functions, which can be linear or nonlinear, transform fuzzy data into membership degrees and grades.

Evolutionary genetic algorithms (GAs) can improve prediction efficiency and make it easier to choose an optimum membership function for air pollution applications. In this paper, we used a GA to determine the threshold parameters for a fuzzy membership kriging function based on preprocessed data from Tehran, Iran. We measured particulate matter with a mass median aerodynamic diameter of less than 10 μm (PM10) concentrations at 52 sample stations in Tehran to identify areas that are dangerous for human health. After we predicted the PM10 data, our results showed that GAs reduce the estimated error (3.74) compared to linear functions (8.94 and 12.29). This study indicates that using a GA for optimizing membership functions can get higher estimated accuracy than fuzzy membership kriging for modeling uncertainty in the prediction process of PM10 data.

1. Introduction

Air quality agencies in various countries have tried to improve air quality management policy by mapping, estimating and monitoring air pollution based on particulate matter (PM) levels (Beaulant et al., 2008). This is because a major factor in public health relates to air quality and depends on the concentrations of particulate matter, which has been supported by comparing PM concentrations with life expectancy (Pope et al., 2002). The concentration of particulate matter with a mass median aerodynamic diameter of less than 10 μm (PM10), an indicator for life expectancy, consists of small liquid and solid particles that can easily be inhaled deeply. Based on previous scientific studies, the current standard for the annual allowable average of PM10 is not to exceed 50 $\mu g/m^3$ (Guo, Guo, & Thiart, 2007).

For people with emphysema, asthma and chronic bronchitis, high concentrations of PM10 can cause breathing difficulties. In addition, for older people with heart problems and respiratory diseases, increasing PM10 levels can cause premature death. Therefore, PM10 is commonly considered one of the major factors contributing to problems caused by air pollution (Bealey et al., 2007); thus, it appears that obtaining measurements of air pollutants based on PM10 observations in urbanized regions is essential.

There are some difficulties in accurately using PM10 levels in sample points collected from monitoring stations as an indicator of problems associated with air pollution. For example, when studying the effects of the distribution of PM10 on lung diseases, the use of collected sample data is inadequate to represent the spatial variability of PM10 data within an urban area. Interpolation techniques such as kriging (Krige, 1951) can consider spatial similarities through an interpolation process at unknown locations and thereby overcome this difficulty for health scientists who are studying the spatial variability of air pollution. Moreover, information measured at monitoring stations in the real world is incomplete and imprecise. Thus, it is essential to consider this uncertainty when modeling air pollution. Uncertain geostatistical simulation techniques such as fuzzy membership kriging may provide useful data in this respect. Fuzzy membership kriging includes data of restricted quality in the interpolation procedure and calculates kriged values and estimation variances as fuzzy numbers by their membership functions. Membership functions

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doi:10.1016/j.compenvurbsys.2009.10.004

transform fuzzy data into spatially distributed membership degrees and grades and create an uncertainty measure, which depends both on homogeneity and configuration of the data. Then, the membership function can be extracted from the data. Some authors have proposed semi-statistical membership functions: linear, quadratic or tangent hyperbolic kriging (Guo et al., 2007). A weakness of these methods is that their use depends on case studies and applications. Optimizing fuzzy membership functions with genetic algorithms (GAs) can present a robust way to search efficiently in the large solution spaces of available membership functions in different case studies.

Therefore, the aim of this paper is to optimize the parameters of fuzzy linear membership functions using a GA and evaluating this method for modeling uncertainty in the prediction process of PM10 data. In this way, we predicted and estimated air pollution with a combination of GAs and fuzzy linear membership kriging. Then, we used 52 preprocessed observations of PM10 concentrations in Tehran, analyzed them based on membership functions and estimated the errors for them.

The structure of this paper is as follows. In Section 2, we present several studies based on kriging methods, fuzzy concepts and the advantages of using GAs for prediction. In Section 3, the basic concepts of required kriging algorithms such as indicator, fuzzy membership and GA are defined. In Section 4, we present a case study to demonstrate spatial properties, the reasons for their importance and various characteristics of the data used. Then, in Section 5, we use the case study to evaluate and demonstrate the results of applying the different kriging methods discussed in Section 3. In this section, we analyze the different results obtained when an algorithm is implemented. In Section 6, we discuss and compare the final results to acquire/show conclusions. Finally, the conclusion section outlines the final results and some possibilities for further work.

2. Related work

Kriging is a well-known spatial estimation technique developed by Krige (1951). This method gives an unbiased estimation of unknown locations by minimizing the estimation variance (Stein, Riley, & Halberg, 2001). In other words, kriging is a geostatistical technique to estimate the values of random fields at unobserved points from the observation of values at known locations. Indicator kriging, a variation on kriging, is usually used to approximate the conditional cumulative distribution function at each point of a grid, based on the correlation structure of indicator-transformed data points (Journel, 1983).

Several studies have applied indicator kriging to various domains of application (e.g., Guo et al., 2007; Isaaks & Srivastava, 1989; Ying, 2000). Based on these studies, we think it is clear that combining fuzzy mathematics with kriging (fuzzy kriging) under vague and imprecise conditions can make indicator kriging more efficient. Fuzzy kriging is derived from Zadeh's (1965, 1987) fuzzy theory. The main goal of fuzzy theory is to simplify mathematical models of uncertain situations or indeterminate processes by mapping a two-value crisp function {0, 1} onto an infinite fuzzy function [0, 1].

Some authors have applied this idea to fuzzy kriging (Diamond, 1989; Lee, 2000; Omre, 1987). Guo (2003) generalized Journel's (1983) threshold indicator coding, indicator variogram and indicator kriging methods to the fuzzy membership grade, fuzzy membership grade variogram and fuzzy membership grade kriging methods. Guo simplified these treatments by predicting air pollution based on three semi-statistical membership functions for fuzzy membership grade kriging. In this study, optimal membership functions must be extracted from the data.

GAs make it easier to find the optimal thresholds of membership functions in the complex fuzzy data modeling, improve the accuracy of fuzzy algorithms used in the prediction processes and facilitate fuzzy spatial programming, which is difficult to implement in geographic information system (GIS). Chang, Lo, and Yu (2005) estimated precipitation with GAs and fuzzy inverse distance weighting (IDW). His results confirm that his method is flexible and usually much better than traditional methods. Thus, the main goals of the present paper are to suggest ways to increase the precision of fuzzy membership kriging with GAs and thereby to improve the prediction of PM10-based air pollution in Tehran.

3. Methods

3.1. Indicator kriging

Kriging is an interpolation technique that estimates unknown values from known sample values and semivariograms. The key tool of this method is the variogram, which relates half of the average squared difference between paired data values to the distance between them. Indicator kriging is a nonlinear indicator coding kriging technique that uses the distribution of grades at different thresholds (Journel, 1983). This method can overcome the limitations (normality and independence of estimation variance) of conventional kriging analysis by transforming data into a set of binary variables (Goovaerts, 1997). In fact, indicator kriging transforms data values into crisp indicators as follows:

$$\chi(U(x_i, y_i)) = \begin{cases} 1 & U(x_i, y_i) > T \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

where T is the cut-off (threshold) value and $U(x_i, y_i)$ is a sampled value at the ith spatial location (x_i, y_i). This nonlinear equation can improve predictions substantially. The indicators are analyzed to determine spatial directional variability with a series of experimental variograms as follows:

$$\gamma(d, T) = \frac{1}{2N_d} \sum_{i=1}^{n} [\chi(U((x_i, y_i) + d)) - \chi(U(x_i, y_i))]^2 \tag{2}$$

where d is the distance between two spatial positions $\{(x_i, y_i), (x_i, y_i)+d\}$, T is the predefined cut-off value, N_d is the number of pairs separated by lag distance d and $U(x_i, y_i)$ is an observed sample datum at (x_i, y_i). Inspection of Eq. (2) allows us to select the orientation of greatest and least spatial distribution. Therefore, the indicator values are ordinarily kriged using the variograms to determine the probability of exceeding the cut-off values by replacing Eq. (2) in Eq. (3) and estimating the coefficient W_i.

$$\sum_{i=1}^{n} W_i = 1$$
$$\gamma((x_l, y_l) - (x_i, y_i), T) = \sum_{j=1}^{n} W_j \gamma((x_i, y_i) - (x_j, y_j), T) + \varepsilon \quad i = 1 \ldots n$$
$$\chi(U(x_l, y_l)) = \sum_{i=1}^{n} W_i \chi(U(x_i, y_i)) \tag{3}$$

where (x_l, y_l) is an unknown location, W_i is the desired coefficient value and ε is a LaGrange multiplier to ensure that $\chi(x_l, y_l)$ is unbiased. Thus, the estimated indicator values are a linear function of W_i at known positions. In Eq. (3), W_i is an unknown weight for a measured value at ith location. This parameter depends on the semivariogram, the distance to the prediction location and the spatial relationships among the measured values around the prediction location. The constraint $\sum W_i = 1$ assures us that the predictor is unbiased for unknown measurement. Using this constraint, the difference between the true value and predicated value will be as small as possible. The next two equations work together to measure an empirical semivariogram, fit a model to it, calculate W_is and predict unknown values.

3.2. Fuzzy membership kriging

Phrased in terms of fuzzy logic, kriging is the grade of membership (between 0 and 1) of the probability of exceeding a certain threshold. Converting the distribution of crisp threshold values into fuzzy thresholds gives us a powerful tool for modeling uncertainty in the prediction process. In our application, hazardous effects on the human body can begin at a very low level, e.g., 30 $\mu g/m^3$, and rise to a very severe level, e.g., 50 $\mu g/m^3$ and above (Guo et al., 2007). Fuzzy sets can represent this imprecision and observational vagueness of PM10 data. In fuzzy set theory, the membership function $\mu_Z(x, y)$ can determine the degree to which the value (x, y) belongs to the fuzzy set Z on the universe set U.

$$\mu_Z(x, y) : U \rightarrow [0, 1] \tag{4}$$

The membership degrees and grades can be defined in different ways (e.g., triangular, trapezoidal or Gaussian), based on experience and application characteristics. For example, the trapezoidal membership functions of four parameters $[l_1, l_2, r_1, r_2]$ can model uncertainty on interval observations, as in Fig. 1 and Eq. (5).

$$\mu(x) = \begin{cases} \frac{x-l_1}{l_2-l_1} & l_1 \leqslant x < l_2 \\ 1 & l_2 \leqslant x < r_1 \\ \frac{r_2-x}{r_2-r_1} & r_1 \leqslant x < r_2 \\ 0 & \text{Otherwise} \end{cases} \tag{5}$$

The triangular membership function is a special case of the trapezoidal function when $l_2 = r_1$.

In membership kriging, a typical linear triangular membership function can be defined by:

$$\mu_{T=\{T_1,T_2,T_3\}}(U(x_i, y_i)) = \begin{cases} 0 & U(x_i, y_i) < T_1 \\ \frac{U(x_i,y_i)-T_1}{T_2-T_1} & T_1 \leqslant U(x_i, y_i) < T_2 \\ \frac{-U(x_i,y_i)+T_3}{T_3-T_2} & T_2 \leqslant U(x_i, y_i) < T_3 \\ 0 & T_3 \leqslant U(x_i, y_i) \end{cases} \tag{6}$$

where $\mu_Z(U(x_i, y_i))$ is the membership degree of $U(x_i, y_i)$, and $T = \{T_1, T_2, T_3\}$ is the set of predefined threshold values. Higher values of PM10 cause a higher rate of disease in a given location. Thus, the membership function is defined by Eq. (7) (Fig. 2).

$$\mu_{T=\{T_1,T_2,T_3\}}(U(x_i, y_i)) = \begin{cases} 0 & 0 \leqslant U(x_i, y_i) < T_1 \\ \frac{U(x_i,y_i)-T_1}{T_2-T_1} & T_1 \leqslant U(x_i, y_i) < T_2 \\ 1 & T_2 \leqslant U(x_i, y_i) \end{cases} \tag{7}$$

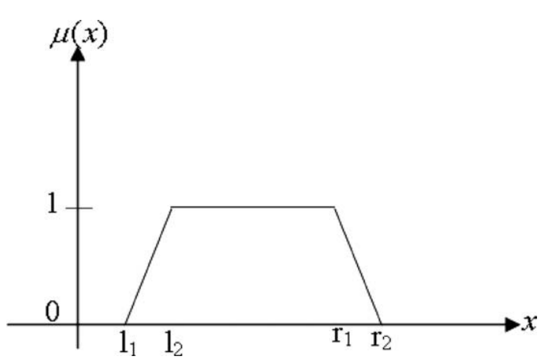

Fig. 1. A trapezoidal fuzzy membership function.

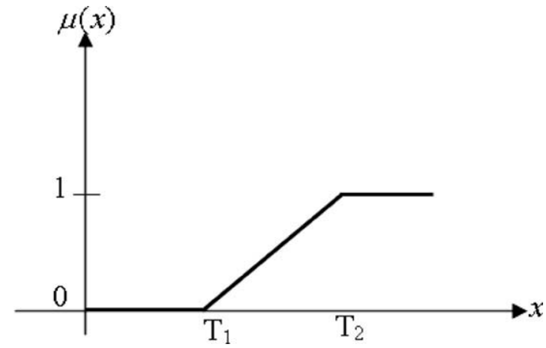

Fig. 2. PM10 sample membership function.

With respect to the above, Eqs. (2) and (3) can be made fuzzy as with Eqs. (8) and (9).

$$\gamma(d, T) = \frac{1}{2N_d} \sum_{j=1}^{n} [\mu_T(U(x_i, y_i) + d) - \mu_T(U(x_i, y_i))]^2 \tag{8}$$

$$\sum_{i=1}^{n} W_i = 1 \tag{9}$$

$$\gamma((x_l, y_l) - (x_i, y_i), T) = \sum_{j=1}^{n} W_j \gamma((x_i, y_i) - (x_j, y_j), T) + \varepsilon \quad i = 1, \ldots, n$$

$$P(\mu_T, C) = \sum_{i=1}^{n} W_i \mu_T(U(x_i, y_i))$$

where $T = \{T_1, T_2, T_3\}$ is the threshold set and $P(\mu_T, C)$ is a fuzzy set that determines fuzzy membership values for each unknown (x_l, y_l) crisp location. Eqs. (7)–(9) show that the definition of the fuzzy membership function can directly affect the prediction process of fuzzy membership kriging. Therefore, adjusting the thresholds of membership functions in an evolutionary procedure to find optimum grading is an essential requirement for validating predictions.

3.3. Genetic algorithms

GAs are a family of computational techniques inspired by evolutionary theory. These algorithms can encode a solution to a specific problem such as a chromosome and apply some selection and recombination operators (such as crossover and mutation) to preserve critical information. This algorithm is often used to optimize functions in various geocomputational applications (Chang et al., 2005).

To implement a GA, you need to begin with a population of random chromosomes. In each generation, the "goodness" of a solution is typically defined with respect to the current population. Selection and recombination operators can generate new sample points within a search space. The "search space" refers to some collection of candidate solutions with a notion of distance between them. Recombination operators can generate progressively better offspring within the search space; crossover and mutation are the most common such operators (Tung, Hsu, Liu, & Li, 2003). The objective function is a mathematical formula that assigns a score of fitness to each chromosome in the current population. The fitness of a chromosome depends on how well that chromosome solves the problem at hand.

In our study, we used a GA to adjust the fuzzy membership function of fuzzy linear membership kriging. For this purpose, the important questions were how to encode each solution, how to evaluate the solutions and how to create new solutions from existing ones (Lee & Pan, 2004). Thresholds are the main component of the membership function that was encoded in our application. Thus, the desired chromosome comprised T_1, T_2, and T_3 genes. In this case, the encoding restriction was defined by Eq. (10). This restriction preserved the meaning of fuzzy sets.

$$C = \{T_1, T_2, T_3\} \quad T_1 \leqslant T_2 \leqslant T_3 \tag{10}$$

The initial population comprised original and randomized C parts. The GA initialized the population by encoding schemata and restrictions and then setting the current population to be the initial population (Fig. 3). The objective function evaluated a chromosome in the current population. If the chromosome did not satisfy the objective function, then the algorithm applied the elitism mechanism to it and selected a new population using the selection mechanism.

Then, the algorithm applied the single-point crossover operation to the chromosomes to form the offspring with a probability between 0.6 and 1. After that, the mutation operator altered each offspring individually with a probability of less than 0.1. Finally, the new population was converted to the current population and evaluated by the objective function. The probability of crossover and mutation depended on the objective function, which was defined based on the mean square error (MSE) as follows:

$$\text{MSE} = \frac{1}{N}\sum_{l=1}^{N}(U_T(x_k, y_k) - U_T^d(x_k, y_k))^2 \tag{11}$$

where N denotes the number of training sample data, $U_T(x_k, y_k)$ represents the result of fuzzy genetic linear membership kriging on training datum (x_k, y_k) and $U_T^d(x_k, y_k)$ denotes the desired output at training datum (x_k, y_k).

4. Study area

Our study area, the city of Tehran, which is located in northern Iran (between 35.56–35.83N and 51.20–51.61E), is a polluted Middle Eastern city. Tehran is bordered by the Alborz mountain range to the north, and it lacks perennial winds. Thus, smoke and other particulate materials cannot escape from the city. Atmospheric pollution in Tehran is primarily due to motor vehicles and heavily polluting industries. Therefore, this area is affected by anthropogenic emissions, and a thick layer of particulate matter is usually found in the atmosphere. Atmospheric pollution, one cause of which is PM10, can affect people's health in many forms. Concentration of PM10 causes deep-lung diseases and directly affects quality of life, so it is important for residents and municipal managers to know which areas of Tehran are safe and which are unsafe.

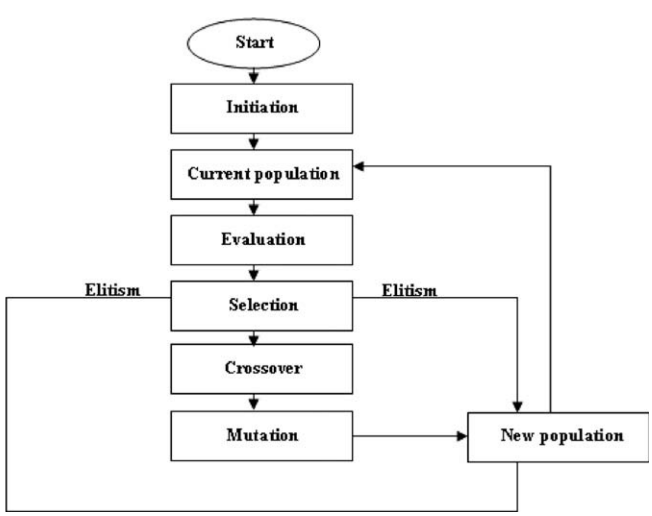

Fig. 3. GA flowchart.

PM10 concentration data have been reported by several air pollution monitoring sample stations in Tehran, and were recorded at 52 locations as positive crisp values (Fig. 4). The measurements of particulate matter were made in urban and suburban sites in the greater region of Tehran. Both kinds of sites were affected by local emissions, and measurements were performed based on surrogate mass collection and simultaneous sampling. In our study, the average of 1-year trajectories of emissions was computed for the year 2007. Thus, the methods mentioned above can be evaluated based on data recorded at the monitoring stations. To evaluate the performance of each model, the mean square error (MSE) was adopted in the implementation phase.

5. Implementing results

This section presents the implementation results of applying the proposed geostatistical methods. For this purpose, a graphical user interface was developed to assist the GIS analysts in evaluating PM10 concentrations using indicator kriging, fuzzy membership kriging and fuzzy genetic membership kriging functions. The interface performs advanced algorithms written in VB.NET and Arcobjects programming languages, and allows users to access different spatial layers.

PM10 sample data, which are stored in ASCII format, were entered into a designed spatial database. A spatial data engine (SDE) allowed the user interface to connect and formulate queries in the spatial database. Therefore, users were able to evaluate all analyzed information, see the required reports and summarize the data in various output forms.

To predict the surface map of PM10 concentrations, the 2007 annual records, which were reported as real positive crisp values at each location, were connected to the interface using an SDE. Then, the ordinary, indicator, fuzzy and genetic kriging algorithms were applied and evaluated on these data. Here we evaluate the usability of 2007 annual records for predicting PM10 concentrations and determining high hazard levels of PM10 in Tehran. These data were preprocessed and corrected based on the accuracy of the measurement tool used before entering them in the prediction process. It is important to know that only 42 sample points were involved in the prediction process, and the others were considered for residual checking using the MSE function we mentioned previously (Eq. (11)).

5.1. Applying ordinary kriging

This prediction method is a classical kriging estimator, which can be applied directly to the spatial observations for modeling linear treatments by linear predictors. Using this method with large smoothing parameters can help to even out some potential errors in the original information. Ordinary kriging is a stochastic interpolation technique that considers two sources of information regarding the attribute: the variation and the distance between points (Alsamamra, Ruiz-Arias, Pozo-Vazquez, & Tovar-Pescador, 2009). This paper focuses on the simple ordinary kriging method for comparison with the proposed models. In this method, we assume $\{U(x, y) = \mu(x, y) + \varepsilon(x, y), (x, y) \in D\}$, where "$(x, y)$" is a spatial location, $\mu(x, y)$ is the simulation output mean over the experimental data and $\varepsilon(x, y)$ is the additive noise with zero mean that represents the variation around the mean. Then, we can say that the expected difference (E) for two sample points (x, y) and $(x, y) + d$ is zero. Accordingly, at an unsampled location (x_0, y_0), ordinary kriging can estimate data values by expressing $U(x_0, y_0)$ as a linear combination of $U(x_i, y_i)$ as follows:

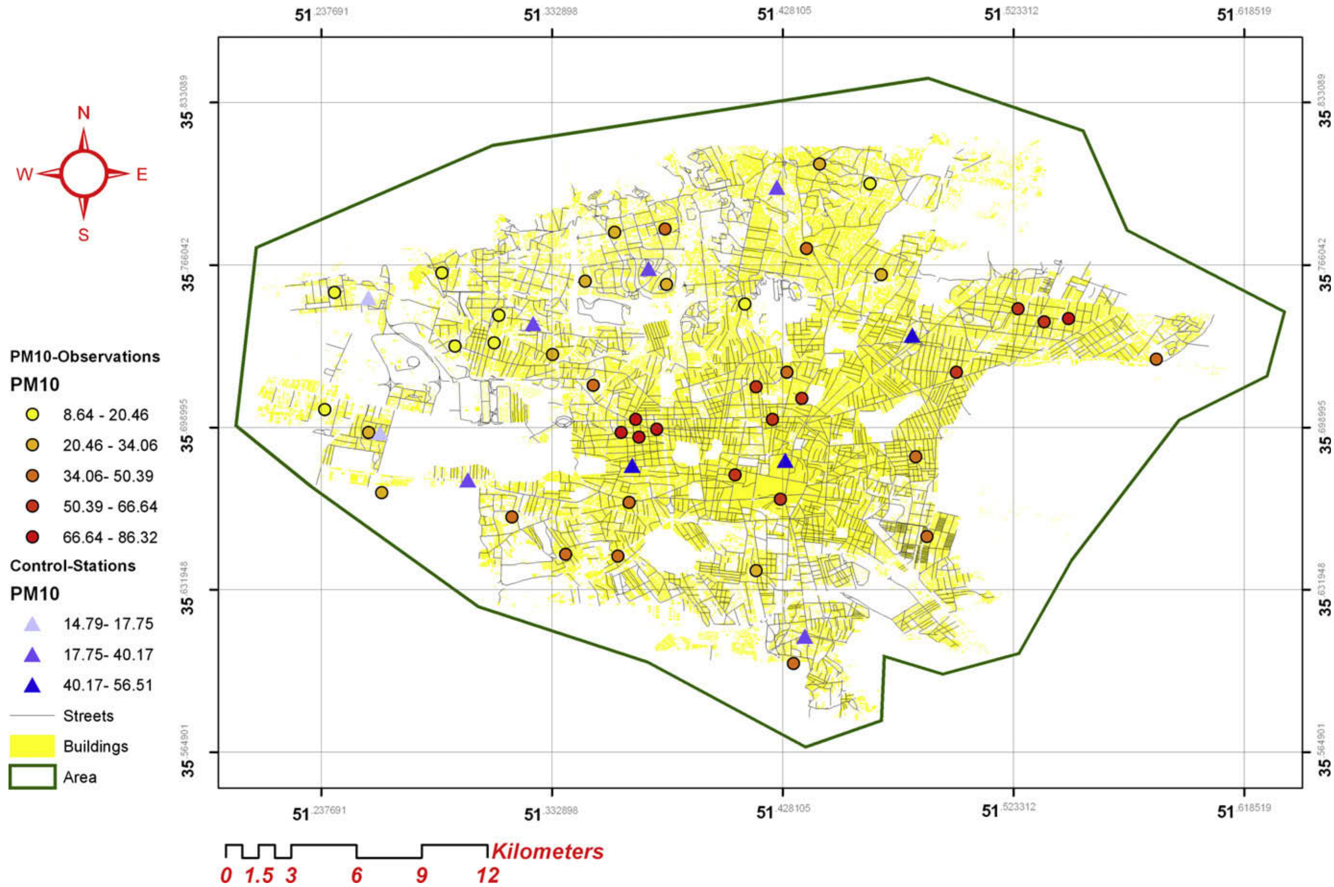

Fig. 4. PM10 data collected in Tehran.

$$U(x_0, y0) = \sum_{i=1}^{n} W_i U(x_i, y_i) + \varepsilon(x_0, y_0) \sum_{i=1}^{n} W_i = 1 \quad (12)$$

where $U(x_0, y_0)$ is the random variable to predict at location (x_0, y_0) and $\varepsilon(x_0, y_0)$ is the noise at position (x_0, y_0) such that $E[\varepsilon(x_0, y_0)] = 0$. In Eq. (12), kriging minimizes the mean square prediction error of sample variations to select weights W_i. For this purpose, the variation between points is measured using semivariograms (Eq. (2)). Fig. 5 demonstrates the semivariogram obtained based on PM10 data for ordinary kriging. In this figure, γ is the semivariogram value plotted on the dependent axis, and h is the separation distance between a pair of points. Ordinary kriging makes use of the best-fit line in the semivariogram (the yellow line in Fig. 5[1]) to predict attribute values at locations where the attribute has not been measured. The equation for this line is the empirical relationship between separation distance and attribute difference. A spherical model (Burrough & McDonnell, 1998) has been used to fit the sample semivariogram in our study.

According to Martin-cob (1996) and Cressie (1993), the assumption needed to perform the spatial prediction and fitting of the theoretical model to the experimental semivariogram is based on how the nugget, range and sill affect the predictor. The nugget effect as an estimate of noise was approximately 0.45, the range value or the distance where the model first flattens out was determined to be 0.34 KM and the sill value or the value at which the semivariogram model attains the range was set 3.28 for the spatial variability of PM10 data. Here, the nugget effect

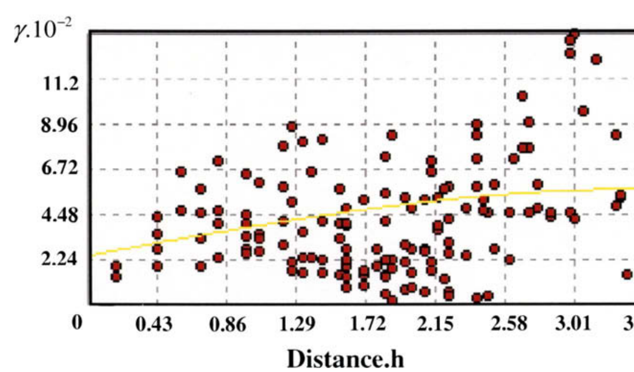

Fig. 5. The semivariogram obtained based on PM10 data.

was attributed to measurement errors or spatial sources of variation at distances smaller than the sampling interval.

Based on the ordinary kriging prediction map in Fig. 6, we can analyze the high hazard levels of PM10 in Tehran. In Fig. 6, the last two classes, which are higher than the threshold value for health concerns (about 50), are considered hazard areas. Actually, the darkest areas in the map, which shows the center of Tehran and some areas to the east, indicate the highest PM10 concentrations and are considered to be hazardous to public health. The population density is high in these areas; thus, quantities of polluting sources have increased dramatically. The lightest parts of the prediction map (i.e., the north, the west and the northwest areas of Tehran), probably due to their higher altitude, are protected from contamination, and thus are the safest areas. The blue text boxes

[1] For interpretation of color in Figs. 4–9,11, the reader is referred to the web version of this article.

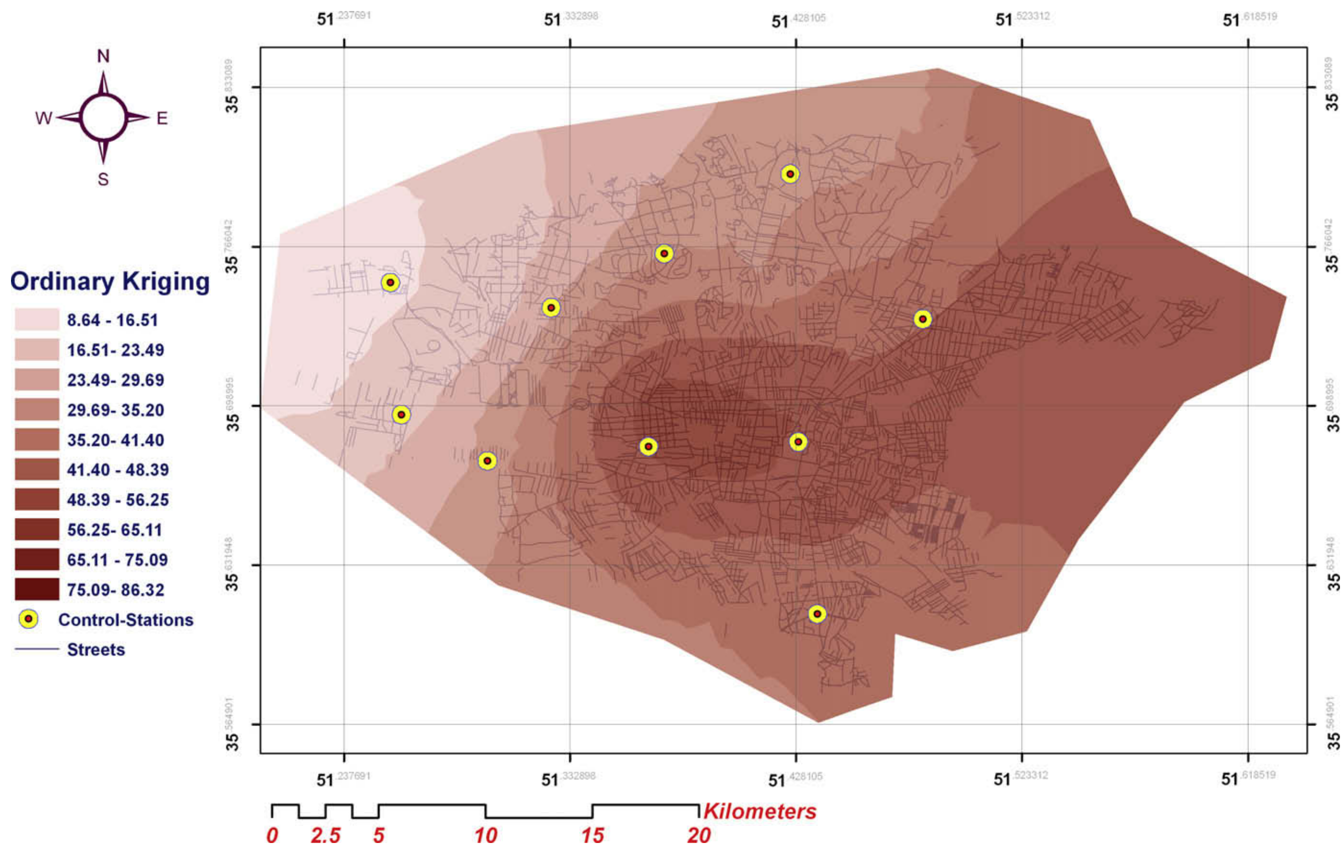

Fig. 6. Map of PM10 concentrations estimated by traditional ordinary kriging.

Table 1
MSEs obtained by applying classic ordinary kriging.

ID	Observed data	Predicted data
3	52.78460	57.41290
4	35.69730	30.73152
5	17.75438	22.85381
10	30.94040	33.90613
12	47.92727	48.66592
14	56.51600	54.31720
24	27.23726	30.41053
25	33.45690	35.59161
33	40.17762	44.81093
50	14.79440	15.56910
		MSE = 12.295455

present the predicted values of ordinary kriging in the control points examined. Ten control points, scattered all over the city, are sorted in Table 1 for MSE calculation and residual checking. These points were selected based on various parameters such as the following:

- Uniform distribution of control points, which is a function of the sample size and the configuration of the sampling location of the observed data.
- Different criteria that are used based on expert knowledge, such as wind direction, water condensation, weather variation, humidity and the position of local anthropogenic pollution sources, influence the accuracy of the 10 control points. For example, in the selected locations, dust particles can be deposited on the measuring device and impact the quality of observations, or wind directions can strongly influence the transport of particulate matters at required points.

The MSE (12.295455) shows less accuracy when applying ordinary kriging for predicting PM10 concentrations in Tehran. This may be a result of vagueness, imprecision of information and insufficient hypothesis-testing issues for modeling nonlinear treatments. It is necessary to point out that modeling vagueness and imprecision in this prediction technique is difficult to implement in spatial environments because of complex mathematical operations.

5.2. Applying fuzzy membership kriging

Applying kriging to indicator data opened a different way to perform spatial predictions. Indicators characterize the spatial variability of categorical variables (Goovaerts, 1997). To use indicator kriging in the prediction process, the information collected from the samples is converted to binary data, with the value 1 assigned to safe areas and the value 0 assigned to unsafe areas. Fig. 7 shows the result of applying indicator kriging (with threshold value of 50 $\mu g/m^3$) on PM10 data for predicting air pollution in Tehran. The blue boxes show the predicted values of indicator kriging at the control points.

Fig. 7 demonstrates significant clustering around the mean, with a smoothing of the results. This indicator map indicates that, in general, the areas where the prediction levels are above the threshold are smaller than those in the result obtained from ordinary kriging. Thus, the distinction of the hazardous regions in Fig. 7 is more evident. In this map, the uncertain zones are associated with the values within the interval of the threshold value around the mean. Therefore, there is not enough confidence in the data to determine whether a location is polluted. However, indicator kriging lacks practical application to the threshold ranges in the prediction process. For example, the hazardous impact of PM10

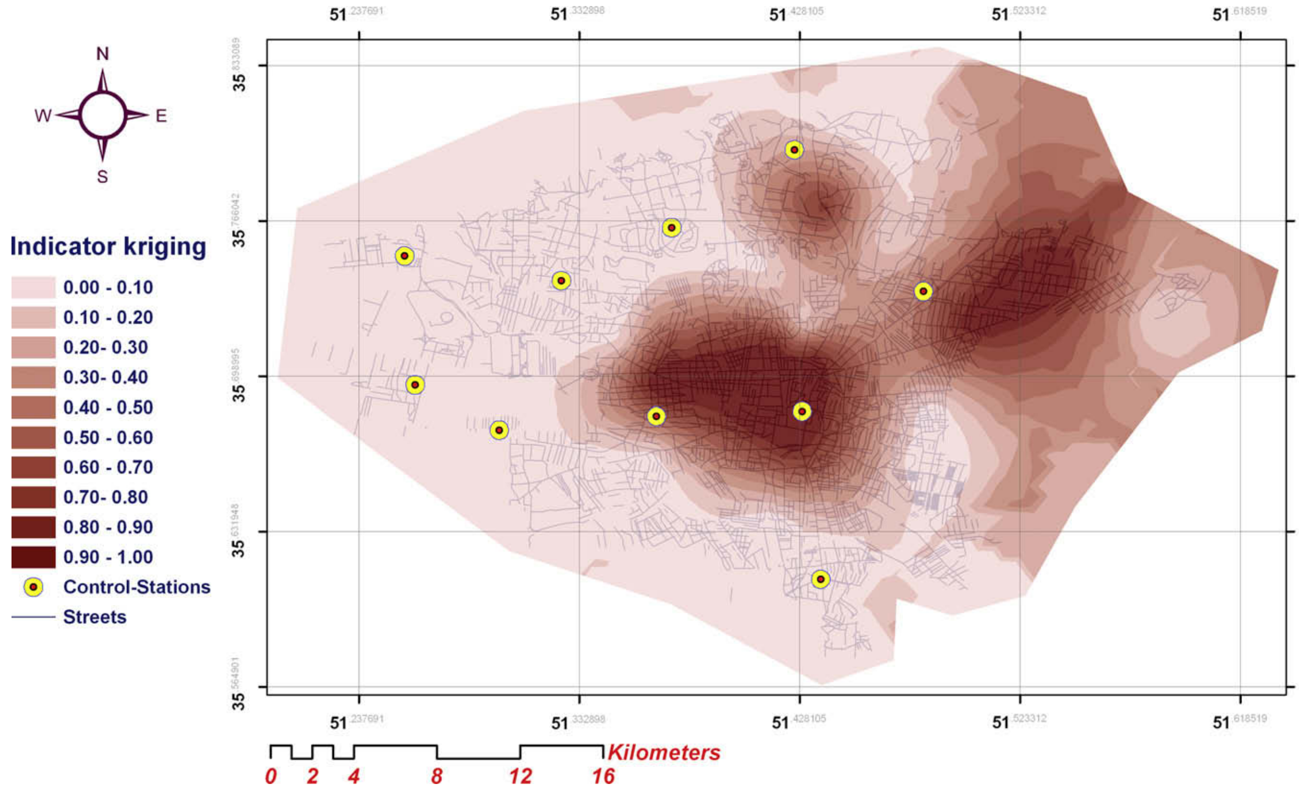

Fig. 7. Map of PM10 concentrations estimated by crisp indicator kriging.

on the human body can start at a very low level, for example, 30, and evolve to a very severe level, 50 and above (Guo et al., 2007). For this uncertain and indeterminate threshold value indicator, kriging transformation is not an effective method because of its crisp properties. Fuzzy membership kriging, which is used in this paper, is a direct extension of indicator kriging, which can extend the {0, 1} of a Cantor set into membership function on [0, 1]. To apply the algorithm of fuzzy membership kriging, it is essential to define a fuzzy membership function. It is easy to find the linear behavior of this membership function in the studied phenomenon that the higher PM10 content in the air, the higher the degree of membership in the fuzzy set of hazardous impact of PM10 on the human body. Then, based on the work of Guo et al. (2007), threshold values for the linear membership function can be defined as follows:

$$T = 0, 9, 90.3\mu_T(U(x_i, y_i)) = \begin{cases} 0 & 0 \leqslant U(x_i, y_i) < 9 \\ \frac{U(x_i, y_i) - 9}{81.3} & 9 \leqslant U(x_i, y_i) < 90.3 \\ 1 & 90.3 \leqslant U(x_i, y_i) \end{cases} \quad (13)$$

Eq. (13) fuzzifies the observed PM10 values for the fuzzy prediction process. In this function, the PM10 membership value at $U(x_i, y_i) = 0$ is designated as 0, and at $U(x_i, y_i) = 90.3$ is designated as 1. The membership function, introduced using expert knowledge, has the large middle bin because of inaccurate resources in the sample data. Raw PM10 data, monitored in 2007, were disturbed by water condensation, weather variation and high absolute humidity. Analyzing the time series based on the total number of trajectories reveals the large variation of PM10 values during different series. The features of each station were quite distinct; the average PM10 concentrations over the period studied exhibited a seasonal variation. Therefore, a broad middle bin is considered for defining linear membership function in Eq. (13). Fig. 8 indicates the result of applying fuzzy linear membership kriging based on Eq. (13). The class divisions show different levels of safety regarding PM10 concentration. In the orthogonal axes of Fig. 8, the darker colored zones represent higher membership values and hazardous PM10 concentrations, and the lighter areas represent lower membership grades and safer PM10 concentrations. Areas of higher PM10 concentrations are located in the middle and northeast of Tehran, and the safest PM10 concentration zones are located in the northwest of Tehran. It is obvious that the interpretation of fuzzy values in this map is a difficult task for users. Therefore, the predicted values were converted back to PM10 to calculate MSE based on control points using membership function.

For this purpose, we need to use a single ‡ – *cut* level to link between fuzzy membership sets and Cantor sets of 10 predicted points as follows:

$$C_\alpha(x_i, y_i) = \{U(x_i, y_i) : \quad \mu_T(U(x_i, y_i)) = \alpha\} \quad (14)$$

where $\alpha \in [0, 1]$ determined by the fuzzy prediction process and $C_\alpha(x_i, y_i)$ is a Cantor set at location (x_i, y_i). Table 2 represents the estimated results of MSE for 10 checkpoints.

The calculation of the total MSE (8.9404) presents more accurate performance of fuzzy membership kriging than the traditional ordinary kriging algorithm. This shows that the spatial variation of PM10 is closer to the mathematical function used in this method for modeling uncertain behaviors of sample data. However, use of this method is limited because thresholds are defined using expert knowledge. To solve this problem, genetic optimization is applied and proposed in the next section.

5.3. Applying fuzzy genetic membership kriging

In this part of the study, we applied a GA to generate fuzzy linear membership kriging to check and evaluate the accuracy of final PM10 prediction results. Then, the proposed GA was implemented for optimum threshold determination. This algorithm can learn and adapt to different components of the defined membership

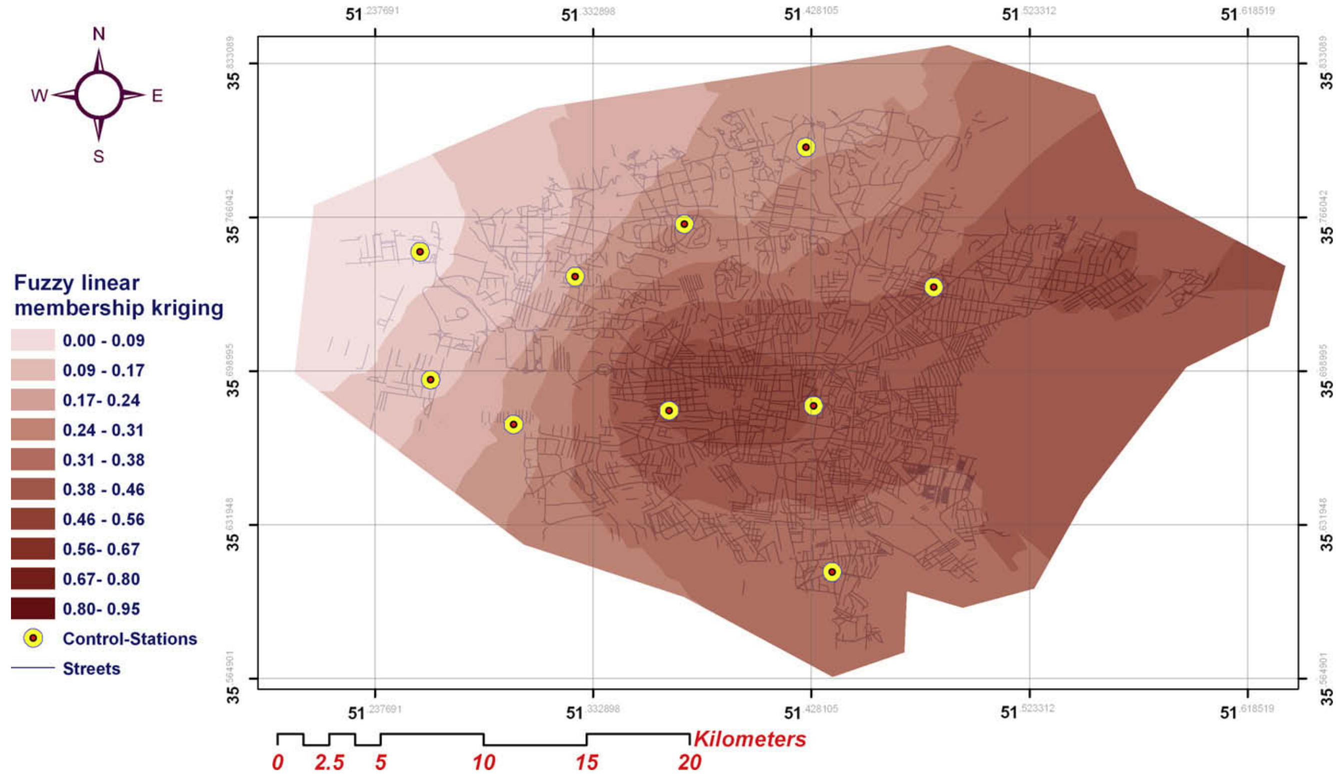

Fig. 8. Map of PM10 concentrations estimated by fuzzy linear membership kriging.

Table 2
MSEs obtained by applying fuzzy membership kriging.

ID	Observed data	Predicted data
3	52.78460	56.97919
4	35.69730	31.12254
5	17.75438	21.35679
10	30.94040	33.57862
12	47.92727	49.35813
14	56.51600	54.47282
24	27.23726	30.21361
25	33.45690	34.59893
33	40.17762	43.92567
50	14.79440	15.50888
		MSE = 8.9404

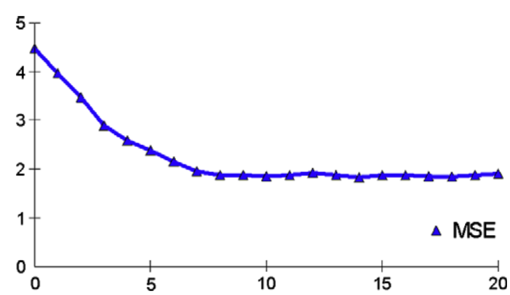

Fig. 9. Convergence curve obtained by applying a GA.

function by optimizing the parameters. For this purpose, it was necessary to encode thresholds using Eq. (10). Then, the initial population was constructed and the algorithm continued to apply, using the designed GA flowchart in Fig. 3. For formation of offspring, some parts of two adjacent chromosomes were exchanged. In this mode, the probability rate between 0.6 and 1 was implemented as the optimum crossover probability. Finally, a mutation operator randomly modified each gene with a probability of less than 0.1. Fig. 9 shows the convergence curve of learning T_1, T_2 and T_3 from a population of eight items, and Table 3 represents results of computing the MSE for data from 10 checkpoints.

The results of the GA showed that $T_g = \{0, 11.25, 86.98\}$ is the best threshold set, with a crossover probability of 0.95 and a mutation rate of 0.1. Implementing fuzzy genetic linear membership kriging using T_g provided the PM10 spatial concentration shown in Fig. 10. In Fig. 10, the last two classes are hazardous areas. The areas of higher PM10 concentration are located in the middle region of Tehran (near Azadi Square) and the northeast of Tehran (near Tehran-pars Square). This method shows the minimum MSE (3.74528), compared with the others.

Table 3
MSEs obtained by applying fuzzy genetic membership kriging.

ID	Observed data	Predicted data
3	52.78460	55.54401
4	35.69730	32.00700
5	17.75438	20.79013
10	30.94040	31.94052
12	47.92727	47.47245
14	56.51600	55.69023
24	27.23726	28.00983
25	33.45690	32.04716
33	40.17762	41.05380
50	14.79440	13.46640
		MSE = 3.74528

6. Discussion and conclusion

The proposed fuzzy genetic membership kriging develops the fuzzy linear membership kriging method and traditional indicator kriging to predict air pollution based on PM10 data. This algorithm improves prediction efficiency and makes it easier to choose and

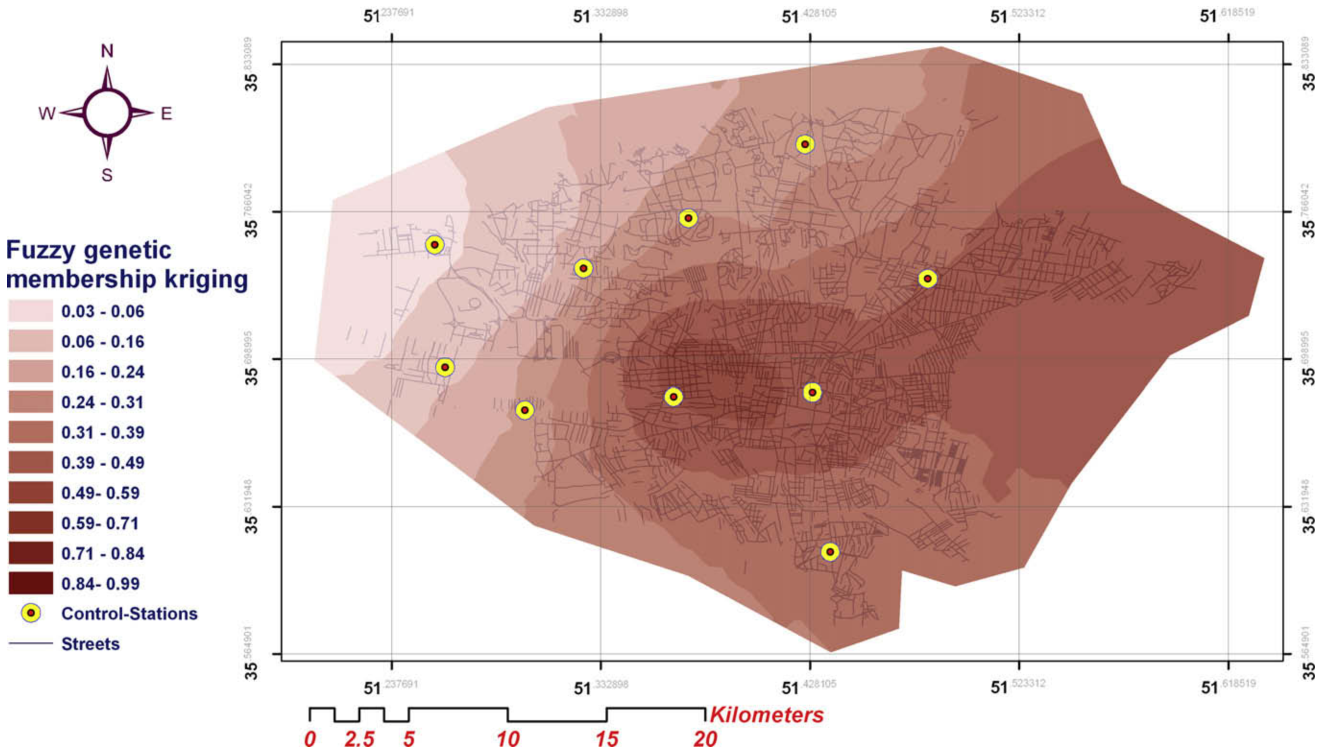

Fig. 10. Map of PM10 concentrations estimated by fuzzy genetic membership kriging.

generate an optimum membership function to find areas where PM10 levels are of high hazardous impact for humans in urban areas. In addition, to define a suitable membership function, the expert's role is reduced, and the user interface is freed from the limitation of different case studies. This approach makes it easy to implement and run the algorithm in a GIS environment and can suggest a flexible way to perform spatial predictions in automatic fuzzy genetic systems. In this way, automatic fuzzy genetic intelligent systems can predict hazardous levels of PM10 data effectively, based on online reports of monitoring stations. This

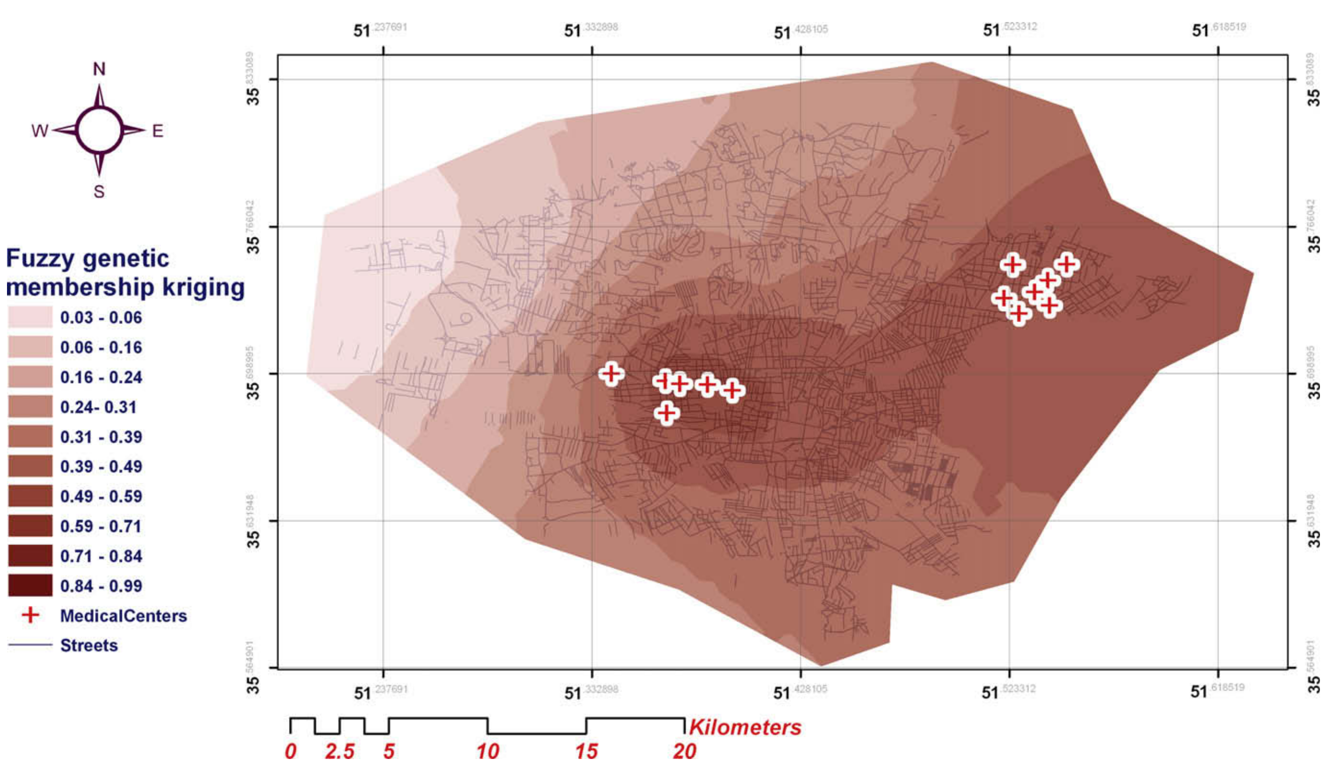

Fig. 11. Investigated hospitals and clinics.

ability to predict enables planners to warn the population against potentially dangerous atmospheric conditions; in addition, it enables decision-makers to examine the possibility of reducing PM10 concentrations in hazardous areas or to improve areas with poor air quality.

Here, fuzzy genetic membership kriging with crossover probability of 0.95 and mutation rate of 0.1 was implemented on 42 recorded PM10 data in Tehran and compared with ordinary and fuzzy membership kriging methods using an MSE calculation of 10 control points. From the data in Tables 1–3, we can determine that the final MSE of 10 control points of GA (0.95, 0.1) was less accurate than ordinary and fuzzy linear membership (with predefined thresholds) for predicting PM10 concentrations. This shows that the spatial variation and treatment of PM10 are closer to the mathematical function used in this method for modeling inaccurate and imprecise behaviors. Furthermore, the uncertainty about spatial variability of PM10 data can be reduced by generating a fuzzy membership function. Therefore, it is reasonable to say that the fuzzy membership relation, which reflects an expert's opinion, can be learned from data sets using genetic algorithms.

From the total MSE shown in Table 1, we can argue that less accuracy of ordinary kriging corresponds to both indeterminate properties of information and insufficient hypothesis-testing issues (the auxiliary information is not spatially exhaustive) for modeling nonlinear relations. Therefore, modeling uncertainty in ordinary kriging is computationally demanding and difficult to implement using GIS. However, this algorithm with large smoothing parameters can help even out some of the potential errors.

Table 2 shows the result of implementing fuzzy membership kriging with linear membership function on PM10 data. The final MSE in Table 2 presents lower rates of error than that in Table 1. This subject indicates higher efficiency, compared to ordinary kriging methods, and suggests the potential of using the fuzzy membership kriging method to predict hazardous areas based on specifying a suitable membership function. The membership function, which plays a key role in the fuzzy membership kriging algorithm, is hypothesized based on an expert's knowledge of sample data. This subject creates a limitation of using different parameters for diverse case studies.

GA makes it easier to find the optimum parameters of membership functions and makes the uncertain prediction process more precise. Moreover, we have shown that the GA can flexibly optimize threshold values and extract optimized membership functions. This is evinced by the high rate of deep-lung diseases among people who live or work in the study areas. In Fig. 10, implementing the GA using T_g indicates that the central part of Tehran (near Azadi Square) and some eastern parts of Tehran (near Tehran-pars Square) are the most dangerous areas for public health. This result is confirmed by statistics on lung diseases from hospitals and clinics around these areas. The spatial distribution of investigated clinics and hospitals (see Fig. 11) and the annual number of lung disease patients (see Table 4) confirms the result of the fuzzy genetic membership kriging for identifying the PM10 concentrations in Tehran. Consequently, the presented fuzzy genetic membership kriging is determined to be ideal for handling uncertainty that depends on vague specification of fuzzy membership function for predicting PM10 data.

In future research, we will adopt various genetic methods for more effective and efficient learning of membership functions, and thereby propose to health scientists an automatic fuzzy genetic system based on predicting PM10 data. In this system, the membership functions will be defined for data and semivariogram parameters. Then, we will use GAs to develop various uncertain kriging methods such as fuzzy ordinary, fuzzy Bayesian and fuzzy indicator to determine the membership functions of data and semivariograms.

Table 4
Number of investigated patients (lung diseases).

ID	Name	Annual patients
120	Emamkhomeini(Near Tehranpars)	3290
121	Nader(Near Tehranpars)	1207
122	Arash(Near Tehranpars)	1505
123	Kadus(Near Tehranpars)	2146
125	Taminejtemaei(Near Tehranpars)	1630
127	Tehranpars(Near Tehranpars)	2403
129	Shahidsamarghandi(Near Tehranpars)	970
68	Farmanfarma(Near Azadi)	2190
70	Pastor(Near Azadi)	3128
102	Azadi(Near Azadi)	2602
104	Babak(Near Azadi)	2210
105	Lola(Near Azadi)	1094
106	Karoon(Near Azadi)	1100

References

Alsamamra, H., Ruiz-Arias, J., Pozo-Vazquez, D., & Tovar-Pescador, J. (2009). A comparative study of ordinary and residual kriging techniques for mapping global solar radiation over southern Spain. *Agricultural and Forest Meteorology, 149*, 1343–1357.

Bealey, W. J., McDonald, A. G., Nemitz, E., Donovan, R., Dragosits, U., Duffy, T. R., et al. (2007). Estimating the reduction of urban PM10 concentrations by trees within an environmental information system for planners. *Journal of Environmental Management, 85*, 44–55.

Beaulant, A. L., Perron, G., Kleinpeter, J., Weber, C., Ranchin, T., & Wald, L. (2008). Adding virtual measuring stations to a network for urban air pollution mapping. *Environment International, 34*, 599–605.

Burrough, P., & McDonnell, R. (1998). *Principles of geographic information systems*. New York: Oxford University Press.

Chang, C. L., Lo, S. L., & Yu, S. L. (2005). Applying fuzzy theory and genetic algorithm to interpolate precipitation. *Journal of Hydrology, 314*, 92–104.

Cressie, N. A. C. (1993). *Statistics for spatial data*. New York: Wiley.

Diamond, P. (1989). Fuzzy kriging. *Fuzzy Sets and Systems, 33*, 315–332.

Goovaerts, P. (1997). *Geostatistica for natural resources evaluation*. New York: Oxford University Press.

Guo, D. (2003). Integrating GIS with Fuzzy logic and Geostatistics: Predicting air pollution PM10 for California using Fuzzy Kriging. MSc thesis, Department of geography, University of Durham, UK.

Guo, D., Guo, R., & Thiart, C. (2007). Predicting air pollution using fuzzy membership grade Kriging. *Computers, Environment and Urban Systems, 31*, 33–51.

Isaaks, E. H., & Srivastava, R. M. (1989). *An introduction to applied geostatistics*. New York: Oxford University Press.

Journel, A. G. (1983). Nonparametric estimation of spatial distributions. *Mathematical Geology, 15*(3), 445–468.

Krige, D. G. (1951). A statistical approach to some basic mine valuation problems on the Witwatersrand. *Journal of the Chemical, Metallurgical and Mining Society of South Africa, 52*, 119–139.

Lee, E. S. (2000). Nero-fuzzy estimation in spatial statistics. *Journal of Mathematical Analysis and Applications, 249*, 221–231.

Lee, C. S., & Pan, C. Y. (2004). An intelligent fuzzy agent for meeting scheduling decision support system. *Fuzzy Sets and Systems, 142*, 467–488.

Martin-cob, A. (1996). Multivariate geostatistical analysis of evapotranspiration and precipitation in mountainous terrain. *Journal of Hydrology, 174*, 19–35.

Omre, H. (1987). Bysian kriging–merging observations and qualified guesses in kriging. *Mathematical Geology, 19*, 25–39.

Pope, C. A., Burnet, R. T., Thun, M. J., Calle, E. E., Krewski, D., Ito, K., et al. (2002). Lung cancer, cardiopulmonary mortality and long-term exposure to fine particulate air pollution. *JAMA, 287*, 1132–1141.

Stein, A., Riley, J., & Halberg, N. (2001). Issues of scale for environmental indicators. *Agriculture, Ecosystems and Environment, 87*, 215–232.

Tung, C. P., Hsu, S. Y., Liu, C. M., & Li, J. S. (2003). Application of the genetic algorithm for optimizing operation rules of the LiYuTan reservoir in Taiwan. *Journal of the American Water Resources Association, 39*, 649–657.

Ying, Z. (2000). IKSIM: A fast algorithm for indicator kriging and simulation in the presence of inequality constraints, hard and soft data. *Computer and Geosciences, 26*(5), 493–507.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.

Zadeh, L. A. (1987). *Fuzzy sets and applications*. New York: Wiley.