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FREE VIBRATION OF AN EMBEDDED FLUID-FILLED SINGLE WALLED CARBON NANOTUBE UNDER AXIAL LOAD USING SHELL CONTINUUM MODEL

Payam Soltani, Roja Bahramian, Javad Saberian

Department of Mechanical Engineering, Islamic Azad University- Semnan branch, Semnan, Iran.

e-mail:p.soltani@semnaniau.ac.ir; payam.soltani@gmail.com

Anoushiravan Farshidianfar

Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

Carbon nanotubes (CNTs) have attracted from scientists and engineers because of their wide range of applications and the superior mechanical properties. Fluid-filled Carbon nanotubes may be used as gas storage tanks or as nanopipes for conveying medicines to a person’s blood stream. Hence, the transport properties of CNTs could be sensitive to their vibrational behavior. Hence, it is essential to consider the mechanical properties of fluid-filled CNTs. In this Paper, the free vibration of an embedded fluid-filled single walled carbon nanotube with simply supported ends is investigated based on the simplified Donnell shell model. When the nanotube is fluid-filled, the fluid is assumed to be an ideal non compression, non rotation and in viscid type and the fluid-structure interaction is described by linear potential flow theory. Furthermore, in this study, the stability of simply supported SWCNT subjected to the axial load is analyzed. The amplitude and frequency of the free vibration are obtained and the effects of internal fluid on the coupling vibration of the model with the different aspect ratios and wave numbers are discussed in detail. Since, the Winkler-type and the Pasternak-type foundation models have been widely used to analyze the effects of the medium stiffness on the vibrational behaviour of CNTs, the surrounding medium with different Winkler and Pasternak constants are considered.

1. Introduction

Carbon nanotubes (CNTs) have attracted from scientists and engineers because of their wide range of applications. With the perfect hollow cylindrical geometry and the superior mechanical properties, these tubes can be used in a variety of technological and biomedical applications to hold fluid such as gas storage tanks[1] or drug-delivery devices[2-3]. There are two major categories for simulating the mechanical properties of Carbon nanotubes: The molecular dynamics approaches (MD) and the continuum mechanics. The molecular simulations are computationally expensive and limited to study the small systems. Hence, the continuum modeling is an efficient method for considering the CNTs characterization. There are two different modeling which are used in continuum mechanics: the beam theories and the shell theories.

In last two decades many continuum structural models have been proposed for considering the Carbon nanotubes characterization. For the first time, Yokobson et al., (1996) [4] used a traditional continuum shell model to predict the buckling of a single walled Carbon nanotube and compared it with the molecular dynamic simulation. Because, transport properties of carbon nanotubes conveying fluid could be extremely sensitive to their vibration mode and the frequencies, considering the mechanical and physical properties of fluid-filled CNTs or carbon nanutubes with conveying fluid was the researchable topic in recent years. Yoon et al. [5] studied the vibration and instability of CNTs conveying fluid with the method of beam mode. Yan et al. [6] and Wang et al. [7] discussed the dynamical stability behaviors of fluid-conveyed CNTs, and found that the natural resonant frequencies depend on the fluid flow velocity and that instability of the CNTs occurs at a critical flow velocity. Although many researches have been done on the dynamic characteristics of CNTs conveying fluid, there are few reports on fluid-filled CNTs vibration and the dynamic behavior of fluid-filled carbon nanotubes still remain many unexplored in the literatures. Dong et al. (2008) [8] studied and calculated characteristics of wave propagation in fluid-filled multi-walled carbon nanotube. Yan et al. (2010) [9] studied the noncoaxial vibration in CNTs and found that the resonant frequencies are decreased due to the effect of flow.

This study, therefore, focus on the free vibration and stability of an embedded empty or fluid-filled single walled carbon nanotube with simply supported ends by using Donnell's cylindrical shell model and the effects of internal fluid on the coupling vibration of the SWCNT-fluid system with the different aspect ratios, the different wave numbers and the different Winkler and Pasternak constants are discussed in detail.

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2. Donnell shell model

Consider a thin-walled simply supported cylindrical shell with radius R, thickness h and length L, as shown in Fig. 1. The linear equation of motion for cylindrical shell is derived by means of variational techniques. [9, 13, 14]

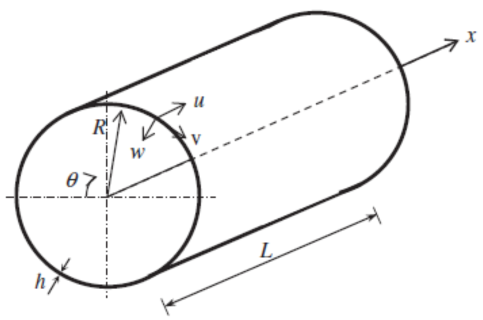


Figure 1. Cylindrical Shell representation of a SWCNT with the coordinate system used

$$\frac{w}{R}+\frac{(1-\nu^2)}{E.h}D.\nabla^4 w=-(\frac{1-\nu^2}{E.h}).\rho_t.\frac{\partial^2 w}{\partial^2 t}. \tag{1}$$

$$\nabla^4 w=(\frac{\partial^2 w}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2})^2$$

X and θ are the axial and circumferential (angular) coordinates respectively, w(x,t) is the radial displacement, t is the time, D is the bending rigidity, ρ_t is the mass density and E is the elastic modulus .

3. Fluid structure interaction

The shell is assumed completely filled with the dense fluid. Furthermore, the fluid is assumed to be an ideal non compression, non rotation and inviscid type. Nonlinearities in the dynamic pressure and in the boundary conditions at the fluid–structure interface are neglected, because fluid movements of the order of the shell thickness may be considered to be small; and hence a linear formulation is valid. Indeed, these nonlinear effects have been found to be negligible by Gonc-alves and Batista (1988) [10]. In addition, pre-stress in the shell due to fluid weight (Hydrostatic effect) is neglected. With these assumptions, the fluid–structure interaction can be described by potential flow theory. [13]

q_f is the flow pressure which is considered as:

$$q_f = \rho_f \left(\frac{L}{m\pi}\right) \frac{I_n\left(\frac{m\pi R}{L}\right)}{I_n'\left(\frac{m\pi R}{L}\right)} \frac{\partial^2 w}{\partial t^2}. \tag{2}$$

Where ρf is the mass density of the internal fluid I_n is the modified Bessel Function of order n and $I_n^{'}$ is the derivative of I_n with respect to R.

4. Elastic Medium

The Winkler spring model and the Pasternak foundation have been widely used to analyze the mechanical properties of embedded CNTs. Fig. 2 shows the analysis model of CNTs embedded in an elastic medium.

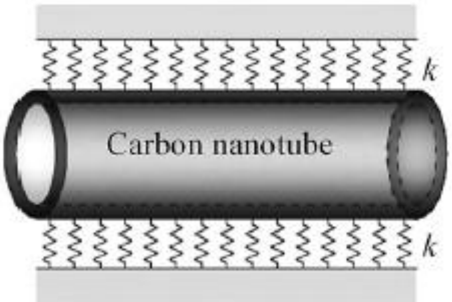


Figure 2. Analysis model of CNTs embedded in elastic medium.

The pressure F acting on the outermost layer due to the surrounding elastic medium can be given by:

$$F = Kw - G\nabla^2 w. \tag{3}$$

K [11] and G [12] are the Winkler spring constant and the Pasternak shear constant respectively.

5. Modelling and formulation

The coupled shell model with fluid-filled and the elastic medium is stated as:

$$\frac{(1-\upsilon^2)}{E.h}D\nabla^4 w+\frac{w}{R^2}+\frac{(1-\upsilon^2)}{E.h}(Kw-G\nabla^2 w)=-\frac{(1-\upsilon^2)}{E.h}(\rho h\frac{\partial^2 w}{\partial t^2}+q_f). \tag{4}$$

The vibration solution for simply supported CNT is approximated by:

$$w=Ae^{J\omega t}\sin(\frac{m\pi x}{L})\cos(n\theta). \tag{5}$$

Where A represents the amplitude of the tube, $J=\sqrt{-1}$, m is the axial half wave number and n is the circumferential wave number.

Substitution of Eq. (5) into Eq. (4), the related frequency can be obtained.

$$a=\frac{1}{R^2}+\frac{(1-\upsilon^2)}{E.h}D((\frac{m\pi}{L})^2+(\frac{n}{R})^2)^2+\frac{(1-\upsilon^2)}{E.h}[K+G((\frac{m\pi}{L})^2+(\frac{n}{R})^2)].$$

$$b=\frac{(1-\upsilon^2)}{E.h}(\rho h+\gamma) \tag{6}$$

$$\gamma=\rho_f(\frac{L}{m\pi})\frac{I_n(\frac{m\pi R}{L})}{I'_n(\frac{m\pi R}{L})}$$

The related frequency is obtained as:

$$.\ \omega^2=\frac{a}{b}\longrightarrow\omega=\sqrt{\frac{a}{b}} \tag{7}$$

When the fluid is not considered, the frequency of the SWCNT is:

$$.\ a=\frac{1}{R^2}+\frac{(1-\upsilon^2)}{E.h}D((\frac{m\pi}{L})^2+(\frac{n}{R})^2)^2+\frac{(1-\upsilon^2)}{E.h}[K+G((\frac{m\pi}{L})^2+(\frac{n}{R})^2)]$$

$$.\ b'=\frac{(1-\upsilon^2)}{E.h}(\rho h) \tag{8}$$

$$\omega'^2=\frac{a}{b'}\longrightarrow\omega'=\sqrt{\frac{a}{b'}}$$

6. *Results and discussion*

The effects of the fluid and the medium on the frequency of SWCNT are investigated using the proposed method. As a case study, mechanical and dimensional properties of the SWCNT and the fluid are shown in Table 1.

Table 1. The parameters used for model (SWCNT and Fluid) used as case study.

	Fluid	SWCNT			
Parameter	ρ_f	ρ_t	Eh	D	h
Dimension	$Kg.m^{-3}$	$Kg.m^{-3}$	J/m^2	eV	nm
Value	1×10^3	2.3×10^3	360	0.85	0.34×10^{-9}

6.1 Frequencies

Various parameters influence the frequency of the fluid-filled and empty SWCNT. We have focused on the different wave numbers, aspect ratios and the mechanical behaviour of the surrounding medium.

6.1.1 The effect of the different wave numbers

It is observed that the wave numbers (m,n) plays an important role in the frequencies, especially for the case that SWCNT is fluid-filled. Table 2.Shows the more detail for the different frequencies.

Table 2. Frequency in the different wave number.

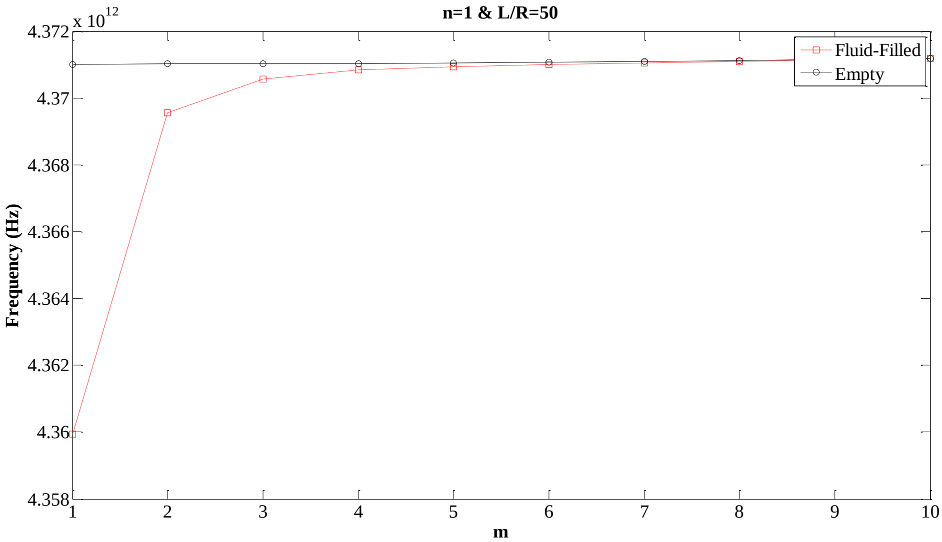
R nm	m	n	L/R	Fluid-Hz $\omega\ (\times 10^{12})$	Without Fluid-Hz $\omega'(\times 10^{12})$
5	1	1	10	4.371	4.371
5	1	5	10	4.4941	4.4944
5	1	10	10	6.0433	6.044
5	5	1	10	4.3732	4.3732
5	5	5	10	4.5184	4.5184
5	5	10	10	6.1126	6.1126
5	1	1	10	4.371	4.371
5	5	1	10	4.3732	4.3732
5	10	1	10	4.3943	4.3943
5	1	5	10	4.4941	4.4944
5	5	5	10	4.5184	4.5184
5	10	5	10	4.6064	4.6064

6.1.2 The effect of different aspect ratios

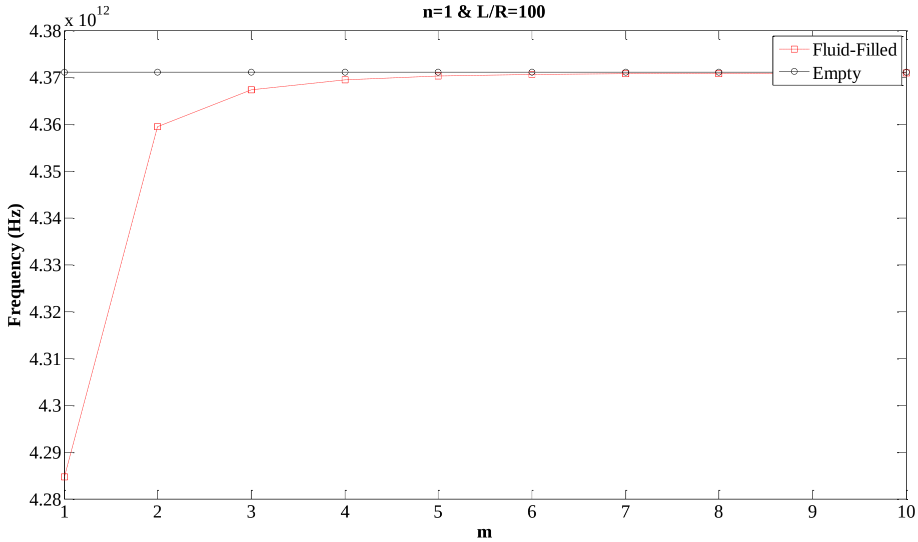
In all cases the frequency increases by increasing the aspect ratios. Fig.3 shows the dependence of the frequency on the axial half wave number with and without fluid. It is observed that when fluid is not existed, the frequencies are around a constant. It means that fluid decreases frequencies in general. Specially, the frequencies change very sharply when m=1 and then rise slowly with the growing of axial half wave number.

6.1.3 The effects of elastic medium

In the third case study, the effects of the foundation constants are observed on the frequencies of the fluid-filled and empty SWCNT. With omitting the Pasternak foundation and changing the Winkler constant, it is seen that the related frequencies are approximately constant up to $K = 1\times 10^{14} N/m^3$. With increasing the K from $2\times 10^{14} N/m^3$, the frequencies increase steadily up to $K = 1\times 10^{19} N/m^3$. But, by increasing the Winkler constant, the frequency plummets. However, the whole shape of diagrams is the same as Fig. 3. It means that when fluid is not existed, the frequencies are around a constant and they change suddenly when m=1 and then level off with the increment of the axial half wave numbers. Furthermore, the growth of the Winkler constant cause the frequencies stabilized. Fig. 4. With ignoring the Winkler foundation and changing the Pasternak constant, it is observed that a little changing in Pasternak constant increase the frequencies. By raising the G from $4\times10^{-3}\,N/m$, the frequencies grow up slowly. The whole shape of the related diagram remains same as Fig. 3 and Fig.4 up to $G=5\times10^{-1}\,N/m$. After that the frequencies rise dramatically. Fig. 5



(a)



(b)

Figure 3. The frequencies on the axial half wave number m SWCNT with and without fluid in the innermost tube

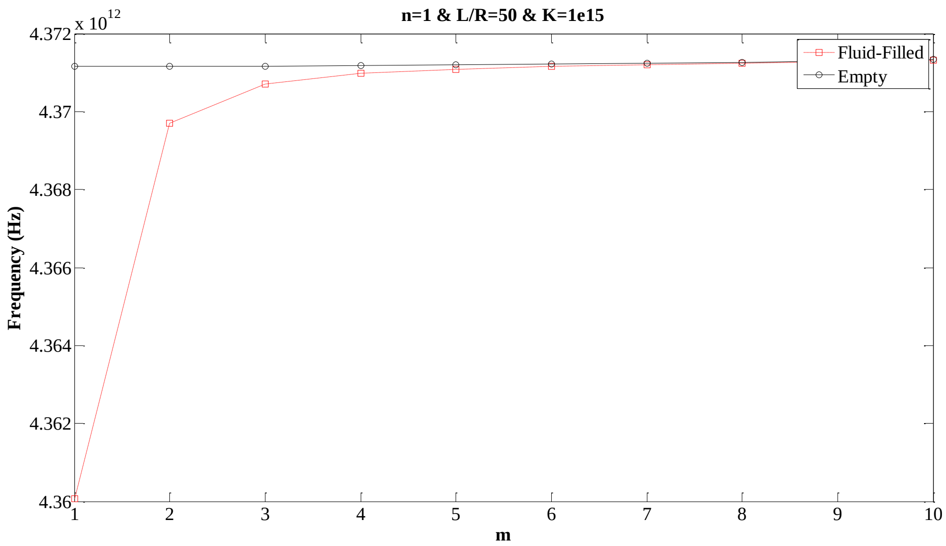


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Figure 4. The effect of Winkler foundation on the SWCNT frequencies with and without fluid in the innermost tube

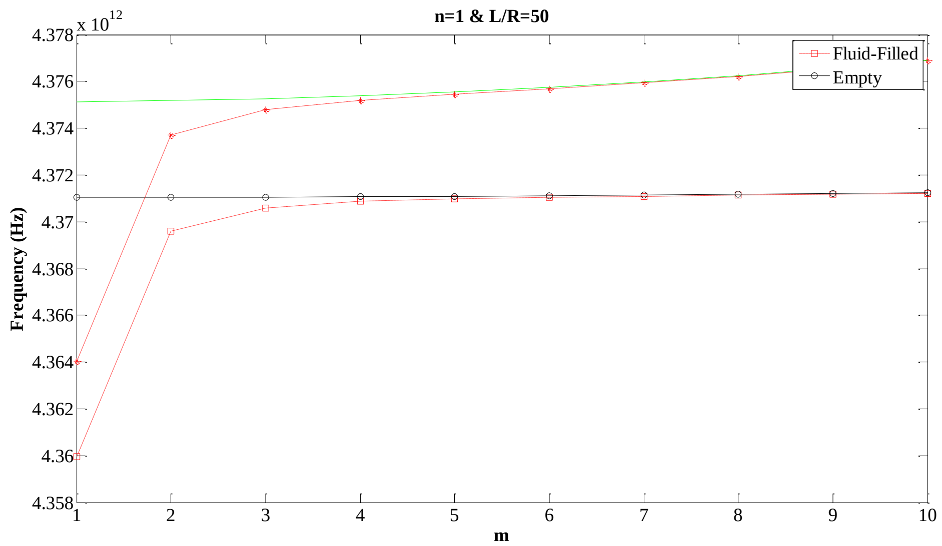

Figure 5. The effect of different Pasternak constants on the SWCNT frequencies with and without fluid in the innermost tube

7. Conclusion

This paper studies the vibration of an embedded fluid-filled SWCNT with the Donnell shell model. The effects of the fluid, different foundation constants and the different aspect ratios on the frequencies are discussed in detail. The results show that fluid makes the frequencies decrease. Also, it is observed that the frequencies increase by increasing the aspect ratios. Furthermore, it is seen that different medium constants cause some changes in the related frequencies. For the Winkler foundation, the frequency changes when K is greater than $1\times10^{14}\,N/m^3$ and rises steadily up to $K = 1\times10^{19}\,N/m^3$. Then at $K = 1\times10^{20}\,N/m^3$, the related frequencies plummet. In addition, with ignoring the Winkler foundation and changing the Pasternak constant, it is observed that a little changing in Pasternak constant increase the frequencies.

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