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Free vibration of carbon nanotubes conveying viscous fluid using nonlocal Timoshenko beam model

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Abstract

The flexural vibration of carbon nanotubes (CNTs) conveying viscous fluid is investigated by the nonlocal Timoshenko beam model. The frequencies are obtained using the Galerkin method. Studying the influence of the fluid viscosity on the fundamental frequency and critical flow velocity of CNTs, is the main aim of this paper. Also, the effects of fluid flow velocity, nonlocal parameter and aspect ratio on the fundamental frequency and critical flow velocity of CNTs are elucidated. Furthermore, different boundary conditions such as clamped-clamped, pinned-pinned, clamped-pinned end conditions have been considered.

Keywords: Carbon nanotube; Viscous fluid; Nonlocal Timoshenko beam theory; Vibration; Instability.

1. Introduction

Carbon nanotubes (CNTs) are cylindrical macromolecules of carbon in a periodic hexagonal arrangement [1]. These cylindrical carbon molecules have novel mechanical, thermal and electrical properties, which make them potentially useful in a wide range of applications such as nanopipes for conveying fluid [2,3], fluid storages, nano-electromechanical systems (NEMS) [4], nanofluidic devices [5], molecular and biological sensors [6], drug-delivery devices [7] and superfibres for composite materials [8]. The most potential applications of the CNTs are dependent on our understanding of their mechanical behaviour. Thus, the mechanical analysis of the CNTs has become a subject of primary interest in recent studies [9–14].

Since the CNTs are extremely small, direct measurement of their mechanical properties is quite difficult. Therefore, computational simulations have been regarded as a powerful tool for the study of their properties. There are two major categories for simulating the mechanical properties of the CNTs: molecular dynamics (MD) simulation and continuum mechanics. It should be noted that MD simulation is very expensive, complex and time consuming, especially for large-sized atomic systems. Recently, several continuum theories have been widely and successfully used to study the mechanics of the CNTs, such as static deflection [15], buckling [16], wave propagation [17], vibration [18] and instability analysis [19]. These continuum theories include, but not limited to, the classical or local continuum theory [13–14], nonlocal continuum theory [17], surface stress theory, couple stress theory [20] and strain gradient theory [21].

In particular, the induced vibration due to fluid flow inside the CNTs raises a significant and challenging research topic because it is a critical issue in the design of the CNT-based fluidic devices. The first contribution to the vibration analysis of the CNTs conveying fluid was probably made by Yoon et al. [19–22]. They developed the Euler–Bernoulli beam model for vibrating CNTs containing fluids, both for the cantilevered and supported systems. It was concluded that the frequencies depended on the fluid flow velocity and, the structural instability of the CNTs could occur at a critical flow velocity. Moreover, they found that the critical flow velocity could fall within the range of practical significance. Wang and Ni [23] further analyzed the fluid-conveying CNT which is developed by Yoon et al. [19] and explored some new phenomena existing in the same dynamical model. Yan et al. [24] studied the instability of triple-walled carbon nanotubes (TWCNTs) conveying fluid based on the Euler–Bernoulli beam model. For the first time in the literature, the Timoshenko beam model has been developed by Khosravian and Rafii-Tabar [25] to study the vibration of multi-walled carbon nanotubes (MWCNTs) conveying a non-viscous fluid. The thermal effects on the vibration and instability of the CNTs conveying fluid have been studied using the Euler–Bernoulli classical beam theory [26].

As the size of CNTs is sufficiently small, the material microstructure becomes more important and cannot be ignored anymore. The classical local elastic models may be no longer accurate enough. Therefore, the theory of nonlocal elasticity has been used to analyze the vibration of fluid-conveying CNTs. The influences of nonlocal effect, viscosity effect, aspect ratio and elastic medium constant on the fundamental frequency of a viscous-fluid-conveying single-walled CNT embedded in an elastic medium have been investigated by Lee and Chang [27]. Based on theory of nonlocal elasticity, a nonlocal beam model has been developed for the vibration analysis of double-walled carbon nanotubes conveying fluid [28] and tubular micro- and nano-beams conveying fluid [29]. Recently, based on thermal elasticity-mechanics and nonlocal elasticity theory, an elastic beam model has been obtained for analysis of dynamical behaviour of fluid-conveying single-walled carbon nanotubes [30].

In this paper, we propose the nonlocal Timoshenko beam model to analyze the vibration and instability of the CNTs conveying viscous fluid. The numerical solutions of the equation of motions are obtained based on the Galerkin method. Influences of the fluid viscosity, nonlocal parameter, aspect ratio and boundary conditions on the fundamental frequency and critical flow velocity of the CNT are examined.

2. Equations of motion

The coupled equations of motion for the nonlocal Timoshenko beam model in displacement form are [31]

$$kGA\left(\frac{\partial^2 w}{\partial x^2}-\frac{\partial \psi}{\partial x}\right)+\left[p_w-\left(e_0 a\right)^2\frac{\partial^2 p_w}{\partial x^2}\right]=m_c\frac{\partial^2}{\partial t^2}\left[w-\left(e_0 a\right)^2\frac{\partial^2 w}{\partial x^2}\right], \tag{1}$$

$$EI\frac{\partial^2 \psi}{\partial x^2}+kGA\left(\frac{\partial w}{\partial x}-\psi\right)+\left[p_\psi-\left(e_0 a\right)^2\frac{\partial^2 p_\psi}{\partial x^2}\right]=I_c\frac{\partial^2}{\partial t^2}\left[\psi-\left(e_0 a\right)^2\frac{\partial^2 \psi}{\partial x^2}\right]. \tag{2}$$

where w is the transverse displacement and ψ is the rotation angle of cross section of the beam. p_w and p_ψ are the distributed forces which induce lateral and angular motion, respectively. A is the cross section area of the beam, m_c is the mass density of the beam per unit length, I is the moment of area of the beam per unit length, I_c is the mass moment of area of the beam per unit length, k is the shear correction coefficient equal to $\pi^2/12$, a is an internal characteristic length, and e_0 is a constant for adjusting the model in matching some reliable results by experiments or other models. E and G is Young's and shear modulus of the beam, respectively.

In this paper, we have assumed that the fluid flow inside the nanotube, considered as a continuum beam, is a plug flow, i.e. the fluid is considered to be an infinitely flexible rod-like structure flowing through the nanotube [32]. Furthermore, the fluid is considered to be a viscous fluid. The plug flow models the interaction of the fluid with the wall. The plug flow induces two forces on the Timoshenko beam. One of these forces, per unit length, is given by [33]

$$p_w = -m_f \left[\frac{\partial^2 w}{\partial t^2} + 2U \frac{\partial^2 w}{\partial x \partial t} + U^2 \frac{\partial^2 w}{\partial x^2} \right] + \mu A_f \left[U \frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x^2 \partial t} \right]. \tag{3}$$

and the other force, per unit length, is given by [32]

$$p_\psi = -I_f \frac{\partial^2 \psi}{\partial t^2}. \tag{4}$$

And produces an angular acceleration in the beam. In the above equations, U is the flow velocity of the fluid, m_f is its mass density per unit length, I_f is its mass moment of area per unit length, A_f is the cross section area of the fluid and μ is the fluid viscosity.

Subtituting Eqs. (3) and (4) into Eqs. (1) and (2), the equations of motion are obtained as

$$\begin{aligned} &\left(m_c + m_f\right) \frac{\partial^2 w}{\partial t^2} + 2m_f U \frac{\partial^2 w}{\partial x \partial t} + m_f U^2 \frac{\partial^2 w}{\partial x^2} - \mu A_f \left(U \frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x^2 \partial t} \right) - kGA \left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \psi}{\partial x} \right) \\ &-\left(e_0 a\right)^2 \left[\left(m_c + m_f\right) \frac{\partial^4 w}{\partial x^2 \partial t^2} + 2m_f U \frac{\partial^4 w}{\partial x^3 \partial t} + m_f U^2 \frac{\partial^4 w}{\partial x^4} - \mu A_f \left(U \frac{\partial^5 w}{\partial x^5} + \frac{\partial^5 w}{\partial x^4 \partial t} \right) \right] = 0, \end{aligned} \tag{5}$$

$$EI \frac{\partial^2 \psi}{\partial x^2} + kGA \left(\frac{\partial w}{\partial x} - \psi \right) - \left(I_c + I_f\right) \left[\frac{\partial^2 \psi}{\partial t^2} - \left(e_0 a\right)^2 \frac{\partial^4 \psi}{\partial x^2 \partial t^2} \right] = 0. \tag{6}$$

3. Solution of the problem

3.1 Boundary conditions

We have considered a beam of length L under three different boundary conditions, namely a beam simply-supported at both ends:

$$w(0,t) = w(L,t) = 0, \frac{\partial \psi}{\partial x}(0,t) = \frac{\partial \psi}{\partial x}(L,t) = 0. \tag{7}$$

a beam clamped at both ends:

$$w(0,t) = w(L,t) = 0, \psi(0,t) = \psi(L,t) = 0. \tag{8}$$

and a beam clamped at one end and simply-supported at the other end:

$$w(0,t) = w(L,t) = 0, \psi(0,t) = \frac{\partial \psi}{\partial x}(L,t) = 0. \tag{9}$$

3.2 Galerkin metod

In this study, the Galerkin method is used to approximate the partial differential equations by a finite set of coupled ordinary differential equations, with the solution expressed as

$$w(x,t) = \sum_{n=1}^{N} X_n(x) W_n(t). \tag{10}$$

$$\psi(x,t) = \sum_{n=1}^{N} \Theta_n(x) \varphi_n(t). \tag{11}$$

where $X_n(x)$ and $\Theta_n(x)$ are orthogonal functions satisfying the boundary conditions and, W_n and φ_n are generalized coordinates. The family of the orthogonal functions are determined in relation to the boundary conditions, and for a pinned-pinned beam are given by

$$X_n(x) = \sin\left(\frac{n\pi x}{L}\right), \Theta_n(x) = \cos\left(\frac{n\pi x}{L}\right). \tag{12}$$

and for a clamped-pinned beam are given by

$$X_n(x) = \sin\left(\frac{n\pi x}{L}\right), \Theta_n(x) = \cos\left(\frac{(2n-1)\pi x}{2L}\right). \tag{13}$$

and for a clamped-clamped beam are given by

$$X_n(x) = \sin\left(\frac{n\pi x}{L}\right), \Theta_n(x) = \sin\left(\frac{n\pi x}{L}\right). \tag{14}$$

Substituting Eqs. (10) and (11) into Eqs. (5) and (6), multiplying the resultant equations by X_m and Θ_m, respectively, and integrating them from x = 0 to L, the following system of equations can be obtained:

$$[\mathbf{M}]\{\ddot{\mathbf{d}}\} + [\mathbf{C}]\{\dot{\mathbf{d}}\} + [\mathbf{K}]\{\mathbf{d}\} = \mathbf{0}. \tag{15}$$

where [**M**], [**C**] and [**K**] denote the symmetric mass matrix, symmetric damping matrix and non-symmetric stiffness matrix, respectively. Also, $(\dot{\cdot})$ denotes $\partial/\partial t$ and $\mathbf{d} = [\mathbf{W}^{\mathrm{T}}\ \boldsymbol{\varphi}^{\mathrm{T}}]^{\mathrm{T}}$.

Substituting the solution

$$\{\mathbf{d}\} = \{\bar{\mathbf{d}}\}\exp(\omega t). \tag{16}$$

into equation (15) we obtain the natural frequencies of the CNT from

$$\left(\omega^2[\mathbf{M}] + \omega[\mathbf{C}] + [\mathbf{K}]\right)\{\mathbf{d}\} = \mathbf{0}. \tag{17}$$

where Im(ω) and $\{\bar{\mathbf{d}}\}$ denote, respectively, the frequency and an undetermined function of amplitude.

To obtain a non-trivial solution of the above equation, it is required that the determinant of the coefficient matrix vanishes, namely,

$$\det\left(\omega^2[\mathbf{M}] + \omega[\mathbf{C}] + [\mathbf{K}]\right) = 0. \tag{18}$$

Therefore, one can compute the eigenvalue numerically from Eq. (18) and obtain the eigenfrequencies of the CNT with various parameter values.

4. Results and discussion

In the following calculations, the parameters of the CNT are the outer radius R_{out} = 0.5 nm, the thickness h = 0.34 nm, the aspect ratio L/R_{out} = 20, the mass density of ρ_{CNT} = 2.3 g/cm^3, Young's modulus E = 1 TPa and poison ratio ν = 0.2. The mass density of fluid is ρ_f = 1 g/cm^3 and the fluid viscosity is μ = 0 Pa.s. The nonlocal parameter is e_0a = 0.

The viscosity effect of fluid on the fundamental frequency of the pinned-pinned CNT is shown in Fig. 1(a). The viscosity effect on the frequency is obvious, especially at large flow velocity. On the contrary, the effect is zero when u = 0. Furthermore, it can be found that increasing the fluid viscosity increases the frequency of CNT. This is because increasing fluid viscosity implies increasing shear force to move and that makes a smaller vibration displacement. In addition, the critical flow velocity increases as the fluid viscosity increases.

The fundamental frequencies of the pinned-pinned CNT are plotted as a function of the internal flow velocity with three different nonlocal parameters e_0a = 0, 1.5 and 2 nm in Fig. 1(b). According to this figure, the nonlocal Timoshenko theory predicts smaller frequencies compared with the local continuum results (e_0a = 0). Actually, the nonlocal theory introduces a more flexible model as the CNT can be viewed as atoms linked by elastic springs while the local model assumes spring constants to take on an infinite value. Consequently, the frequency reduction in the nonlocal model is physically justifiable. Furthermore, the difference between the nonlocal and local results will be significant for higher nonlocal parameter and thus, the small-scale effects can not be ignored. As the flow velocity increases the frequency decreases until it becomes zero. This corresponds to the buckling instability of the CNT. The flow velocity at which buckling instability occurs is called critical flow velocity. As shown in Fig. 1(b), the critical flow velocity decreases when the nonlocal parameter increases.

The effect of the aspect ratio, $L/2R_{out}$, on the fundamental frequency of the pinned-pinned CNT is shown in Fig. 1(c). It can be seen that the frequency and critical flow velocity decrease as the aspect ratio increases.

The fundamental frequencies of a CNT are shown as a function of the internal flow velocity for different boundary conditions in Fig. 1(d). According to this figure, the frequency and critical flow velocity increase as the stiffness of the end conditions of CNT grows up, from pinned-pinned to clamped-clamped.

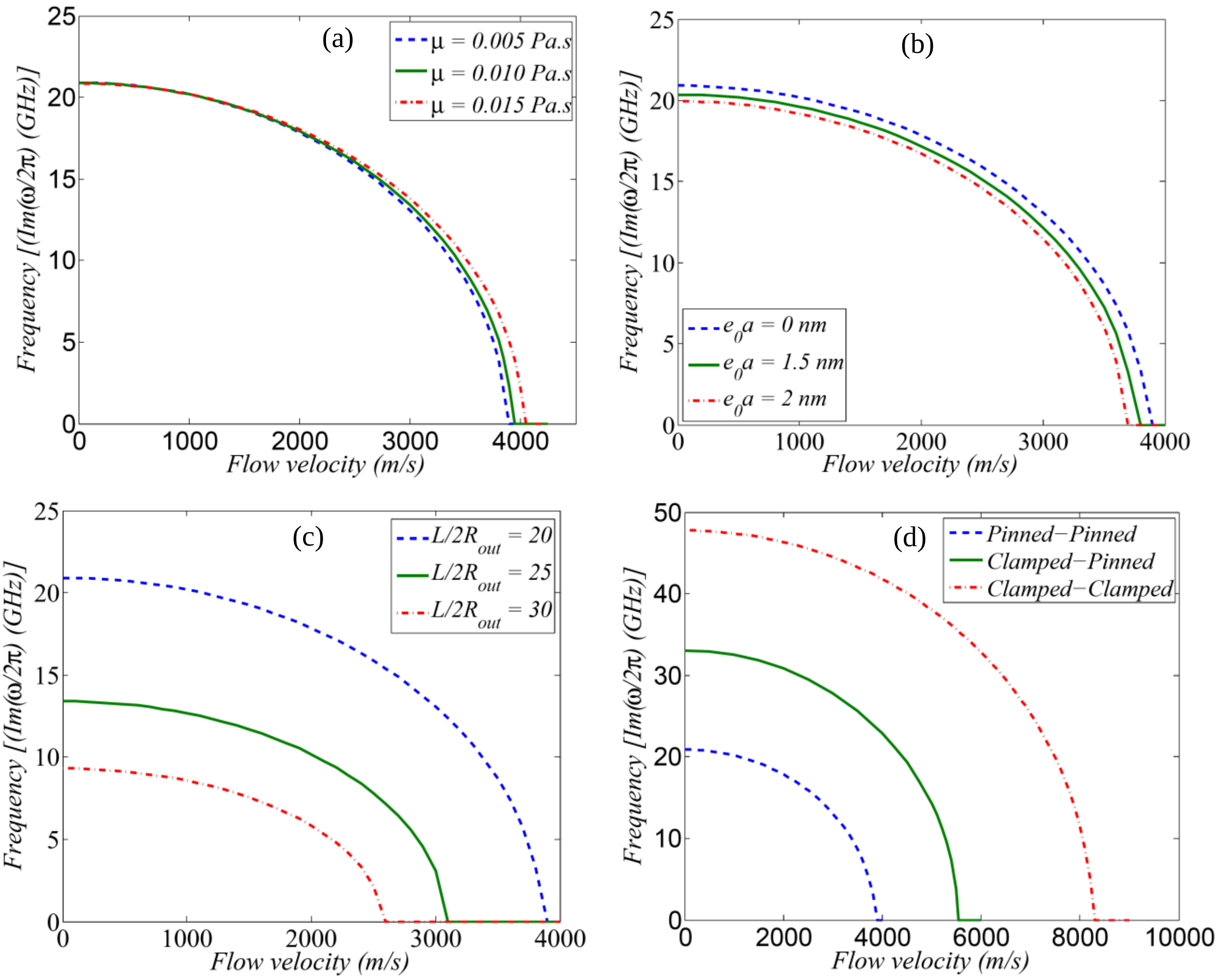

Figure 1. Fundamental frequencies of a pinned-pinned CNT as a function of flow velocity for different, (a) viscous parameters, (b) nonlocal parameters, (c) aspect ratios and (d) boundary conditions.

5. Conclusion

On the basis of the nonlocal Timoshenko beam theory, transverse vibration of CNTs conveying viscous fluid was studied. Considering the viscosity effect of fluid was the main aim of the present paper. It was observed that the influences of fluid flow velocity, nonlocal parameter, aspect ratio and boundary condition on the fundamental frequency and critical flow velocity of CNTs were significant. Based on the numerical results, the following main conclusions were drawn. It was found that the nonlocal Timoshenko theory predicted smaller frequencies compared with the local continuum results and the critical flow velocity decreased when the nonlocal parameter increased. Our results indicated that the fundamental frequency and critical flow velocity of the CNTs increased as the fluid viscosity and stiffness of the boundary conditions of CNTs increased and decreased as the aspect ratio increased. Moreover, increasing the fluid velocity decreased the fundamental frequency.

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