ISAV2011
1st International Conference on Acoustics and Vibration
21-22 Dec 2011
Amirkabir University of Technology
Iranian Society of Acoustics and Vibration

Vibration and instability analysis of viscoelastic single-walled carbon nanotubes conveying viscous fluid embedded in a viscoelastic medium

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Abstract

The vibration and instability behavior of viscoelastic single-walled carbon nanotubes (SWCNTs) conveying viscous fluid and embedded in a viscoelastic medium is analyzed based on the nonlocal Euler–Bernoulli beam model. The equation of motion is derived by employing Newton's method. The frequencies are obtained using the Galerkin method. Investigating the influences of the viscoelastic property of SWCNTs and fluid viscosity on the frequency and critical flow velocity is the main aim of this paper. Furthermore, two additional terms resulting from combination of fluid viscosity and nonlocal parameter are considered in the presented equation of motion in this work, which are neglected in previous equations. Moreover, the effects of fluid flow velocity, nonlocal parameter, aspect ratio, elastic modulus constant and damping factor of the viscoelastic medium on the frequency and critical flow velocity of SWCNTs are elucidated.

Keywords: Viscoelastic carbon nanotube; Viscous fluid; Viscoelastic medium; Vibration; Instability.

1. Introduction

Nowadays, carbon nanotubes (CNTs) are one of the most exciting new materials and have been of great scientific interest to many potential applications in nanobiological devices and nanomechanical systems such as fluid conveyance and drug delivery [1–3]. This is because they exhibit exceptional mechanical, chemical, electronic, and thermal properties [4–5].

In recent years, several types of continuum-based elasticity theories, modelled as an elastic cylindrical tube, have been used to study the nanomechanics and vibration responses of CNTs [6–9]. For example, the Euler–Bernoulli classical beam theory and the Timoshenko beam theory have been employed to the investigation of such mechanical and structural properties of CNTs as buckling stress and strain [6], wave characteristics [8], and resonance frequency [9]. Recently, the continuum-based theories have also been extensively applied to study the vibration behaviours of CNTs filled with fluids by several researchers [10–14]. For instance, Yoon et al. [10] studied the influence of internal moving fluid on free vibration and flow-induced buckling instability of cantilever CNTs based on a continuum elastic model. Reddy et al. [11] examined the free vibration analysis of fluid-

conveying SWCNTs and also studied the effects of the flow velocity on the natural frequency and mode shape. Lee and Chang [13] analyzed the vibration of the SWCNTs conveying viscous fluid with nonlocal Euler-Bernoulli beam model. Ghavanloo and Fazelzadeh [14] investigated the flexural vibration of viscoelastic CNTs conveying fluid and embedded in viscous fluid by the nonlocal Timoshenko beam model.

In this paper, the nonlocal Euler–Bernoulli beam theory is applied to study vibration and instability of viscoelastic SWCNTs conveying viscous fluid and embedded in a viscoelastic medium. The numerical solutions of the equation of motion are obtained using Galerkin method. This study includes the influences of the viscoelastic property of the SWCNTs, fluid flow velocity, fluid viscosity, nonlocal parameter, aspect ratio, elastic modulus constant and damping factor of the viscoelastic medium on the frequency and critical flow velocity of SWCNTs.

2. Equation of motion

Consider the system of Fig. 1, a viscoelastic pipe of length L, internal perimeter S, flow-area A, mass per unit axial length m, and effective flexural rigidity EI, conveying viscous fluid of viscosity μ, mass per unit axial length M, with a steady axial flow velocity U and embedded in viscoelastic medium of elastic modulus constant K and damping factor C. It is assumed that the effect of gravity is negligible. The pipe is considered to be slender, and its lateral motion, $w(x, t)$, to be small.

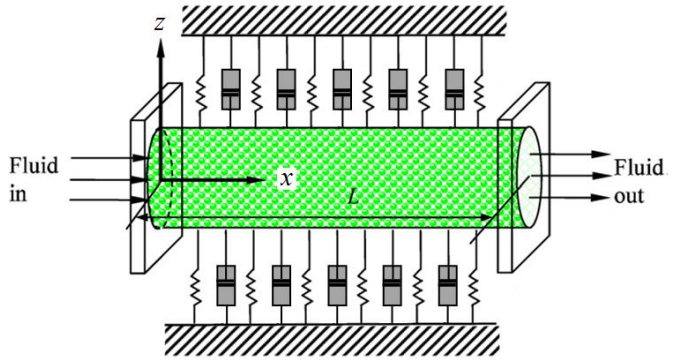

Figure 1. A viscoelastic SWCNT conveying viscous fluid and embedded in viscoelastic medium.

The force exerted on the pipe walls due to the fluid viscosity is given by [15]

$$F_\mu = \mu A\left(\frac{\partial^3 w}{\partial x^2 \partial t} + U\frac{\partial^3 w}{\partial x^3}\right). \tag{1}$$

In what follows, the Newton's laws will be applied to formulate the equation of motion. For that purpose, the fluid and pipe elements will be treated separately. By analyzing the forces and moments acting on the element of the fluid (Fig. 2(a)), the applying of Newton's second law in the x- and z-directions yields

$$-A\frac{\partial p}{\partial x} - qS + F\frac{\partial w}{\partial x} = Ma_{fx}. \tag{2}$$

$$-F - A\frac{\partial}{\partial x}\left(p\frac{\partial w}{\partial x}\right) - qS\frac{\partial w}{\partial x} + F_\mu = Ma_{fz}. \tag{3}$$

where q is the wall-shear stress on the internal surface of the pipe, F is the transverse force per unit length between the pipe wall and fluid, and p is the internal pressure. a_{fx} and a_{fz} are the accelerations of the fluid element in the x- and z- directions, respectively, which are given by [16]

$$a_{fx} = \frac{dU}{dt}, a_{fz} = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 w, a_{pz} = \frac{\partial^2 w}{\partial t^2}. \tag{4}$$

the last equation above is the lateral acceleration of the pipe.

Similarly, for the pipe element of Fig. 2(b) one obtains

$$\frac{\partial T}{\partial x} + qS - F\frac{\partial w}{\partial x} = 0\,. \tag{5}$$

$$\frac{\partial Q}{\partial x} + F + \frac{\partial}{\partial x}\left(T\frac{\partial w}{\partial x}\right) + qS\frac{\partial w}{\partial x} - C\frac{\partial w}{\partial t} - Kw = ma_{pz}\,. \tag{6}$$

$$Q = \frac{\partial \bar{M}}{\partial x}\,. \tag{7}$$

in which T is the longitudinal tension, Q the transverse shear force, and $\bar{M}$ the bending moment. Moreover, $\bar{M}$ can be obtained by [17]

$$\bar{M} = \int_{A_b} z\sigma dA_b\,. \tag{8}$$

where σ is the nonlocal axial stress.

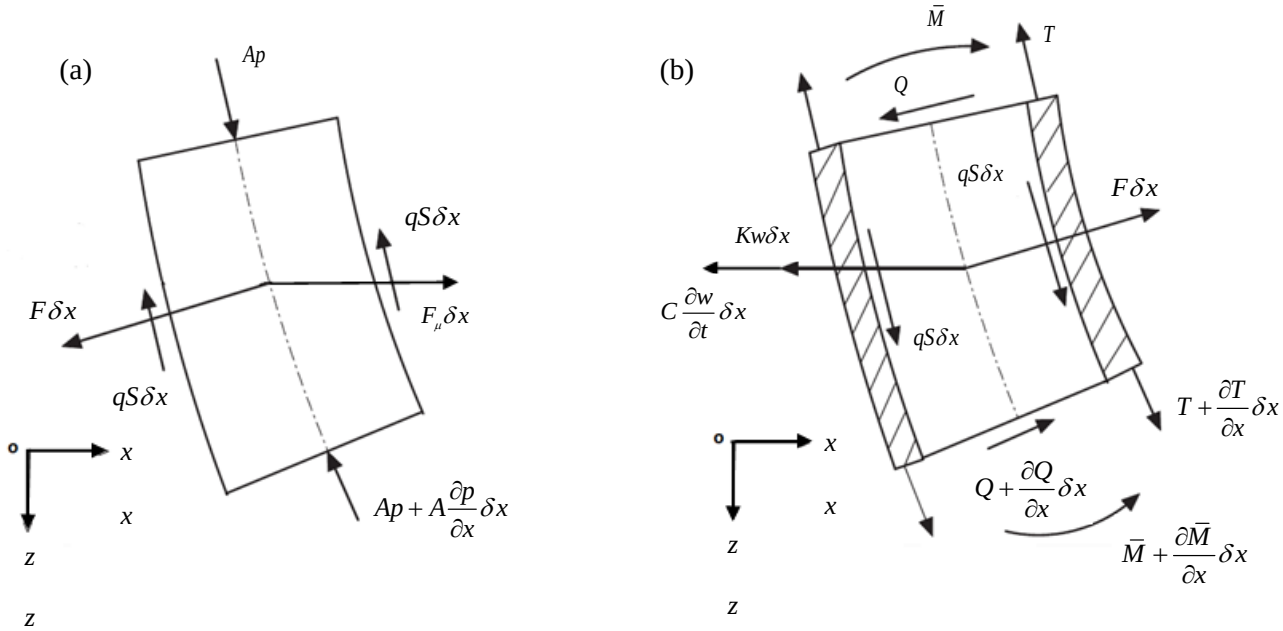

Figure 2. Forces and moments acting on elements of the pipe and the fluid, (a) Fluid element and (b) Pipe element.

According to the theory of nonlocal elasticity, the classic Hooke's law for a uniaxial stress state is given by

$$\sigma - (e_0 a)^2 \frac{\partial^2 \sigma}{\partial x^2} = E\varepsilon = -Ez\frac{\partial^2 w}{\partial x^2}\,. \tag{9}$$

where ε is the axial strain, e_0 a constant appropriate to the pipe material, and a an internal characteristic length (e.g., lattice spacing, granular distance). Here, it ought to be noted that the value of e_0 needs to be determined for each material.

Now substituting Eq. (8) into Eq. (9) yields

$$\bar{M} = \left(e_0 a\right)^2 \frac{\partial^2 \bar{M}}{\partial x^2} - EI\frac{\partial^2 w}{\partial x^2}\,. \tag{10}$$

Based on Kelvin's model on elastic materials with viscoelastic structural damping coefficient g, Young's modulus E is replaced with the operator $E(1+g\,\partial/\partial t)$ [18, 19]. Application of the above operator in the nonlocal constitutive relationships (10) gives

$$\bar{M}=(e_0a)^2\frac{\partial^2\bar{M}}{\partial x^2}-EI\frac{\partial^2 w}{\partial x^2}-EIg\frac{\partial^3 w}{\partial x^2\partial t}. \tag{11}$$

Combining Eqs. (1) – (7) and (11) one obtains

$$\begin{aligned}&EI\frac{\partial^4 w}{\partial x^4}+EIg\frac{\partial^5 w}{\partial x^4\partial t}-\frac{\partial}{\partial x}\left[(T-pA)\frac{\partial w}{\partial x}\right]+M\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2+m\frac{\partial^2 w}{\partial t^2}-\mu A\left(\frac{\partial^3 w}{\partial x^2\partial t}+U\frac{\partial^3 w}{\partial x^3}\right)\\&+C\frac{\partial w}{\partial t}+Kw-(e_0a)^2\left[-\frac{\partial}{\partial x^3}\left[(T-pA)\frac{\partial w}{\partial x}\right]+C\frac{\partial^3 w}{\partial x^2\partial t}+K\frac{\partial^2 w}{\partial x^2}+(m+M)\frac{\partial^4 w}{\partial x^2\partial t^2}+MU^2\frac{\partial^4 w}{\partial x^4}\right.\\&\left.+2MU\frac{\partial^4 w}{\partial x^3\partial t}-\mu A\left(\frac{\partial^5 w}{\partial x^4\partial t}+U\frac{\partial^5 w}{\partial x^5}\right)\right]=0.\end{aligned} \tag{12}$$

If the externally imposed tension and pressurization effect are either absent or neglected, the above equation may be written as

$$\begin{aligned}&EI\frac{\partial^4 w}{\partial x^4}+EIg\frac{\partial^5 w}{\partial x^4\partial t}+C\frac{\partial w}{\partial t}+Kw+MU^2\frac{\partial^2 w}{\partial x^2}+2MU\frac{\partial^2 w}{\partial x\partial t}+(m+M)\frac{\partial^2 w}{\partial t^2}-\mu A\left(\frac{\partial^3 w}{\partial x^2\partial t}+U\frac{\partial^3 w}{\partial x^3}\right)\\&-(e_0a)^2\left[C\frac{\partial^3 w}{\partial x^2\partial t}+K\frac{\partial^2 w}{\partial x^2}+MU^2\frac{\partial^4 w}{\partial x^4}+(m+M)\frac{\partial^4 w}{\partial x^2\partial t^2}+2MU\frac{\partial^4 w}{\partial x^3\partial t}-\mu A\left(\frac{\partial^5 w}{\partial x^4\partial t}+U\frac{\partial^5 w}{\partial x^5}\right)\right]=0.\end{aligned} \tag{13}$$

It can be seen that, when $e_0a = 0$, Eq. (13) may be reduced to the equation of motion of classical Euler–Bernoulli pipes conveying fluid. Also, it is noted that fluid viscosity has been combined with the nonlocal parameter of pipe material (see the last two terms appearing in the square-brackets given in Eq. (13)). It is convenient to introduce dimensionless parameters and variables upon which Eq. (13) becomes

$$\begin{aligned}&\frac{\partial^4 y}{\partial\xi^4}+\alpha\frac{\partial^4 y}{\partial\xi^3\partial\tau}+c\frac{\partial y}{\partial\tau}+ky+u^2\frac{\partial^2 y}{\partial\xi^2}+2m_0^{1/2}u\frac{\partial^2 y}{\partial\xi\partial\tau}+\frac{\partial^2 y}{\partial\tau^2}-\eta\left(m_0^{1/2}\frac{\partial^3 y}{\partial\xi^2\partial\tau}+u\frac{\partial^3 y}{\partial\xi^3}\right)\\&-e_n^2\left[c\frac{\partial^3 y}{\partial\tau\partial\xi^2}+k\frac{\partial^2 y}{\partial\xi^2}+u^2\frac{\partial^4 y}{\partial\xi^4}+\frac{\partial^4 y}{\partial\xi^2\partial\tau^2}+2m_0^{1/2}u\frac{\partial^4 y}{\partial\xi^3\partial\tau}-\eta\left(m_0^{1/2}\frac{\partial^5 y}{\partial\xi^4\partial\tau}+u\frac{\partial^5 y}{\partial\xi^5}\right)\right]=0.\end{aligned} \tag{14}$$

The corresponding dimensionless parameters and variables are defined as

$$\begin{aligned}&y=w/L,\xi=x/L,\tau=\left[EI/(M+m)\right]^{1/2}t/L^2,u=(M/EI)^{1/2}UL,m_0=M/(M+m)\\&e_n=e_0a/L,\alpha=g/L^2\sqrt{EI/(M+m)},\eta=\mu A/\sqrt{EIM},k=KL^4/EI,c=CL^2/\sqrt{EI(M+m)}.\end{aligned} \tag{15}$$

Before closing this section, it should be mentioned that the equation of motion of SWCNTs conveying viscous fluid has been given previously, by and Lee and Chang [13]. However, the last two terms represented in the square-bracket of Eq. (14) have been omitted in their equation. Therefore, the current equation of motion is more reliable.

3. Solutions of the problem

We have considered a SWCNT clamped at both ends. The boundary conditions in this case are given by

$$y(0,\tau)=y(1,\tau)=0,\frac{\partial y}{\partial \xi}(0,\tau)=\frac{\partial y}{\partial \xi}(1,\tau)=0. \tag{16}$$

In equation (14), by applying the Galerkin's method [16], the flexural displacement $y(\zeta,\tau)$ is expanded into the first N vibrational modes,

$$y(\xi,\tau)=\sum_{n=1}^{N} X_n(\xi)\times\psi_n(\tau). \tag{17}$$

where $\psi_n(\tau)$ is an unknown function of time, and $X_n(\zeta)$ is an unknown mode-shape function.

Substituting equation (17) into equation (14), and multiplying the resulting equation by the orthogonality X_m of X_n, and integrating it from $x = 0$ to L, we finally obtain the equation involving $\psi_n(\tau)$ as

$$[\mathbf{A}]\{\ddot{\boldsymbol{\psi}}\}+[\mathbf{B}]\{\dot{\boldsymbol{\psi}}\}+[\mathbf{C}]\{\boldsymbol{\psi}\}=\mathbf{0}. \tag{18}$$

Substituting the solution

$$\{\boldsymbol{\psi}\}=\{\overline{\boldsymbol{\psi}}\}\exp(\beta\tau). \tag{19}$$

into equation (18) we obtain the natural frequencies of the SWCNT from

$$\left(\beta^2[\mathbf{A}]+\beta[\mathbf{B}]+[\mathbf{C}]\right)\{\boldsymbol{\psi}\}=\mathbf{0}. \tag{20}$$

where Im(β) and $\{\overline{\boldsymbol{\psi}}\}$ denote, respectively, the frequency and an undetermined function of amplitude.

To obtain a non-trivial solution of the above equation, it is required that the determinant of the coefficient matrix vanishes, namely,

$$\det\left(\beta^2[\mathbf{A}]+\beta[\mathbf{B}]+[\mathbf{C}]\right)=0. \tag{21}$$

Therefore, one can compute the eigenvalue numerically from Eq. (21) and obtain the eigenfrequencies of the SWCNT with various parameter values.

4. Results and discussion

The geometric and material non-dimensional parameters used in the calculation are as follows: $\alpha = 0$, $\eta = 0.05$, $e_0a/L = 0.05$, $k = 20$, $c = 5$ and $L/d = 100$. The problem is solved with $N = 4$.

Fig. 3 shows the dimensionless frequency β as a function of dimensionless flow velocity u for with different viscoelastic structural damping coefficients, for $c = 5$ and 25. As the flow velocity increases the frequency decreases until it becomes zero. This corresponds to the inducing of buckling instability of the SWCNT. The flow velocity at which buckling instability occurs is called critical flow velocity. It can be found that increasing the viscoelastic structural damping coefficient decreases the frequency and critical flow velocity. This was reported by Ghavanloo and Fazelzadeh [14]. Furthermore, the viscoelastic structural damping effect on the frequency and critical flow velocity increases as damping factor of the viscoelastic medium increases. This wasn't reported before.

The effect of two additional terms in the presented equation of motion on the frequency and critical flow velocity is shown in fig. 4, for $\eta = 0.5$, $c = 0$, 15 and 30. For $c = 0$ and 15, the effect of these terms on the frequency is obvious, especially at large flow velocity and is zero when $u = 0$. Furthermore, it can be found that considering these terms increases the frequency and critical flow velocity. For $c = 30$, the effect of these terms is negligible. But for $c = 40$, considering these terms decreases the frequency and critical flow velocity. Therefore, the effect of two additional terms on

the frequency and critical flow velocity depends on damping factor of the viscoelastic medium. This wasn't reported before.

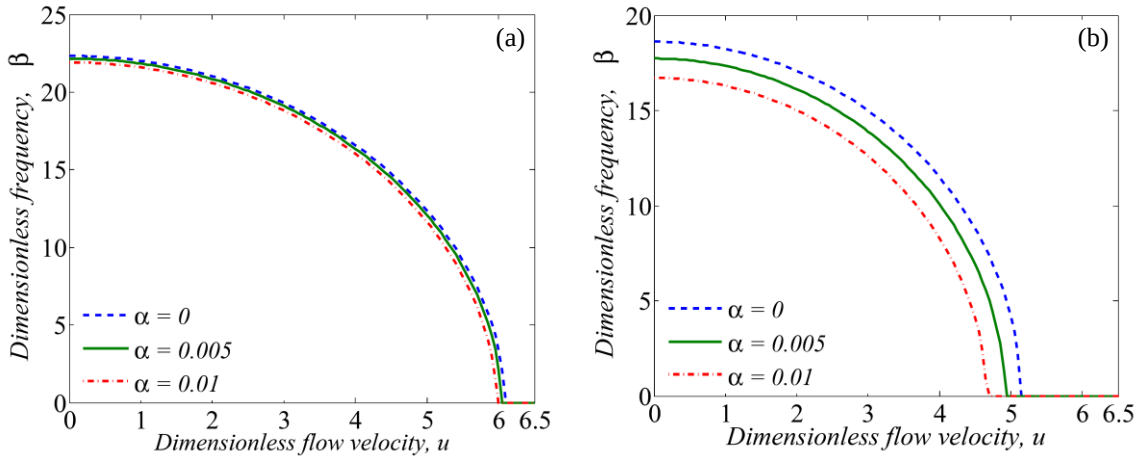

Figure 3. Dimensionless frequency as a function of dimensionless flow velocity with different viscoelastic structural damping coefficients, (a) c = 5 and (b) c = 25.

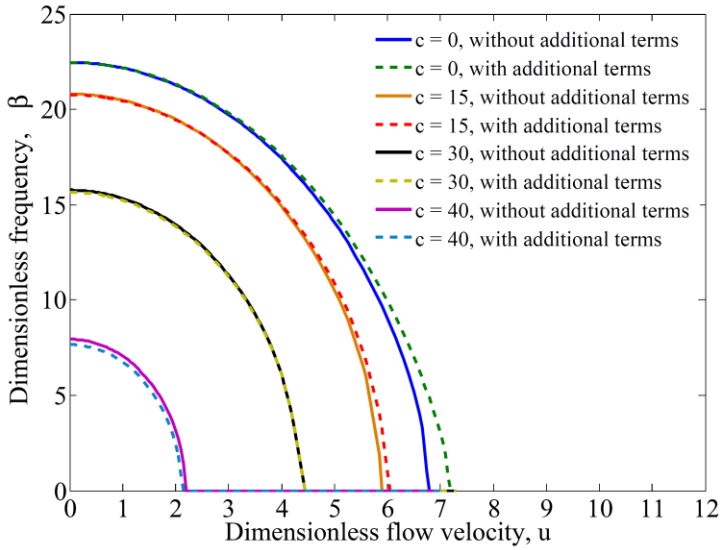

Figure 4. Dimensionless frequency as a function of dimensionless flow velocity, with and without two additional terms for η = 0.5 and c = 0, 15 and 30.

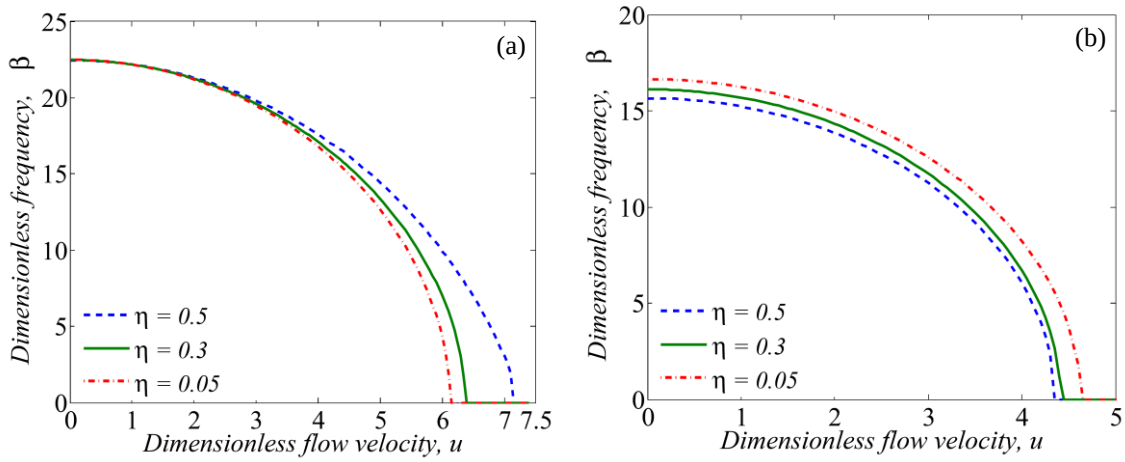

Figure 5. Dimensionless frequency as a function of dimensionless flow velocity with different viscous parameters, (a) c = 0 and (b) c = 30.

The viscosity effect of fluid on the frequency is shown in Fig. 5 for c = 0 and 30. For c = 0, the viscosity effect on the frequency is obvious, especially at large flow velocity and is zero when u = 0. Furthermore, it can be found that increasing the viscous parameter increases the frequency and critical flow velocity. This was reported by Lee and Chang [13]. But, for c = 30, increasing the viscous parameter decreases the frequency and critical flow velocity. Therefore, the viscosity effect on the frequency and critical flow velocity depends on damping factor of the viscoelastic medium. This wasn't reported before.

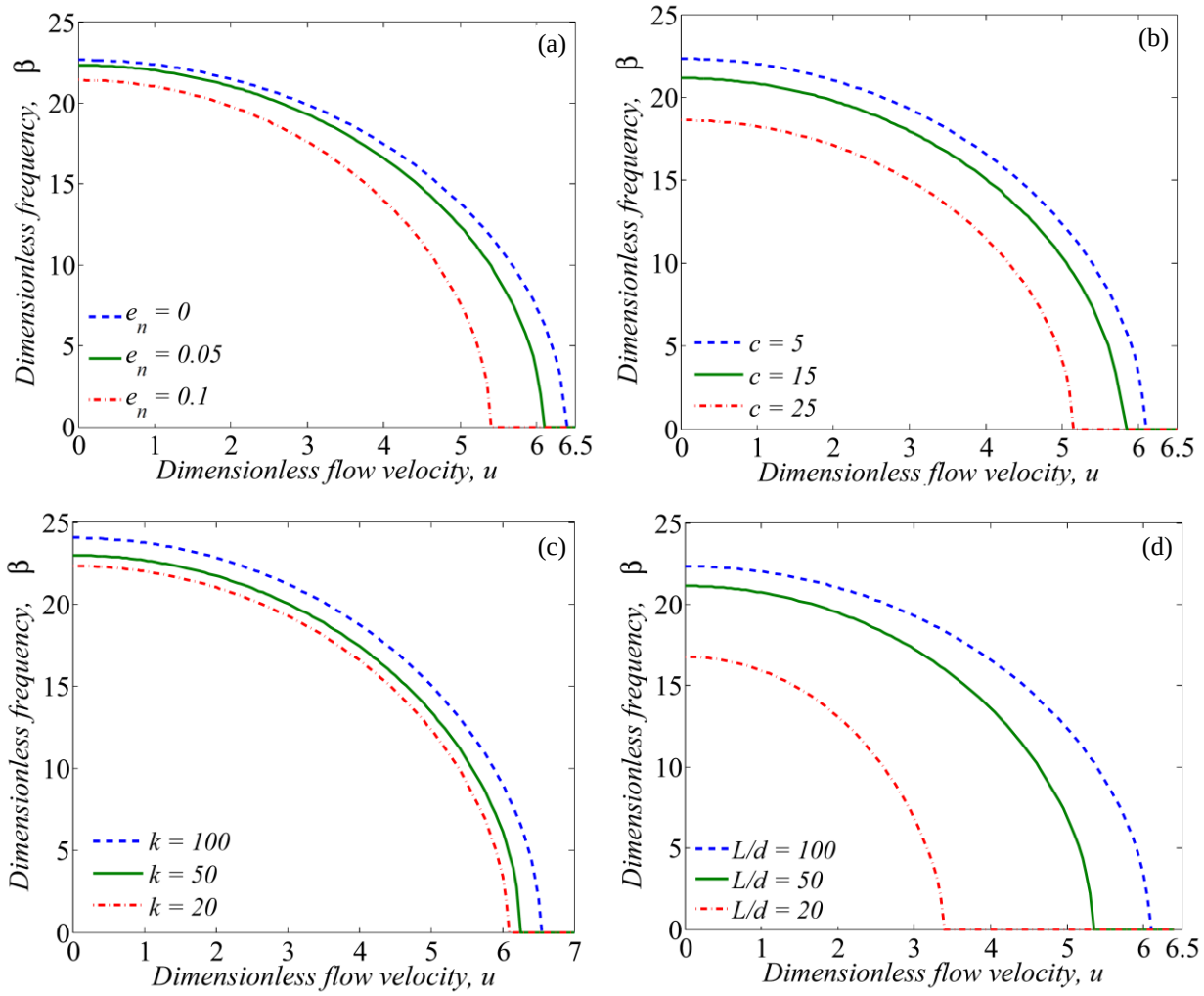

Figure 6. Dimensionless frequency as a function of dimensionless flow velocity with different, (a) nonlocal parameters, (b) dampoing factors of the viscoelastic medium, (c) elastic modulus constants of the viscoelastic medium and (d) aspect ratios.

Fig. 6(a) shows the dimensionless frequency β as a function of dimensionless flow velocity u with different nonlocal parameters. The frequency is significantly influenced by the nonlocal parameter. It can be found that the nonlocal effect on the frequency increases as the flow velocity increases. Furthermore, increasing the nonlocal parameter decreases the frequency and critical flow velocity.

Fig. 6(b) shows damping factor effect of the viscoelastic medium on the frequency. It is clear that the frequency and critical flow velocity decrease as the damping factor increases.

Fig. 6(c) depicts the dimensionless frequency as a function of dimensionless flow velocity with different elastic modulus constants of the viscoelastic medium k. With an increase of elastic modulus constant, the frequency also increases. This is because increasing the elastic modulus constant makes the nanotube become stiffer. Furthermore, as the elastic modulus constant increases, the critical flow velocity increases.

The effect of aspect ratio, *L/d*, on the frequency is shown in Fig. 6(d). With an increase of aspect ratio, the frequency and critical flow velocity increase.

Conclusions

Vibration analysis of viscoelastic SWCNTs conveying viscous fluid embedded in a viscoelastic medium was studied with nonlocal Euler-Bernoulli beam model. Based on the analysis, it was observed that the influences of viscoelastic structural damping of SWCNTs, fluid viscosity, fluid flow velocity, nonlocal parameter, aspect ratio, elastic modulus constant and damping factor of the viscoelastic medium on the frequency and critical flow velocity of clamped-clamped SWCNTs were significant. The results showed that the frequency and critical flow velocity of the SWCNTs decreased as the structural damping coefficient of SWCNTs, damping factor of the viscoelastic medium and nonlocal parameter increased and increased as the aspect ratio and elastic modulus constant of the viscoelastic medium increased (under the same velocity conditions). Furthermore, the effects of structural damping coefficient of SWCNTs and fluid viscosity on the frequency and critical flow velocity depended on the damping factor of the viscoelastic medium. Also, the frequency decreased as the fluid flow velocity increased.

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