Amirkabir University of Technology

ISAV2011
1st International Conference on Acoustics and Vibration
21-22 Dec 2011
Amirkabir University of Technology

Iranian Society of Acoustics and Vibration

# Consideration of Lock-in Using a Modified Wake Oscillator in Vortex Induced Vibrations about a Cylinder

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## Abstract

In the present study, a Modified-Landl model is introduced to estimate the structural oscillation amplitude of a circular cylinder, which is subjected to a fluid flow, during lock-in. The Modified-Landl model is exactly the same as Landl model with regard to the van der Pol and forth order terms while just the coefficients are modified and corrected in a different way than Landl and with a new approach. Here, the displacement, velocity and acceleration couplings are used to solve the Modified-Landl equation for wake oscillators. The response of the coupled equations is assumed to be harmonic. A linear approach is adopted to simplify and derive the solutions algebraically. The results for amplitude during lock-in versus the reduced velocity and maximum structural oscillation amplitude versus the Skop-Griffin parameter are plotted and compared with those of Facchinetti and de Langre. The present modified model evinces a better compliance with experiment with respect to van der Pol model.

**Keywords**: VIV; wake oscillator model; van der Pol equation; Skop-Griffin plot.

## 1. Introduction

Vortex-induced vibration (VIV) is a six degree of freedom self-regulated nonlinear phenomenon. When a bluff body is subjected to the fluid flow, a wake is formed beyond the body. The flow velocity inside the wake is slower than the ambient velocity. This difference in the velocity of fluid particles makes the particles in the ambient current plunge into the wake and then vortices appear. The formation of vortices induces a pressure fluctuation on the surface of the cylinder. Thus, the lift force varies and the body sets out to oscillate in the transverse direction to the flow. VIV occurs in many engineering circumferences such as offshore structures, bridges, tall buildings, airplane control surfaces, power transmission lines, etc. According to the literature, the destructive characteristic of VIV projects the importance of considering and scrutinizing this phenomenon more accurately. Therefore, exact mathematical models are needed to manifest the behaviour of fluid-structure interaction.

According to the literature, diverse attempts are done to express VIV behaviour in terms of mathematical models following the idea of a wake oscillator. Gabbai and Benaroya [1] reviewed the previous literature on modelling of VIV. Facchhinetti and de Langre [2] presented a successful model, which includes a van der Pol term in the wake oscillator equation. Nayfe [3] considered the attributes of a self-excited van der Pol oscillator and determined the finite amplitude of wake oscillation through the elimination of the mixed-secular term. Facchinetti and de Langre [2] showed that van der Pol model matches the experiment more accurately with regard to the maximum structural oscillation amplitude compared to the previous results of Hartlen and Currie [4] and Krenk and Nielsen [5].

In 1975, Landl [6] worked out a model, which includes not only the van der Pol terms but also a forth order term to represent the wake oscillator. To derive the values of coefficients, he adopted a curve fitting method to span the hysteric effects shown by experimental results of Parkinson, Feng and Ferguson [7], Fig. 1. Although this method modifies the hysteric effect conformity of the named model, the amplitude behaviour is still in question; since it is not as effectual as van der Pol.

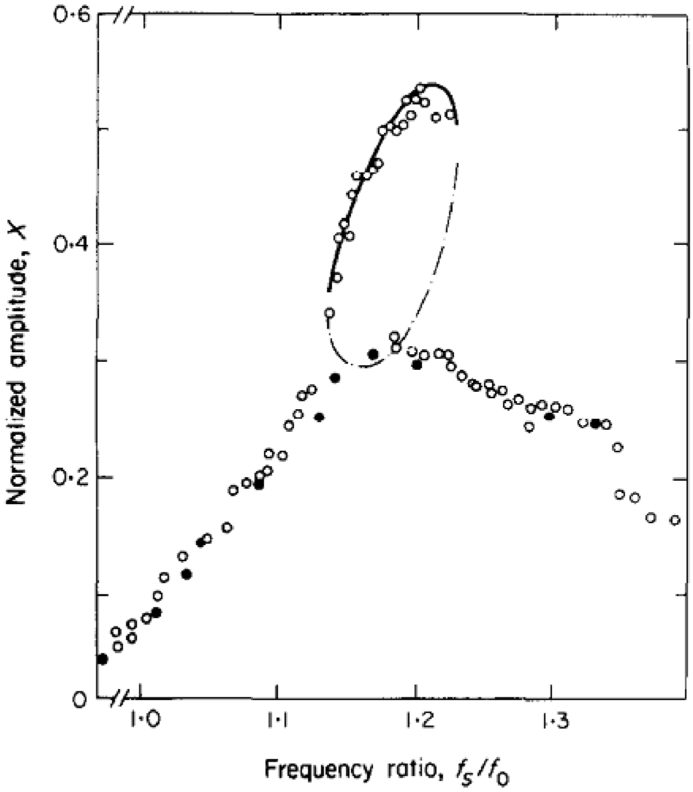


**Figure 1.** Experimental results for amplitude taken from Parkinson, Feng and Ferguson [7] and the theoretical curve obtained by Landl [6].

In the present work, a similar Landl model is studied ignoring the conformance with hysteric effects. Instead, the proper coefficients are introduced to make the maximum structural oscillation amplitude obey the experimental data from Skop-Griffin plot [8]. It is predicted that the Modified-Landl model will enable us to model amplitude more accurately while it is as efficient as van der Pol in the modelling of hysteresis. Also, the effectuality of different couplings is studied together to find out which one gives a better prediction of maximum structural oscillation amplitude in Modified-Landl model.

## 2. VIV model

Since, the oscillatory structure is a circular cylinder, it is assumed to be rigid, but elastically supported with 1 degree of freedom and diameter, *D*, which is allowed to oscillate in transverse direction to the flow, Fig. 2.

The dimensionless structural equation is defined as follows, Eq. (1):

$$\ddot{y} + (2\xi\delta + \frac{\gamma}{\mu})\dot{y} + \delta^2 y = Mq \qquad (1)$$

Where $y$ and $q$ denote dimensionless amplitude of structure and reduced vortex lift coefficient, respectively. The parameters $\xi$, $\delta$, $\gamma$, $\mu$ and $M$ are orderly the structure reduced damping, the reduced angular frequency of the structure ($\Omega_{structure}/\Omega_{fluid}$), the fluid-added damping coefficient, dimensionless mass ratio and a mass number that scales the effect of the wake on the structure.

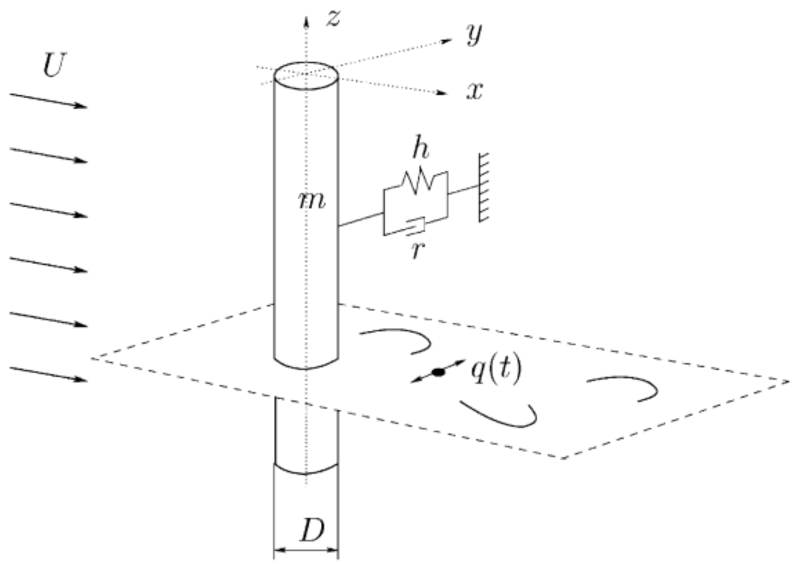


**Figure 2.** Model of coupled structure and wake oscillators for 2-D vortex induced vibrations [2]: *U*, ambient velocity; *m*, structure mass plus fluid added-mass; *h*, Support stiffness; *r*, damping of system; *D*, diameter of circular cylinder; *q(t)*, reduced lift vortex lift coefficient.

## 2.1 van der Pol oscillator

This model is defined as Eq. (2).

$$\ddot{q}+\varepsilon(q^2-1)\dot{q}+q=Ay \text{ or } A\dot{y} \text{ or } A\ddot{y} \tag{2}$$

Here, three force terms of $Ay$, $A\dot{y}$ and $A\ddot{y}$, sequentially indicate displacement, velocity and acceleration models for inertial force. $\varepsilon$ is the coefficient of nonlinearity defined by Nayfeh [3]. $q$ is described as $2C_L/C_{L0}$, where $C_{L0}$ is the reference lift coefficient. The finite amplitude of the stable quasi-harmonic oscillation, $q_0$, is determined in such way to eliminate the mixed-secular term in the solution of wake oscillator equation. For van der Pol oscillator, $q_0$ equals 2. $A$ is a parameter that is derived with $\varepsilon$ through experiments.

## 2.2 Modified-Landl oscillator

Landl [6], represented his model as Eq. (3), which is not in reduced form.

$$\ddot{C}_L+(\alpha'-\beta' C_L^2+\lambda' C_L^4)\dot{C}_L+\delta^2 C_L=A\dot{y} \tag{3}$$

The coefficients $\alpha'$, $\beta'$ and $\lambda'$ are determined through curve fitting such that the whole wake oscillator equation traces the experimental data for hysteresis. As described before, this model fails to depict a true scale of amplitude based on Skop-Griffin plot [8]. In this study, the Landl model is simplified to a dimensionless pattern and the coefficient of nonlinearity factor, $\varepsilon=\alpha'$, is extracted, Eq. (4).

$$\ddot{q}+\varepsilon(1-\beta q^2+\lambda q^4)\dot{q}+q=Ay \text{ or } A\dot{y} \text{ or } A\ddot{y} \tag{4}$$

Now, we plan to work out appropriate $\beta$ and $\lambda$ values to refine the amplitude behaviour. Since, this case study is a comparison between van der Pol and Modified-Landl, the $\beta$ coefficient is set equal to 1 in order to have similar terms in both models. By adopting the same approach of Nayfe [3] to omit the mixed-secular term, $\lambda$ is obtained equal to 0.125. Therefore, the final Modified-Landl equation is presented in the form of Eq. (5).

$$\ddot{q} + \varepsilon(1 - q^2 + 0.125q^4)\dot{q} + q = Ay \text{ or } A\dot{y} \text{ or } A\ddot{y} \tag{5}$$

### 2.3 Values of model parameters

The values of introduced parameters in sections 2.1 and 2.2 as well as those related to the structural oscillator are set here. As the matter of similarity between two models, the parameters are given the same values in Facchinetti and de Langre's study [2]. Thus $\xi$, $\gamma$ and $M$ are, respectively, $3.1\times10^{-3}$, 0.8 and $2\times10^{-4}$, Balasubramanian et al. [9]. $\mu$ can be determined by the value of $M$ as $\mu = 0.05/M$.

Vickery and Watkins [10], Bishop and Hassan [11], King [12], Griffin [13], Pantazopolous [14], set out experiments to plot lift magnification, $K$, versus structural amplitude, $y_0$. Facchinetti and de Langre [2] studied these results to find the best proportion of $A/\varepsilon$ for all acceleration, velocity and displacement conditions. They concluded that $A/\varepsilon = 40$ best fits the experimental data, where $A$ is 12 and $\varepsilon$ is 0.3.

## 3. Dynamics

For displacement and velocity couplings in van der Pol model, some of the dynamics are explained in Krenk and Nielsen [5], and Balasubramanian and Skop [15]. Acceleration with two other couplings are described in Facchinetti and de Langre [2]. Thus, we avoid further explanations for van der Pol model. Instead, the dynamics of Modified-Landl will be revealed, which is similar to that of van der Pol.

We assume that the responses to both wake oscillator and structural equations are harmonic with a relative phase of $\varphi$, Eq. (6).

$$y(t) = y_0 \cos(\omega t),\; q(t) = q_0 \cos(\omega t - \varphi) \tag{6}$$

Here, $\omega$ is the time independent common angular frequency. $q_0$ and $y_0$ are amplitudes. Substitution of assumed responses in the structural oscillator equation yields the amplitude and phase of the linear transfer function, Eqs. (7, 8).

$$\frac{y_0}{q_0} = \frac{M}{\sqrt{(\delta^2 - \omega^2)^2 + (2\xi\delta + \frac{\gamma}{\mu})^2 \omega^2}} \tag{7}$$

$$\tan(\varphi) = -\frac{(2\xi\delta + \gamma/\mu)\omega}{\delta^2 - \omega^2} \tag{8}$$

Now, by substitution of responses into the Modified-Landl wake oscillator equation and adding some algebraic calculations, $q_0$ and $\omega$ equations are concluded as Eqs. (9, 10).

$$q_0 = 2\sqrt{2 + \left[\frac{AM}{\varepsilon}\frac{4C}{(\delta^2 - \omega^2)^2 + (2\xi\delta + \gamma/\mu)^2\omega^2}\right]^{0.5}} \tag{9}$$

$$\omega^6 - \left[1 + 2\delta^2 - (2\xi\delta + \gamma/\mu)^2\right]\omega^4 - \left[-2\delta^2 + (2\xi\delta + \gamma/\mu)^2 - \delta^4\right]\omega^2 - \delta^4 + G = 0 \tag{10}$$

The parameters $C$ and $G$ are the same as those defined by Facchinetti and de Langre [2], which for displacement coupling are defined like Eq. (11).

$$C = -(2\xi\delta + \gamma/\mu),\ G = AM(\delta^2 - \omega^2) \tag{11}$$

For the velocity coupling we have Eq. (12).

$$C = \delta^2 - \omega^2,\ G = AM(2\xi\delta + \gamma/\mu)\omega^2 \tag{12}$$

And for acceleration coupling Eq. (13) is used.

$$C = (2\xi\delta + \gamma/\mu)\omega^2,\ G = AM(\omega^2 - \delta^2)\omega^2 \tag{13}$$

The simultaneous solution of Eqs. 7, 8, 9 and 10 is desired to obtain the result for structural oscillation amplitude. The graphs of results are depicted in Fig. 3 and are compared with those of Facchinetti and de Langre [2]. A glance over the graphs reveals that the Modified-Landl model evinces a better accordance with experiment while maintains the ven der Pol attributes in the aspects of lock-in range and general shape of the curves. Fig. 4 manifests that all three displacement, velocity and acceleration couplings for Modified-Landl model are capable of modelling the maximum amplitude to almost the same extent. It means that, regardless of compatibility to the hysteric effects, the Modified-Landl model is insensitive to the type of coupling. For $S_G \leq 1$, the displacement coupling models the structural oscillation amplitude better than other couplings, while for van der Pol model, the acceleration coupling is effectual. When $S_G > 1$, the velocity coupling for Modified-Landl model is slightly better than the other two couplings. For both models, the acceleration coupling almost reveals a similar behaviour. The sufficiency of displacement and velocity couplings for Modified-Landl model is significantly outstanding and remarkable with respect to those of van der Pol.

## 4. Conclusion

In this work, a Modified-Landl model, based on the classic Landl model, is introduced. A new approach is opted to modify the Landl coefficients. The results are compared with van der Pol model. It is concluded that Modified-Landl model manifests a better agreement with experiment. In contrary to van der Pol model for which acceleration coupling shows the best behaviour, displacement coupling reveals a better trace of maximum structural oscillation amplitude for the presented model. When studying the hysteric effects, the acceleration coupling shows a better compliance. Generally, all displacement, velocity and acceleration couplings, in Modified-Landl model, have got a very similar behaviour and they are varying in the proximity of each other. Thus, the Modified-Landl model, to an acceptable extent, is insensitive to the type of the coupling term.

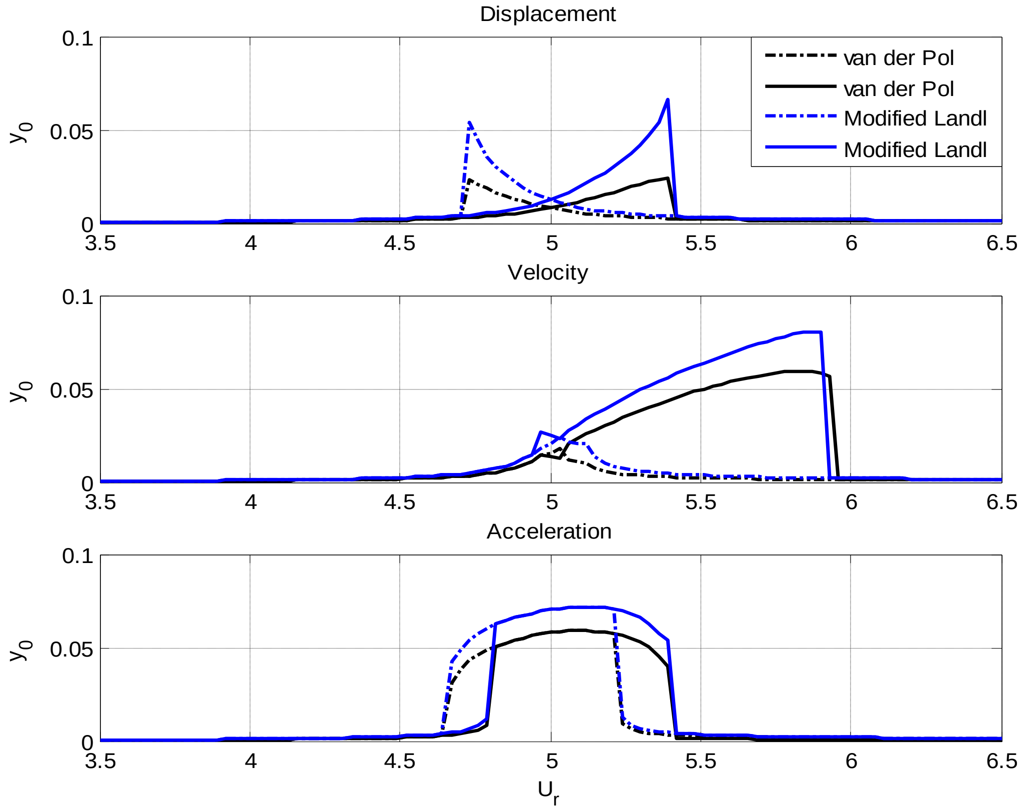


**Figure 3.** Comparison of van der pol with Modified Landl model for three displacement, velocity and acceleration couplings.

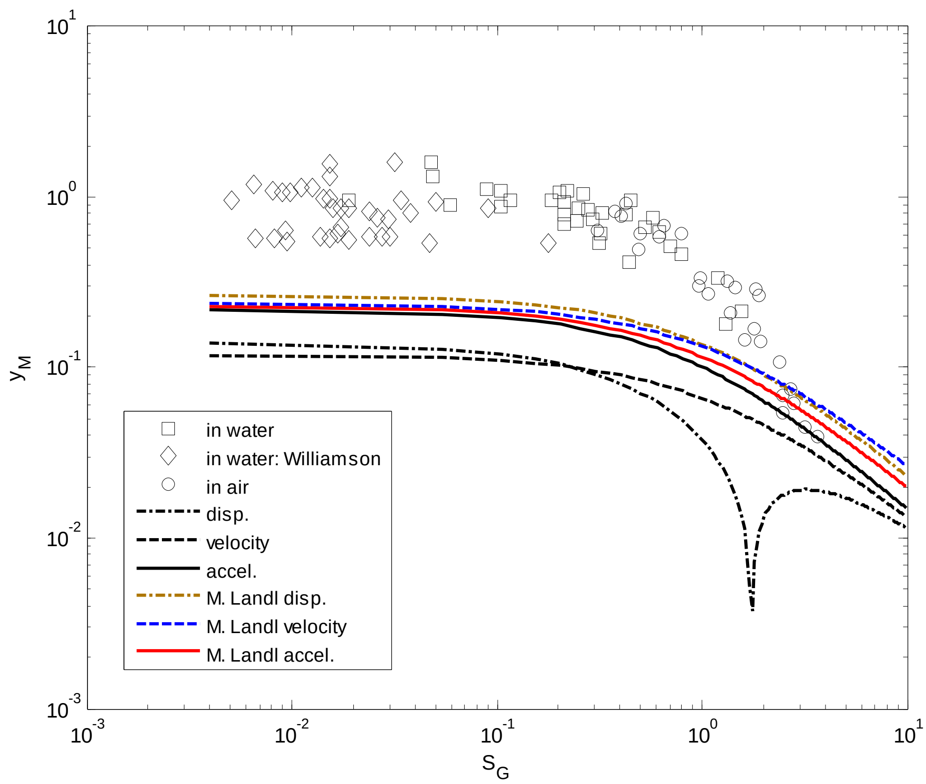


**Figure 4.** Modified-Landl structural oscillation amplitude at lock-in $y_M$ as a function of the Skop-Griffin parameter $S_G$ compared with van der Pol for three displacement, _ . _, velocity, _ _, and acceleration, __, couplings. Experimental data in air: ○, Balasubramanian and Skop, [15]. Experimental data in water: □, Balasubramanian and Skop [15]; ◊, Khalak and Williamson [16].

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