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AN OUTER COMMUTATOR MULTIPLIER OF FINITELY GENERATED ABELIAN GROUPS

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Abstract. We present an explicit structure for the Baer invariant of a finitely generated abelian group with respect to the variety $[\mathfrak{N}_{c_1}, \mathfrak{N}_{c_2}]$, for all $c_2 \leq c_1 \leq 2c_2$.

1. Introduction

An interesting problem connected to the notion of Baer invariants is the computation of Baer invariants for some natural classes of groups with respect to common varieties. The class of finitely generated abelian groups is an appropriate candidate because of their explicit structure theorem.

2. Main results

Let $G \cong \mathbb{Z}^{(k)} \oplus \mathbb{Z}_{n_1} \oplus \mathbb{Z}_{n_2} \oplus \cdots \oplus \mathbb{Z}_{n_t}$ be a finitely generated abelian group with $n_{i+1} \mid n_i$ for all $1 \leq i \leq t-1$, where for any group X, $X^{(n)}$ denotes the group $X \oplus X \oplus \cdots \oplus X$ (n copies). Let $F = F\langle x_1, \ldots, x_k, x_{k+1}, \ldots, x_{k+t}\rangle$ be the free group on the set $\{x_1, \ldots, x_{k+t}\}$. It is easy to see that

$$1 \longrightarrow R \longrightarrow F \longrightarrow G \longrightarrow 1,$$

is a free presentation for G in which $R = \prod_{i=1}^{t} R_i \gamma_2(F)$, where $R_i = \langle x_{k+i}^{n_i}\rangle$, so the Baer invariant of G with respect to $[\mathfrak{N}_{c_1}, \mathfrak{N}_{c_2}]$ is

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$$[\mathfrak{N}_{c_1},\mathfrak{N}_{c_2}]M(G)\cong\frac{R\cap[\gamma_{c_1+1}(F),\gamma_{c_2+1}(F)]}{[R,\ _{c_1}F,\gamma_{c_2+1}(F)][R,\ _{c_2}F,\gamma_{c_1+1}(F)]}.$$

Since $R\supseteq\gamma_2(F)$ we have

$$[\mathfrak{N}_{c_1},\mathfrak{N}_{c_2}]M(G)\cong\frac{[\gamma_{c_1+1}(F),\gamma_{c_2+1}(F)]}{[R,\ _{c_1}F,\gamma_{c_2+1}(F)][R,\ _{c_2}F,\gamma_{c_1+1}(F)]}.$$

Define

$$\begin{aligned}A=\{[\beta,\alpha]\mid \beta \text{ and } \alpha \text{ are basic commutators on } X \text{ such that } \beta>\alpha,\\ wt(\beta)=c_1+1,\ wt(\alpha)=c_2+1\ \}.\end{aligned}$$

Lemma 2.1. *If $c_1\le 2c_2$, then every element of A is a basic commutator on X.*

Now put $H=[R,\ _{c_1}F,\gamma_{c_2+1}(F)][R,\ _{c_2}F,\gamma_{c_1+1}(F)]\cap\gamma_{c_1+c_2+3}(F)$ we have the following.

Lemma 2.2. $[\gamma_{c_1+1}(F),\gamma_{c_2+1}(F)]\equiv\langle A\rangle \pmod H$.

Now we have

Lemma 2.3. *With the above notation and assumptions $[\gamma_{c_1+1}(F),\gamma_{c_2+1}(F)]/H$ is the free abelian group with the basis $\bar{A}$.*

And finally we can prove.

Theorem 2.4. *Let $G\cong\mathbf{Z}^{(k)}\oplus\mathbb{Z}_{n_1}\oplus\mathbb{Z}_{n_2}\oplus\cdots\oplus\mathbb{Z}_{n_t}$ be a finitely generated abelian group with $n_{i+1}\mid n_i$ for all $1\le i\le t-1$. If $c_2\le c_1\le 2c_2$, then*

$$[\mathfrak{N}_{c_1},\mathfrak{N}_{c_2}]M(G)\cong\mathbb{Z}^{(b_k)}\oplus\mathbb{Z}_{n_1}^{(b_{k+1}-b_k)}\oplus\mathbb{Z}_{n_2}^{(b_{k+2}-b_{k+1})}\oplus\ldots\oplus\mathbb{Z}_{n_t}^{(b_{k+t}-b_{k+t-1})}$$

where $b_i=\chi_{c_1+1}(i)\chi_{c_2+1}(i)$, if $c_1>c_2$ and $b_i=\chi_2(\chi_{c_1+1}(i))$ if $c_1=c_2$.

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