Impact of Wind Integration on Electricity Markets: A Chance-Constrained Nash Cournot Model

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Abstract

Wind-powered electricity generation is growing at a high rate around the world, mainly driven by the associated environmental benefits. However, because of the uncertain nature of wind energy, large-scale integration of wind-powered generators into power systems introduces uncertainty of supply in the operation and planning of electric energy systems. A chance-constrained model is presented to study the strategic behavior of power suppliers (firms) with respect to the wind generation uncertainty in an electricity market. The Linear Complementarity Problem (LCP) of a Nash-Cournot competition is extended to account for the wind generation uncertainty. An iterative solution approach is introduced to solve the resultant joint chance-constrained programming problem. Five studies were conducted to show the impact of the amount, location and standard deviation of the wind generation and transmission line limits on the suppliers' profit in the Nash-Cournot game for different confidence levels. The value of stochastic solution index is used to evaluate the suppliers' loss of profit. The numerical results for a 3-bus test system are given to verify the formulation and the solution approach.

Key words: Wind power integration, Wind power uncertainty, Chance-constrained programming

1 Introduction

Wind generation installation, as a renewable energy source, is growing at the rate of 30% annually, with a global installed capacity of 157,900 (MW) in 2009, and is widely used in Europe, Asia and the United states [1]. In some cases such as Alberta, Canada, interest for over 9000 MW of new wind developments has

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been filed with the system operator in a system with 9800 MW peak demand [2]. Integration of wind generators into electricity systems introduces a new source of uncertainty in power systems and impacts the operation of electricity markets [3–5]. In particular, market equilibrium and strategic behavior of market players under uncertainty of wind power generation is the focus of this paper.

Different models, that simulate the strategic behavior of generation companies in electricity markets, have been presented in the literature. An efficient Mathematical Program with Equilibrium Constraints (MPEC) model was presented in [6], in which a strategic gaming model for analyzing an oligopolistic market economy was presented. A Linear Complementarity Model (LCP) of Nash-Cournot competition in a bilateral and pool power market was formulated in [7], where two Cournot models of imperfect competition were formulated as a mixed linear complementarity problem. Another optimization approach for computing market equilibrium was addressed in [8], in which the utilities' profit maximization problems were replaced by a minimization problem under some reasonable assumptions. These proposed Cournot models considered demand and generation as deterministic quantities for the most part. In order to take into account the uncertainty associated with load and generator availabilities, a stochastic Cournot model for market price of electricity was studied in [9, 10]. In [11] a stochastic MPEC problem was introduced, where a supply function game was written in a Chance-Constrained Programming (CCP) formulation.

In this paper, wind generation uncertainty is taken into account in a Cournot model of an electricity market. An MPEC is used to find the market equilibrium. Since all market participants are acting to maximize their profits simultaneously in the presence of intermittent wind generation, a joint chance-constrained formulation is used. The arbitrage model of [7] is used since it was shown that this model has the same equilibrium as a Cournot competition in a pool market model [7]. Also, the fact that it has a unique equilibrium point makes the model suitable for chance-constrained formulation. In addition, applicability of the model of [7] to real-life large systems makes it a good choice for practical analyses. The model includes power system linear (DC load flow) equations and limits on line power flows.

The model considered in this paper is a Cournot model with arbitrage, which is different from the ones used in [9–11]. This work differs from the one presented in [11], where a closed form solution for a chance-constrained formulation was found for a supply function game, and line power flow constraints and bilateral agreements were not considered. The earlier studies in this area mostly fall in the deterministic category. When there are uncertainties, the deterministic solution may not be meaningful and a stochastic approach is needed. The stochastic programming used in [9, 10] is generally based on the expected value of the firms' profit, which has the drawback of not considering the variance and risk. Considering the previous works in this area, the contributions of this work are as follows. The first contribution is to extend the chance-constrained formulation for the Cournot model with arbitrage in the presence of wind generation uncertainty. The second contribution is to enhance an iterative solution approach for the above mentioned problem. The significance of this work is that it provides the characteristic of the strategic behavior of market participants in the presence of uncertainties in the electricity market. It guarantees the market is cleared at the equilibrium (for this specific formulation) with a pre-specified probability, which may provide an insight for electricity market participants.

The remainder of the paper is organized as follows: in Section 2 the background of the Nash-Cournot competition and chance-constrained programming are presented. Nash-Cournot competition under uncertainty is introduced in Section 3. The numerical results and observed characteristics are presented in Section 4, and Section 5 concludes the paper.

2 Background

2.1 Nash-Cournot Competition

In an electricity market, Cournot competition is an economic model that describes the market structure in which generation companies compete on the amount of output power they will produce, which they decide on independently and simultaneously. An essential assumption of this model is that each firm aims to maximize its profit based on the expectation that its own output decision will not affect the decisions of its rivals. Price is a commonly known decreasing function of total output. The firms' optimization problem of such a game may be written as in [7]:

$$\max \sum_i [P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_g x_{gi} + a_{fi})]x_{fi} - \sum_i C_{fi}x_{fi} \tag{1a}$$

$$s.t. \quad x_{fi} \le X_{fi};\ \forall i \tag{1b}$$

$$P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_g x_{gi} + a_{fi}) = p_{Hf} + W_i; \quad \forall i \tag{1c}$$

$$\sum_i a_{fi} = 0 \tag{1d}$$

$$x_{fi} \ge 0;\ \forall i \tag{1e}$$

where, P_{i0} and Q_{i0} are the price and quantity intercepts, respectively; x_{fi} is the power generation, a_{fi} is the amount of arbitrage, X_{fi} is the capacity of the power generation of firm f at bus i. x_{gi} is the power generation of firm g at bus i and C_{fi} is the cost \$/MWh, W_i is the congestion based cost of transferring power from an arbitrary hub node to bus i and p_{Hf} is the hub price.

The transmission pricing policy is modeled based on the congestion pricing scheme [12]. The net MW flow through line k is:

$$\sum_i PTDF_{ik}y_i = f_k \tag{2}$$

where Power Transfer Distribution Factor (PTDF), is a matrix describing the linear sensitivity relation between changes in injections or loads (typically restricted to active power components), and changes in flows on lines. The owner of the grid is assumed to manage the limited interface capacity to maximize the value of transmission services y_i, as expressed by the generator's willingness to pay. This behavior is equivalent to having the grid choose values of y_i to maximize its revenue $\sum_i W_i y_i$ as if W_i are fixed, while respecting the interface constraints. The grid linear programming problem may be written as:

$$\max \sum_i W_i y_i \tag{3a}$$

$$s.t. \quad -T_{k-} \leq \sum_i PTDF_{ik} y_i \leq T_{k+}; \tag{3b}$$

Equilibrium for the arbitrage model requires the following balance to be maintained between the transmission services provided by the grid and the service demanded by the arbitragers:

$$y_i = a_{fi}; \quad ,\forall f, i. \tag{4}$$

Also, the hub price assumed by suppliers must be consistent:

$$p_{Hf} = p_{H1}; \quad ,\forall f > 1. \tag{5}$$

The Karush-Kuhn-Tucker (KKT) optimality condition of the firms and grid owner optimization problems are found. The equilibrium point is the solution of the resultant set of linear complementarity equations with the market clearing equation, which can be found in [7].

2.2 Chance-Constrained Programming (CCP)

Chance-constrained programming is a type of stochastic programming, where randomness is incorporated into the model by means of a probabilistic measure. CCP was first introduced in [13] as a tool for solving the temporal planning problem under uncertainty. In chance-constrained problems, a particular constraint must hold with a prescribed probability. The objective is often an expectation function (*the E-model*), or it may be the variance of some result (*the V-model*) or the probability of some occurrence such as satisfying the constraints (*the P-model*) [14]. A chance-constrained programming problem as a nonlinear program may be presented as in [13, 15]:

$$\min_{\mathbf{x} \in \Re^n} f(\mathbf{x}, \boldsymbol{\xi}) \tag{6a}$$

$$s.t. \quad h_0(\mathbf{x}) = P(g_1(\mathbf{x}, \boldsymbol{\xi}) \geq 0, \ldots, g_k(\mathbf{x}, \boldsymbol{\xi}) \geq 0) \geq \epsilon \tag{6b}$$

$$h_i(\mathbf{x}) \geq p_i, \; \forall i = 1, \ldots, m \tag{6c}$$

$$\mathbf{x} \in \mathbf{D} \tag{6d}$$

where, P is the probability measure, $\boldsymbol{\xi} \in \Omega \subset \Re^q$ is a random vector, and Ω is the sample space; $p_1 \dots p_m$ are constant and ϵ is a confidence level (probability); $g_1 \dots g_k$ and $h_0 \dots h_m$ are functions defined on $\Re^{n+q}$ and $\Re^n$, respectively; and D is a set of feasible solutions determined by other equality constraints.

Many studies have been performed to analyze and efficiently solve CCP problems [16–19]. The solution to a CCP problem is usually obtained using its deterministic equivalent because the probability in the problem does not allow existing methods to solve for a feasible solution [14]. In a CCP problem, the random elements could appear either in the constraints or in the objective function [20]. In this paper, special attention is given to the case where uncertainty is in the objective function (i.e., the constraint (6b) does not exist). There are different approaches that deal with the uncertainty in the objective function of the mathematical programming problem. One approach is to introduce a new constraint and a new objective function so that the stochastic problem is:

$$\min_{\mathbf{x} \in \Re^n} d \tag{7a}$$

$$s.t. \quad P(f(\mathbf{x}, \boldsymbol{\xi}) \leq d) \geq \epsilon,\ \epsilon \in [0, 1] \tag{7b}$$

$$h_i(\mathbf{x}) \geq p_i,\ \forall i = 1, \dots, m \tag{7c}$$

$$\mathbf{x} \in \mathbf{D} \tag{7d}$$

where d is a new variable, which presents an upper limit for the objective function. The case where $f(\mathbf{x}, \boldsymbol{\xi}) = \boldsymbol{\xi}^T \mathbf{x}$ is worth separate consideration under the assumption that the random variable vector $\boldsymbol{\xi}$ has a multivariate normal distribution, because the deterministic equivalence for linear chance-constrained problem is easier to find.

Theorem 2.1 *[20, 21] Assume that the stochastic vector $\boldsymbol{\xi} = (\xi_1, \xi_2, \dots, \xi_n)$ and the function $f(\mathbf{x}, \boldsymbol{\xi})$ has the form $f(\mathbf{x}, \boldsymbol{\xi}) = \xi_1 x_1 + \xi_2 x_2 + \ldots + \xi_n x_n$. If ξ_i are assumed to be independently normally distributed random variables, then $P(f(\mathbf{x}, \boldsymbol{\xi}) \leq d) \geq \epsilon$ if and only if:*

$$\boldsymbol{\mu}_\xi^T \mathbf{x} + \Phi^{-1}(\epsilon) \sqrt{\mathbf{x}^\mathbf{T} \mathbf{C}_\xi \mathbf{x}} \leq d \tag{8}$$

where, $\boldsymbol{\mu}_\xi$ and $\mathbf{C}_\xi$ are the expectation vector and the covariance matrix of $\boldsymbol{\xi}$ respectively. Following Theorem 2.1, the chance-constrained problem (7) may be re-written as:

$$\min_{\mathbf{x} \in \Re^n} \boldsymbol{\mu}_\xi^T \mathbf{x} + \Phi^{-1}(\epsilon) \sqrt{\mathbf{x}^\mathbf{T} \mathbf{C}_\xi \mathbf{x}} \tag{9a}$$

$$s.t. \quad h_i(\mathbf{x}) \geq p_i,\ \forall i = 1, \dots, m \tag{9b}$$

$$\mathbf{x} \in \mathbf{D} \tag{9c}$$

The optimization problem (9) is the deterministic equivalent for (7) and can be solved using optimization algorithms.

3 Nash-Cournot Competition Under Uncertainty

In this section, the impact of wind generation uncertainty on the electricity market equilibrium problem is studied using chance-constrained programming. It is assumed that wind generators are non-strategic and their output is absorbed by the system. This assumption is in line with the practice in Alberta's electricity market where wind facilities are considered "price takers". Considering wind generators presence in the system, the firms' optimization problem (1) may be re-written as:

$$\max \sum_i [P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi} + x_{wi})]x_{fi} - \sum_i C_{fi}x_{fi} \tag{10a}$$

$$s.t. \quad x_{fi} \leq X_{fi} \; \forall i \tag{10b}$$

$$P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi} + x_{wi}) = p_{Hf} + W_i, \quad \forall i \tag{10c}$$

$$\sum_i a_{fi} = 0 \tag{10d}$$

$$x_{fi} \geq 0; \; \forall i \tag{10e}$$

where x_{wi} is the amount of wind generation at bus i. The wind generation firm is not a market player so it doesn't have an optimization problem. Due to the intermittency of the wind generation, the model given in (10) includes stochastic variable x_{wi}, both in the objective function (10a) and the equality constraints (10c). The objective function (10a) is replaced with a probabilistic constraint as in (7b). Because (10c) is an equality constraint, it is not modeled by a probabilistic constraint. The stochastic problem (10) has no meaningful maximum as it is; however, it can be modeled using chance-constrained programming to solve for the optimum solution set. Following the method introduced in Section 2.2, the optimization problem (10) may be written as:

$$\max A_f \tag{11a}$$

$$s.t.$$

$$P\{\sum_i [P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi} + x_{wi})]x_{fi} - \sum_i C_{fi}x_{fi} \geq A_f\} \geq \epsilon \tag{11b}$$

$$x_{fi} \leq X_{fi} \quad \forall i \tag{11c}$$

$$P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi} + x_{wi}) = p_{Hf} + W_i \quad \forall i \tag{11d}$$

$$\sum_i a_{fi} = 0 \tag{11e}$$

$$x_{fi} \geq 0 \quad \forall i \tag{11f}$$

where A_f is the new variable, which represents a lower limit for the objective function, and the objective function (10a) is now a probabilistic constraint in (11b). Constraints (11b) and (11d) contain random variable x_{wi} and probabilistic measure P, which cannot be solved as it is. These constraints must be replaced by deterministic equivalents so that the feasible solution set of the problem remains unchanged. To find the equivalent deterministic formulation for the stochastic problem (11), the objective function (11b) is rewritten as:

$$\begin{aligned} & P\{\sum_i [P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi})]x_{fi} - \\ & \sum_i C_{fi}x_{fi} - A_f \geq \frac{P_{j0}}{Q_{j0}} x_{wj}x_{fj}\} \geq \epsilon \end{aligned} \tag{12}$$

where the uncertain wind generation in bus j, x_{wj}, is transfered to the right side of the inequality. To demonstrate the approach better, in this paper it is assumed that the wind generation is only located at bus j. Using Theorem 2.1 the set of x_{fi} satisfying the probability in (12) is the same as those satisfying the constraint:

$$\begin{aligned} & \sum_i [P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi} + \mu_{x_{wi}} + \Phi^{-1}(\epsilon)\sigma_{x_{wi}})]x_{fi} \\ & - \sum_i C_{fi}x_{fi} \geq 0 \end{aligned} \tag{13}$$

Hence, the objective function (11b) can be replaced by (13) and the resultant deterministic equivalent may be written as:

$$\begin{aligned} \max \; & \sum_i [P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi} + \mu_{x_{wi}} + \\ & \Phi^{-1}(\epsilon)\sigma_{x_{wi}})]x_{fi} - \sum_i C_{fi}x_{fi} && \text{(14a)} \\ s.t. \; & x_{fi} \leq X_{fi} \quad \forall i && \text{(14b)} \\ & P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi} + \mu_{x_{wi}} + \\ & \Phi^{-1}(\epsilon)\sigma_{x_{wi}})) = p_{Hf} + W_i \quad \forall i && \text{(14c)} \\ & \sum_i a_{fi} = 0 && \text{(14d)} \\ & x_{fi} \geq 0 \quad \forall i && \text{(14e)} \end{aligned}$$

where $z = \Phi^{-1}(\epsilon)$ is the normal standard quantile corresponding to the area under the normal curve, which is equal or less than ϵ. Comparing (14) with (10) it is observed that the x_{wi} is replaced by $\mu_{x_{wi}} + \Phi^{-1}(\epsilon)\sigma_{x_{wi}}$, which is a weighted perturbation around the mean $\mu_{x_{wi}}$. The same perturbation is applied to the constraint (14c), because it is an equality constraint and makes the linkage between the firms. It should be noted that different z values result in different probabilities in (11b). The KKT conditions for the model (14) are:

For x_{fi}, $\forall i$:

$$P_{i0} - \frac{P_{i0}}{Q_{i0}}(2x_{fi} + \sum_{g\neq f} x_{gi} + a_{fi} + \mu_{x_{wi}} +$$

$$\Phi^{-1}(\epsilon)\sigma_{x_{wi}}) - C_{fi} + \alpha_{fi}\frac{P_{i0}}{Q_{i0}} - \rho_{fi} \leq 0;\ x_{fi} \geq 0; \tag{15a}$$

$$x_{fi}(P_{i0} - \frac{P_{i0}}{Q_{i0}}(2x_{fi} + \sum_{g\neq f} x_{gi} + a_{fi} + \mu_{x_{wi}} +$$

$$\Phi^{-1}(\epsilon)\sigma_{x_{wi}}) - C_{fi} + \alpha_{fi}\frac{P_{i0}}{Q_{i0}} - \rho_{fi}) = 0; \tag{15b}$$

For a_{fi}, $\forall i$:

$$\frac{P_{i0}}{Q_{i0}}(\alpha_{fi} - x_{fi}) - \beta_f = 0 \tag{15c}$$

For p_{Hf} :

$$\sum_i \alpha_{fi} = 0 \tag{15d}$$

For ρ_{fi}, $\forall i$:

$$\rho_{fi}(x_{fi} - X_{fi}) = 0;\ \rho_{fi} \geq 0;\ x_{fi} \leq X_{fi}; \tag{15e}$$

For $\alpha_{fi}, \forall i$:

$$P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi} + \mu_{x_{wi}} +$$

$$\Phi^{-1}(\epsilon)\sigma_{x_{wi}}) = p_{Hf} + W_i; \tag{15f}$$

For $\beta_f, \forall i$:

$$\sum_i a_{fi} = 0; \tag{15g}$$

The grid owner and market clearing optimization problems are the same as those given in [7]. In cases, where the wind generation is not located at the same bus as the other generation suppliers, the uncertainty does not appear in the objective function (10a), and only appears in the constraint (10c). In the pool model with the presence of the arbitrages this constraint is binding. Therefore, the equality in (10c) can be replaced by an inequality and the probabilistic constraint is:

$$P\{P_{i0} - \frac{P_{i0}}{Q_{i0}}(\sum_{g,g\neq w} x_{gi} + a_{fi} + x_{wi}) \geq p_{Hf} + W_i\} \geq \epsilon \quad \forall i \tag{16}$$

And the deterministic approximation for the chance constraint (16) is:

$$P_{i0} - \frac{P_{i0}}{Q_{i0}} (\sum_{g, g \neq w} x_{gi} + a_{fi} + \mu_{x_{wi}} + \Phi^{-1}(\epsilon)\sigma_{x_{wi}}) \geq p_{Hf} + W_i \tag{17}$$

The constraint (10c) is then replaced by the deterministic approximation (17) and the KKT conditions are derived for the resultant optimization problem.

3.1 Solution approach

This section introduces the solution approach for the chance-constrained model (15). In the market equilibrium problem, each supplier intends to maximize its profit. This maximization problem is a complex problem for suppliers in the sense of their strategic behavior in bidding into the market. When there is uncertainty in the model, the problem becomes more challenging because each market participant tries to solve a chance-constrained problem. In other words, considering the wind generation uncertainty, each supplier attempts to maximize its benefit with a high probability ϵ. It should be noted that the wind generation is supplied into the grid without bidding, which means that wind is not a market player. Each supplier seems to separately solve the chance-constrained optimization (14) to maximize its benefit with probability ϵ; however, the suppliers' maximization problems are linked by the arbitrage constraint (11d). The probability ϵ is assumed to be the same for all the suppliers. It shows a probability by which the optimum solution is the maximum for all the wind generation realizations. The resultant equilibrium problem (15) may have joint or separate forms depending on the location of the wind generation with respect to the other firms' generation. The problem is then to find a scalar quantile z such that the joint probability of the equilibrium point for the market problem becomes ϵ. An iterative approach is used to find the joint probability of the equilibrium problem. The steps are:

1. Choose an initial scalar $z = \Phi^{-1}(\epsilon)$.
2. Solve the LCP problem (15) together with grid owner KKT conditions and market clearing (4).
3. Calculate suppliers' benefit vector $\mathbf{B}_f$.
4. Using a normal distribution [22], generate random variable $x_{wi} = N(\mu_{x_{wi}}, \sigma^2_{x_{wi}})$ with n number of trials.
5. Use Monte Carlo Simulation (MCS) and solve n equilibrium problems using KKT conditions of (10) together with grid and market clearing equations.
6. Calculate each supplier's benefit for each trial and obtain vector $\mathbf{B}_{mcf}$.
7. Find the joint probability of having the maximum profit $P = P(\mathbf{B}_{mcf} \geq B_f)$.

8. If the termination condition, $|\epsilon - P| \leq \Delta\epsilon$, is met, stop; otherwise update the z value using the false position method as explained in appendix A, then go to step 2.

Three possible cases may result depending on the location of the wind generation:

a) Wind generation at the same bus as the other generation units: In this case, the uncertainty appears in the objective functions of those firms' optimization problems that have generation at the same bus as the wind generation. The resultant problem is a joint chance-constrained and may be solved using the proposed iterative method.

b) Wind generation at the same bus as one of the generation units: In this case, the uncertainty appears in the objective function of one firm optimization problem, which has generation at the same bus as the wind generation. The resultant problem is a separate chance-constrained and may be solved using the proposed iterative method. It should be noted that in this case, the probability of only one of the firms is checked in the iterative method.

c) Wind generation at the separate bus: In this case, the uncertainty appears only in the equality constraint (10c). The resultant problem is a joint chance-constrained and may be solved using the proposed iterative method; however the probabilistic constraint (10c) is checked in each iteration instead of probability of the firms' profits.

4 Numerical Simulations and Observed Characteristics

This section presents the numerical results for a three-bus test system as the one used in [7]. This simple example is used to illustrate an application of the presented model and can be used to verify it. The generators are only on buses 1 and 2. The demand functions are $p_i(q_i) = 40 - 0.08q_i$, $i = 1, 2$ and $p_3(q_3) = 32 - 0.0516q_3$, \$/MWh [6]. These functions show that the demand at bus 3 is more responsive to the price. There are two suppliers f=1, 2, each with one generator. Both generators have unlimited capacity on their own bus and a maximum of 35 MW capacity on the other bus. This is different from the case in [7], and is used here to demonstrate the joint chance-constrained application. A constant marginal cost of \$15/MWh is assumed for supplier 1 and \$20/MWh for supplier 2. The only transmission cost is due to congestion. Two different scenarios are considered for the transmission system. In scenario 1, there is no congestion (i.e., infinite line capacity), while in scenario 2 there is 25 MW congestion on the transmission line between buses 1 and 2 (k=1) (T_{1+}=T_{1-}= 25 MW). The deterministic case is presented as the base case for comparing with the stochastic cases. Four studies are performed to examine the model presented. In the first study, the system does not include a transmission line limit. In the second, the transmission line limit is considered, and in the third, the effect of the wind generation standard deviation is demonstrated. Finally, the effect of wind generation location is studied.

4.1 Deterministic case

In this case, the set of KKT conditions of the firms' optimization problems (10), grid optimization and the market clearing problem are solved simultaneously. Two cases are assumed for the wind generation penetration into the system. In one, there is no wind generation μ_{xw}= 0 MW , and in the other, a wind generation with mean value of μ_{xw}= 60 MW and zero variance is considered at bus i=2. In both cases, a limit on the line flow between buses 1 and 2 (k=1) (T_{1+}=T_{1-}= 25 MW) is assumed. To obtain the deterministic solution for the equilibrium problem (10), the random variable x_{wi} is replaced by its expected value μ_{xw}. The prices at the system buses, and firms' profits in both cases are shown in Tables A. The deterministic solution may be used to compare the effect of confidence level and wind generation standard deviation in the equilibrium problem.

4.2 No transmission line limit

Using the probabilistic formulation, the wind generation uncertainty is modeled using CCP as presented in Section 3. The set of KKT conditions of the firms' optimization problems (15), grid optimization and the market clearing equation are solved simultaneously, when the transmission line limits are not taken into accoun. The mean value of the wind generation is considered to be μ_{xw}= 60 MW at bus i= 2 and its standard deviation σ_{xw}= 12 MW. This is the case where the wind and firms' generations are at the same bus. Hence, the uncertainty appears in both firms' objective functions, and the resultant problem is a joint chance-constrrained problem. Using the iterative method presented in Section 3.1, the problem is solved for different confidence levels ϵ. The solid line in Fig. 1 shows the firm 1's profit for different confidence levels. The firm 2's profit follows the same trend. The effect of having higher confidence on the maximized profit can be seen from the figure. The profits are decreasing from \$3035 and \$474 for 90% confidence level to \$2966 and \$447 for 99% confidence level for firms 1 and 2, respectively. This is the trade off between having low confidence higher profit, and high confidence lower profit for the whole realizations of the wind generation in practice. As the confidence level increases, the firms are generating less power, allowing for wind generation to supply the load demand. With the wind generation penetrated into the system, the electricity price slightly decreases, and because the loads are assumed to be elastic, they increase slightly. The smaller generation and smaller price lead to a smaller profit for both firms. The reduction in profit is significant when comparing the deterministic profit for firms (i.e., \$3021 for firm 1) in table A with stochastic results in figure 1 (i.e., \$2966 for 99%). The slope of the profit reduction with respect to the confidence level may assist the firms in their bidding strategies. It can be seen that this slope is sharper for confidence levels higher than 96%, which shows the conservative behavior for the firms.

4.3 The effect of transmission line limits

The same test system is used and a transmission line limit of 25 MW is considered on the transmission line between buses 1 and 2 (k=1) (T_{1+}=T_{1-}= 25 MW). In this case, the incurred congestion cost for each firm plays a big role in having a higher confidence level in their profit. In other words, the congestion charge will decrease the chance of gaining maximized profit over the whole realization of the wind generation. This is because the net profit of the firm, that pays for the congestion to deliver power to the load, decreases as the congestion increases. The dotted line in Fig. 1 shows the firm 1's profit for different confidence levels. The firm 2's profit follows the same trend and is not shown here. Comparing the with and without line limit cases, it can be observed that in the presence of the line limits, the firms are not willing to sell more power due to the congestion cost. Consequently their profit is generally less than the case with no line limit. Another observation is that, in the presence of the line limits, as the confidence level increases, the firms' profits decrease slightly. Because the transmission line limit has already affected the prices in different buses, the firms' profits are already low. The higher confidence levels also cause the firms' profits to fall more.

4.4 The effect of wind generation standard deviation

To presents the effect of the wind generation standard deviation on the firms' profits, a normally distributed wind generation is used. The three bus test system with the line limit (k=1) (T_{1+}=T_{1-}= 25 MW) is assumed, and the wind generation with the mean μ_w = 60 MW and a range of standard deviation from 5% to 25% of its mean value is considered at bus i=2 [22, 23]. The profit of firm 1, for different confidence levels and different standard deviations is shown in Fig. 2. It can be seen that firm 1's profit is decreasing from \$3020 with confidence level 90% and 5% standard deviation, to \$2893 for confidence level 98% and standard deviation 25%. Firm 2 follows the same trend, and its profit decreases from \$468 for confidence level 90% and standard deviation 5%, to \$419 for confidence level 99% with standard deviation 25%. As the standard deviation increases, the firms exhibit more conservative characteristics, allowing for wind generation to serve the load, and the firms will offer lower capacity. The higher standard deviation causes the firms to produce less power and the electricity price decreases slightly. Both decrease in electricity price and generations, lead to less profit for firms for higher confidence levels. The profit reduction slope, along the confidence level axis, for different σ_w shows how the firms behave in the environment with higher uncertainty. This behavior can be better presented using the loss of profit index or normalized Value of Stochastic Solution (VSS). The VSS index is used to calculate the importance of the uncertainty in the optimization problem. It shows the loss of profit due to the presence of uncertainty in the problem, and is defined as [24]:

$$\text{VSS} = \text{EEV - RP} \tag{18}$$

where EEV is the expected result when replacing the random variables by their expected values in the optimization problem (i.e., the deterministic results in table A) , and RP is the result of the CCP. The smaller the VSS the closer the expected value solution to the chance-constrained solution. The firms' loss of profit in the presence of uncertainty for different confidence levels {0.90 to 0.99} and standard deviations {5% to 25%} is shown in Figs. 3 and 4. These figures show that firm 2 is more susceptible to the wind generation standard deviation and confidence levels, because as the standard deviation and confidence levels increase, the loss of profit reaches up to 25% for firm 2 and 4.5% for firm 1. In the environment with higher uncertainty (i.e., larger $\sigma_w\%$), the firms lose more profit for higher confidence levels. Also, firm 2 has a higher VSS than firm 1 because it is a more expensive firm, and located at the same bus as the wind generation.

4.5 The effect of location of wind generation

This section presents the effect of wind generation location on the firms' profits. Wind generation of $\mu_w = 60$ MW is assumed to be located in bus $i = 3$. The difference between this case study and the previous ones is in the problem formulation. Since the wind is not located at the same bus as the other generation, uncertainty does not appear in the objective functions of the firms' optimization problems. Hence, the uncertainty is presented only in the arbitrage constraints (10c). Fig. 5 shows the firms' profits for different confidence levels and standard deviations. Firm 1's profit decreases from \$2215 to \$2071 as the confidence level and standard deviation increase. Firm 2's profit, which is not shown here, follows the same trend as firm 1 and decreases from \$959 to \$865 as the confidence level and standard deviation increase. With respect to the previous case, it can be seen that firm 2's profit is almost double and firm 1's profit decreases. This is because firm 2 is selling more power than firm 1. Also, as the confidence level increases, the firms are selling less power and making less profit.

Figs. 6 and 7 show the normalized VSS (the firms' loss of profit) for different confidence levels and standard deviations. In this case, with the wind generation located at bus $i = 3$, which does not include the other firms' generation, the VSS index is similar for the firms, although firm 2 is affected more because it is an expensive firm. For each standard deviation $\sigma_w\%$, as the confidence level increases the loss of profit increases for both firms. Comparing the VSS index in Figs. 6 and 3, it can be seen that, firm 1's loss of profit is higher than the case where the wind generation is located at bus $i = 2$. This is because firm 1 is willing to produce less power when the wind generation was at bus $i = 3$. The same comparison for firm 2 reveals that firm 2 is willing to produce more power and consequently its loss of profit is less than the case where wind generation is at bus $i = 2$.

5 Conclusions

This paper introduced a chance-constrained model for the Nash-Cournot competition with wind generation uncertainty. The resultant LCP model is solved using an iterative solution approach to find the final z value that guarantees a pre-specified confidence level on each firm's profit. A three-bus test system is used to study the effect of transmission line limits, the amount of wind generation and its standard deviation and location, on the strategic behavior of suppliers. The simulations show that the firms' profits decrease when there are line limits. As the confidence level increases, the firms behavior become more conservative, which allows wind generation to participate more in serving the load. We quantified the effect of uncertainty in the optimization problem using the normalized VSS index to show the percentage of the profit loss for the firms. It is shown that the amount of the firms' loss of profit depended on the location of the wind generation with respect to the other firms, and the relative expensiveness of the firms. The simulations show that increasing the wind generation standard deviation to 25%, leads to 4.3% and 24.5% loss of profit for firms 1 and 2 respectively. The effect of wind generation location with respect to the other firms is also studied. The simulations also show that when the generation is not in the same bus as other firms' generation, the loss of profits are 7.3% and 10.8% for firms 1 and 2 respectively.

A z update algorithm

This appendix introduces the false position method [25] used to update the z value. Upper and lower multivariate bounds (z_u and z_l) on the quantile z are firstly chosen and then the problem is solved for these values. The corresponding multivariate probabilities are found and called P_{up} and P_{lo}, which surround the target probability P_{target}. Assuming that the multivariate z values are changing with the same rate as the univariate z values, the univariate z values corresponding to P_{up}, P_{lo} and P_{target} are found from the normal standard table and called $\hat{z}_u$, $\hat{z}_l$ and $\hat{z}_\epsilon$, respectively. The probability corresponding to z_ϵ^i is found from the MCS and its univariate z_{temp} is found. If $z_{temp} < \hat{z}_\epsilon$ then z_l and $\hat{z}_l$ are swapped with z_ϵ^i and z_{temp}, respectively. Otherwise, if $z_{temp} \geq \hat{z}_\epsilon$ then z_u and $\hat{z}_u$ are swapped with z_ϵ^i and z_{temp}. This shrinks the interval $[z_u, z_l]$ until the final z_ϵ is obtained. Using the multivariate values z_l and z_u and their corresponding univariate values $\hat{z}_l$ and $\hat{z}_u$, interpolation is used to find the updated value of z_ϵ:

$$z_\epsilon^{i+1} = z_l + \frac{\hat{z}_\epsilon - \hat{z}_l}{\hat{z}_u - \hat{z}_l}(z_u - z_l) \tag{19}$$

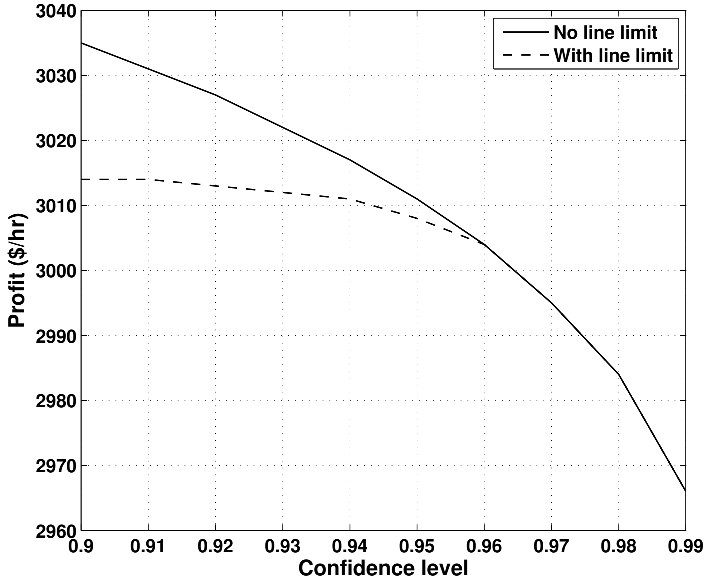

Figure 1: Firm 1's profit for different confidence levels

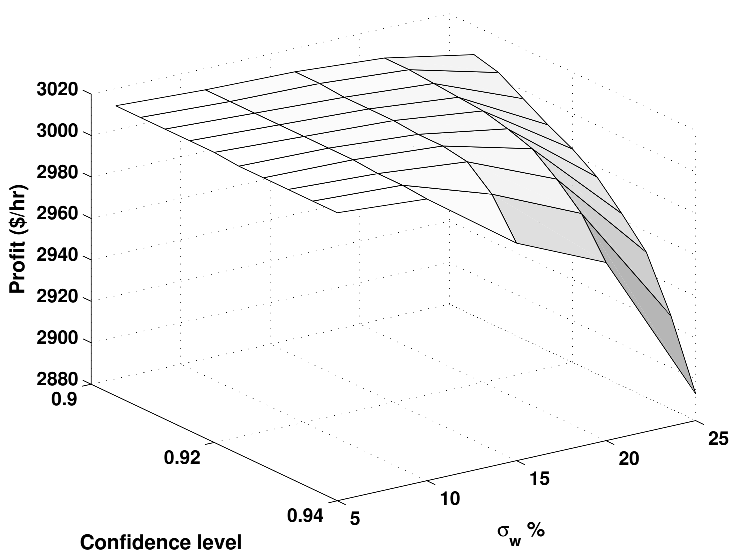

Figure 2: Firm 1's profit for different confidence levels and standard deviations, wind generation at bus 2

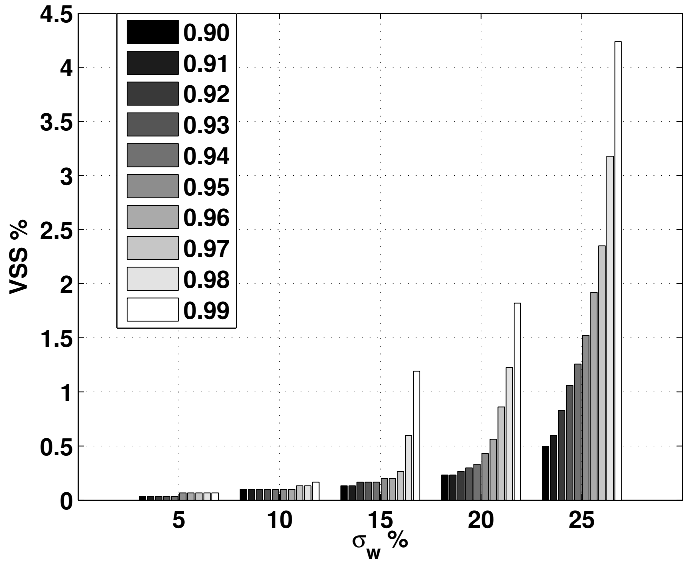

Figure 3: Firm 1's loss of profit

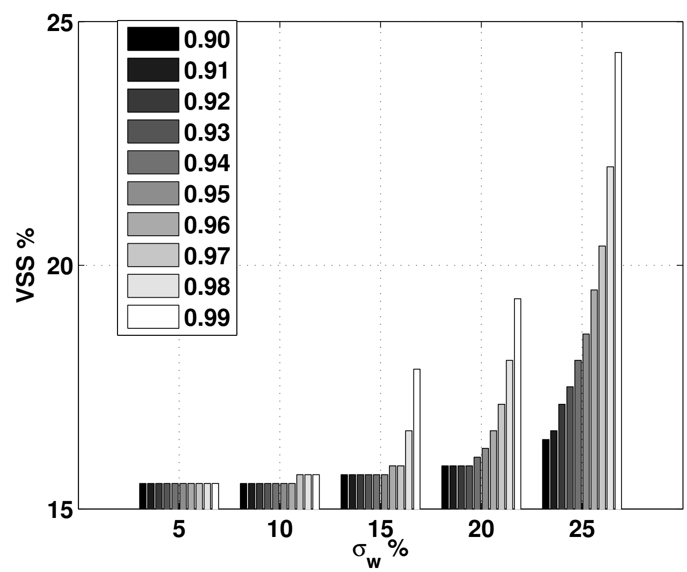

Figure 4: Firm 2's loss of profit

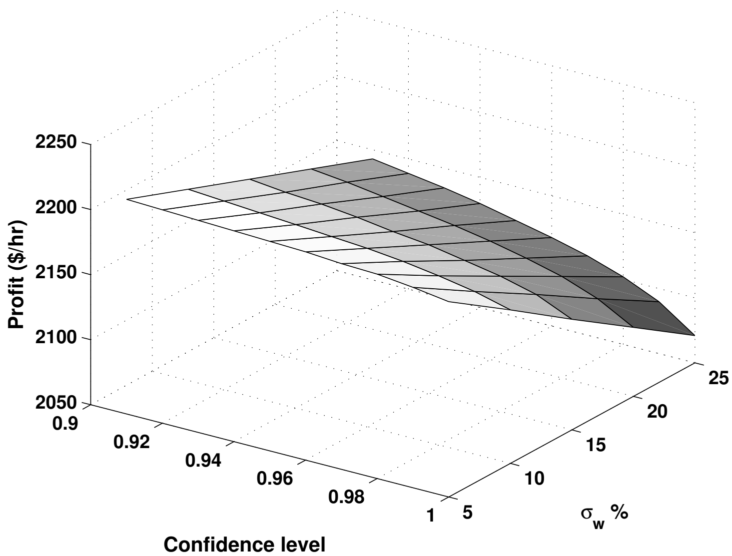

Figure 5: Firm 1's profit for different confidence levels and standard deviations, wind generation at bus 3

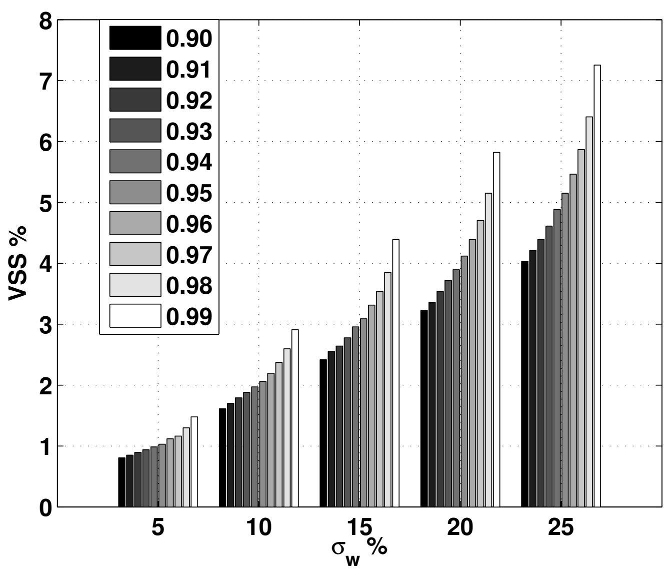

Figure 6: Firm 1's loss of profit

Case	$p_1(\$/hr)$	$p_2(\$/hr)$	$p_3(\$/hr)$	$B_{f_1}(\$/hr)$	$B_{f_2}(\$/hr)$
μ_{xw}= 0 MW	23.16	24.51	23.83	3002	903
μ_{xw}= 60 MW	23.24	23.53	23.38	3021	554

Table 1: Prices and Profits for the 3-bus test system

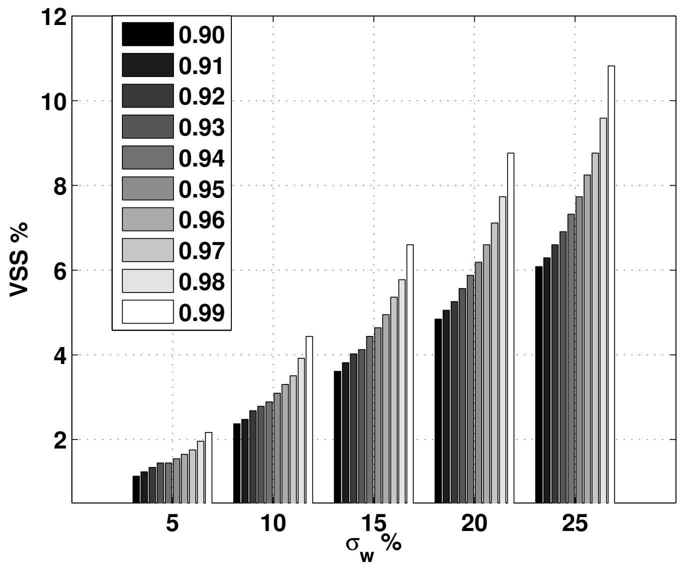

Figure 7: Firm 2's loss of profit

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