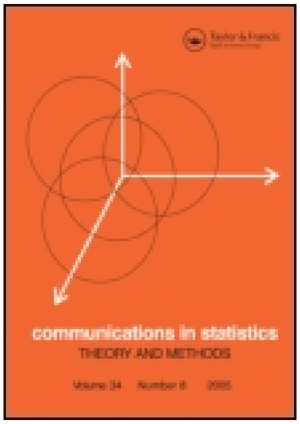

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Testing Goodness-of-Fit for Exponential Distribution Based on Cumulative Residual Entropy

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Testing exponentiality has long been an interesting issue in statistical inferences. In this article, we introduce a new measure of distance between two distributions that is similar Kullback–Leibler divergence, but using the distribution function rather than the density function. This new measure is based on the cumulative residual entropy. Based on this new measure, a consistent test statistic for testing the hypothesis of exponentiality against some alternatives is developed. Critical values for various sample sizes determined by means of Monte Carlo simulations are presented for the test statistics. Also, by means of Monte Carlo simulations, the power of the proposed test under various alternative is compared with that of other tests. Finally, we found that the power differences between the proposed test and other tests are not remarkable. The use of the proposed test is shown in an illustrative example.

Keywords Cumulative residual entropy; Kullback–Leibler divergence; Maximum entropy; Power study; Test for exponentiality.

Mathematics Subject Classification 62G10; 62E10; 94A17; 65C05.

1. Introduction

The notion of entropy is of fundamental importance in different areas such as physics, probability and statistics, communication theory, and economics. In information theory, entropy is a measure of the uncertainty associated with a random variable. This concept was introduced by Shannon (1948). Shannon entropy represents an absolute limit on the best possible lossless compression of any communication. For a random variable X the Shannon entropy is defined as

$$H(X) = -\int_{-\infty}^{\infty} f(x) \ln f(x) \mathrm{d}x,$$

where f is the probability density function (pdf) if X is continuous, probability mass function if X is discrete.

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However, the Shannon entropy has certain disadvantages. For example, it requires the knowledge of density function for non discrete random variables, the discrete Shannon entropy dose not converge to its continuous analogous, and in order to estimate the Shannon entropy for a continuous density, one has to obtain the density estimation, which is not a trivial task. Rao et al. (2004) introduced a new measure of information that extends the Shannon entropy to continuous random variables, and called it cumulative residual entropy (CRE). They showed that it is more general than the Shannon entropy and possesses more general mathematical properties than the Shannon entropy. Its definition is valid for both continuous and discrete cases. It can easily be computed from sample data and its estimation asymptotically converges to the true value. CRE has applications in reliability engineering and computer vision, for more details see Rao (2005). This measure is based on the cumulative distribution function (cdf) F and is defined as follows:

$$CRE(X) = -\int_{R_+^N} P(|X| > \lambda) \ln P(|X| > \lambda) \mathrm{d}\lambda,$$

where $X = (X_1, \ldots . X_N)$ and $\lambda = (\lambda_1, \ldots, \lambda_N)$ and $|X| > \lambda$ means that, for every i, $|X_i| > \lambda_i$, and $R_+^N = \{(\lambda_1, \ldots, \lambda_N); \lambda_i \geq 0, 1 \leq i \leq N\}$. In reliability theory, CRE is based on survival function $\overline{F}(x) = 1 - F(x)$, and is defined as

$$CRE(X) = -\int_0^\infty \overline{F}(x) \ln \overline{F}(x) \mathrm{d}x.$$

Testing for exponentiality still attracts considerable attention and is the topic of a good amount of recent research. Many authors provide test statisics for detecting departures from the hypothesis of exponentiality against specific or general alternatives. Alwasel (2001) and Ahmad and Alwasel (1999) used the lack of memory property of the exponential distribution. Grzegorzewski and Wieczorkowski (1999) and Ebrahimi and Habibullah (1992) make use of the maximum entropy principle. Also, since early work by Sukhatme (1937) and later work by Epstein and Sobel (1953, 1954, 1955) and Epstein (1954, 1960) considerable attention has been given to testing the hypothesis of exponentiality. Park and Park (2003) established the entropy-based goodness of fit test statistics based on the nonparametric distribution functions of the sample entropy and modified sample entropy, and compare their performances for the exponential and normal distributions.

The rest of this article is organized as follows. In Sec. 2, we use a new measure of distance between two distributions that is similar Kullback–Leibler divergence, but using the distribution function rather than the density function. Based on this new measure, a consistent test statistic for testing the hypothesis of exponentiality against some alternatives is developed. In Sec. 3, we consider some power estimates obtained by the method of Mont Carlo simulation. The use of the proposed test is illustrated by an example in Sec. 4.

2. Test Statistics and Its Properties

Suppose X and Y be two non negative and absolutely continuous random variables with cdf F and G and pdf f and g, respectively. As an information distance between two distribution function F and G, Kullback and Leibler (1951) proposed the following discrimination measure, also known as relative entropy of X and Y:

$$I_{X,Y} = \int_0^\infty f(x) \ln \frac{f(x)}{g(x)} \mathrm{d}x.$$

Also, Ebrahimi and Kirmani (1996) defined a measure of discrimination between two residual lifetime distributions.

To construct a goodness-of-fit test for exponentiality, we first define a new measure of distance between two distribution that is similar to Kullback–Leibler divergence (KL), but using the distribution function rather than the density function and call it cumulative Kullback–Leibler (CKL) divergence.

Definition 2.1. If X and Y be two non negative and absolutely continuous random variables with, respectively, cdfs F and G, then CKL between these distributions is defined as

$$CKL(F:G) = \int_0^\infty \overline{F}(x) \ln \frac{\overline{F}(x)}{\overline{G}(x)} \mathrm{d}x - [E(X) - E(Y)],$$

where $\overline{F}(x) = 1 - F(x)$ and $\overline{G}(x) = 1 - G(x)$ are, respectively, cumulative residual distributions.

Lemma 2.1. *$CKL(F:G) \geq 0$, and equality holds if and only if $F = G$, a.e.*

Proof. By the log-sum inequality, we have

$$\int_0^\infty \overline{F}(x) \ln \frac{\overline{F}(x)}{\overline{G}(x)} \mathrm{d}x \geq \int_0^\infty \overline{F}(x)\mathrm{d}x \ln \frac{\int_0^\infty \overline{F}(x)\mathrm{d}x}{\int_0^\infty \overline{G}(x)\mathrm{d}x} = E(X) \log \frac{E(X)}{E(Y)}.$$

The proof is complete if we use the inequality $x \ln \frac{x}{y} \geq x - y$, $\forall x > 0$ and $\forall y > 0$ and note that in the log-sum inequality, equality holds if and only if $\overline{F}(x) = \overline{G}(x)$, *a.e.*

Let $X_1, X_2, \ldots, X_n$ be non negative; independent and identically distributed (iid) random variables from an absolutely continuous cdf F with order statistics, $X_{(1)} \leq \cdots \leq X_{(n)}$, and with finite $\lambda = \frac{E(X_1^2)}{2E(X_1)}$. Let $F_0(x, \lambda) = 1 - e^{-\frac{x}{\lambda}}$, $\lambda > 0$, $x > 0$, denote an exponential cdf, where λ is the unknown mean parameter. The aim of this article is testing the hypothesis

$$H_0 : F(x) = F_0(x, \lambda), \quad \text{vs.} \quad H_a : F(x) \neq F_0(x, \lambda).$$

Under the null hypothesis $CKL(F:F_0) = 0$ and large value of $CKL(F; F_0)$ leads us to reject the null hypothesis H_0 in favor of the alternative hypothesis H_a. Since evaluation of the integral in $CKL(F:F_0)$ requires complete knowledge of F and F_0, then $CKL(F:F_0)$ is not operational. We operationalize $CKL(F:F_0)$ by developing a discrimination information statistics. Toward this end, $CKL(F; F_0)$ is written as

$$\begin{aligned} CKL(F:F_0) &= -CRE(F) - \int_0^\infty \overline{F}(x) \ln \overline{F}_0(x;\lambda)\mathrm{d}x - E(X) + \lambda \\ &= -CRE(F) + \frac{1}{\lambda}\int_0^\infty x\overline{F}(x)\mathrm{d}x - E(X) + \lambda \end{aligned}$$

$$= -CRE(F) + \frac{1}{2\lambda}E(X^2) - E(X) + \lambda$$
$$= -CRE(F) + \lambda. \tag{1}$$

The last equality is obtained by noting that $\lambda = \frac{E(X_1^2)}{2E(X_1)}$. An estimator of $CRE(F)$ is the CRE of the empirical distribution $F_n(x) = \sum_{i=0}^{n-1} \frac{i}{n} I_{[x_{(i)}, x_{(i+1)})}$. Thus,

$$\widehat{CRE}(F) = -\int_0^\infty \overline{F}_n(x)\ln(\overline{F}_n(x))\mathrm{d}x$$
$$= -\sum_{i=1}^{n-1} \frac{n-i}{n}\left(\ln\frac{n-i}{n}\right)(X_{(i+1)} - X_{(i)}),$$

where $\overline{F}_n(x) = 1 - F_n(x)$. By replacing $CRE(F)$ by $\widehat{CRE}(F)$ and λ by $\widehat{\lambda} = \frac{\sum_{i=1}^n X_i^2}{2\sum_{i=1}^n X_i}$ in (1), an estimator of $CKL(F : F_0)$ is obtained as follows:

$$\widehat{CKL}(F : F_0) = \sum_{i=1}^{n-1} \frac{n-i}{n}\left(\ln\frac{n-i}{n}\right)(X_{(i+1)} - X_{(i)}) + \frac{\sum_{i=1}^n X_i^2}{2\sum_{i=1}^n X_i}.$$

Thus, the test statistics is defined as

$$T_n = \frac{\sum_{i=1}^{n-1} \frac{n-i}{n}(\ln\frac{n-i}{n})(X_{(i+1)} - X_{(i)}) + \frac{\sum_{i=1}^n X_i^2}{2\sum_{i=1}^n X_i}}{\frac{\sum_{i=1}^n X_i^2}{2\sum_{i=1}^n X_i}}. \tag{2}$$

We reject H_0 at the significance level α and favor H_a if $T_n \geq T_{n,1-\alpha}$, where $T_{n,1-\alpha}$ is $100(1-\alpha)-$precentile of T_n under H_0.

Rao et al. (2004) proved that $CRE(F_n) \to CRE(F)$ a.s. Thus, $CRE(F_n)$ is a consistent estimator for θ. By consistency of $\frac{\sum_{i=1}^n X_i^2}{2\sum_{i=1}^n X_i}$ for λ and applying Slutsky Theorem, under the null hypothesis, $T_n \xrightarrow{p} 0$. On the other hand, the exponential distribution maximizes CRE among all distributions that have the same coefficient of variation (Rao et al., 2004), so $CRE(F) < CRE(F_0) = \lambda$. Under H_a, $CRE(F_n) \to CRE(F)$ a.e; thus, $T_n \xrightarrow{p} \frac{-CRE(F)+\lambda}{\lambda} > \frac{-CRE(F_0)+\lambda}{\lambda} = 0$. This means that the T_n test is a consistent test. The distribution of T_n under the null hypothesis has not been obtained analytically. To determine the percentage point $T_{n,1-\alpha}$, Monte Carlo simulations were employed.

A Monte Carlo experiment. In order to obtain the percentiles of the null distribution of T_n, 100,000 samples of size n were generated from the standard exponential distribution for selected values $n = 1, \ldots, 39$ and 40 to 60 by 5. For each sample, the T_n statistics as defined in (1) was calculated. The values were then used to determine the critical values $T_{n,0.95}$ and $T_{n,0.99}$. A selection of the 95 and 99% points is presented in Table 1.

The Type I error control using the 0.95 percentiles of the T_n statistics was evaluated by simulating random samples from a spectrum of Exponential populations. A selection of the result is presented in Table 2. It can be seen that the empirical percentiles given in Table 2 provide an excellent Type I error control.

Table 1
Critical values of the test statistic T_n

n	T_n $\alpha = 0.01$	T_n $\alpha = 0.05$	n	T_n $\alpha = 0.01$	T_n $\alpha = 0.05$
1	1	1	23	0.203320	0.145133
2	0.986629	0.930413	24	0.199954	0.140382
3	0.873334	0.716627	25	0.191123	0.136555
4	0.731870	0.568323	26	0.184511	0.1330370
5	0.622731	0.478341	27	0.179066	0.1290080
6	0.546711	0.413322	28	0.177599	0.1246880
7	0.488503	0.365211	29	0.170543	0.1220246
8	0.444109	0.328022	30	0.169259	0.1186599
9	0.407138	0.327022	31	0.164550	0.1165592
10	0.373206	0.274001	32	0.160690	0.1125455
11	0.348556	0.256111	33	0.157976	0.1111602
12	0.325782	0.238452	34	0.153959	0.1082894
13	0.306955	0.224279	35	0.151553	0.1060633
14	0.292023	0.211613	36	0.149443	0.1039062
15	0.277680	0.200307	37	0.146812	0.1015964
16	0.263829	0.191363	38	0.143305	0.0994503
17	0.252797	0.182744	39	0.139728	0.0982132
18	0.244387	0.174967	40	0.138110	0.0959014
19	0.232801	0.166225	45	0.125438	0.0873280
20	0.225033	0.162147	50	0.118886	0.0807222
21	0.217664	0.155650	55	0.110479	0.0748003
22	0.211146	0.150695	60	0.104300	0.0698177

Table 2
Type I error control of T_n test: $\alpha = 0.05$. (Simulation estimates based on 100,000 replications)

Exp(λ)	n = 5	n = 15	n = 25
$\lambda = 2$	0.04954	0.05015	0.05020
$\lambda = 3$	0.05026	0.04877	0.04887
$\lambda = 4$	0.04962	0.05059	0.04942
$\lambda = 5$	0.05041	0.04995	0.04925

3. Power Comparison

The goodness-of-fit test based on the empirical distribution function is widely used as a tool for testing distributional hypotheses. Finkelstein and Schafer (1971) provided the statistics $S,^*$ that tests the fit to an exponential distribution with mean unknown and showed that it is more power than a Kolmogorov-Smirnow type statistics suggested by Lillifors (1969) for the cases tested. Van-Soest (1969) did much the same thing with the Cramer-Von Mises statistics. Recently, Choi et al. (2004) discussed goodness-of-fit tests of the exponential distribution based on Kullback–Leibler information. To construct the test statistics, Correas entropy estimator was used as an estimator of Shannons entropy.

In this section, the performance of the T_n test is investigated using Mont Carlo simulation to the Van-Soest statistics

$$W^2 = \sum_{i=1}^{n} \left\{ F_0(X_{(i)}, \widehat{\theta}) - \frac{2i-1}{2n} \right\}^2 + \frac{1}{12n}$$

and Finkelstein and Schafers statistics

$$S^* = \sum_{i=1}^{n} \max\left\{ \left| F_0(X_{(i)}, \widehat{\theta}) - \frac{i}{n} \right|, \left| F_0(X_{(i)}, \widehat{\theta}) - \frac{i-1}{n} \right| \right\},$$

where $\widehat{\theta} = \frac{1}{n}\sum_{i=1}^{n} X_i = \overline{X}$ and Choi et al. statistics

$$KLC_{mn} = \frac{\exp(C_{mn})}{\exp(\ln \overline{X} + 1)},$$

where $C_{mn} = -\frac{1}{n}\sum_{i=1}^{n} \log\left\{ \frac{\sum_{j=i-m}^{i+m}(X_(j)-\overline{X}_i)(j-i)}{n\sum_{j=i-m}^{i+m}(X_(j)-\overline{X}_i)^2} \right\}$ and $\overline{X}_i = \sum_{j=i-m}^{i+m} \frac{X_{(j)}}{2m+1}$, which are proposed for testing H_0 against H_a. In KLC_{mn} statistics, the windows size m is a positive integer smaller than $\frac{n}{2}$, $X_{(j)} = X_{(1)}$, if $j < 1$ and $X_{(j)} = X_{(n)}$, if $j > n$. H_0 is rejected of large value of W_2 and S^* and of small value of KLC_{mn}. As alternative distributions, the following distributions were selected for power analysis:

(a) a Weibull distribution with density function

$$f(x; \lambda, \beta) = \frac{\beta}{\lambda^\beta} x^{\beta-1} \exp\left(-\left(\frac{x}{\lambda}\right)^\beta \right), \quad \beta > 0, \ \lambda > 0, \ x \geq 0;$$

(b) a gamma distribution with density function

$$f(x; \lambda, \beta) = \frac{x^{\beta-1}\exp(-(\frac{x}{\lambda}))}{\lambda^\beta \Gamma(\beta)}, \quad \beta > 0, \ \lambda > 0, \ x \geq 0;$$

(c) a lognormal distribution with density function

$$f(x; v, \sigma_2) = \frac{1}{x\sigma\sqrt{(2\pi)}} \exp\left\{ -\frac{1}{2\sigma^2}(\ln x - v)^2 \right\}, \quad -\infty < v < \infty, \ \sigma > 0, \ x > 0.$$

For each distribution we set parameters such that $\frac{E(X_1^2)}{2E(X_1)} = 1$, i.e., $\lambda = \frac{2\Gamma(1+\frac{1}{\beta})}{\Gamma(1+\frac{2}{\beta})}$ for the Weibull distribution, $\lambda = \frac{2}{1+\beta}$ for the gamma distribution and $\sigma^2 = \frac{2}{3}(\ln 2 - v)$ for the log-normal case. A total of 100,000 samples of sizes $n = 5, 10, 15, 20, 25$ were generated from each distribution. The statistics T_n, W^2, S^*, KLC_{mn} were calculated for each samples and their powers were recorded in Tables 3–5, by taking the proportion of rejections.

From the result of Table 3–5, we see that the power differences between T_n test and other tests are not remarkable. But calculations of T_n is easier than the

Table 3
Power comparison for the tests T_n, W^2, S^*, and KLC_{mn} when the alternative distribution is Weibull, at the significance levels $\alpha = 0.01$ and $\alpha = 0.05$ and sample sizes are $n = 5, 10, 15, 20, 25$

		T_n		W^2		S^*		KLC_{mn}	
n	β	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$
5	2	0.112	0.345	0.078	0.290	0.074	0.299	0.112	0.368
	3	0.334	0.708	0.254	0.642	0.249	0.659	0.340	0.733
	4	0.595	0.910	0.483	0.864	0.476	0.879	0.595	0.917
10	2	0.338	0.647	0.288	0.614	0.303	0.631	0.374	0.695
	3	0.857	0.978	0.812	0.968	0.831	0.976	0.870	0.981
	4	0.990	0.999	0.978	0.999	0.984	0.999	0.989	0.999
15	2	0.550	0.829	0.537	0.828	0.555	0.837	0.615	0.864
	3	0.983	0.999	0.978	0.998	0.984	0.999	0.987	0.999
	4	0.999	0.999	0.999	1	0.999	1	0.999	1
20	2	0.721	0.919	0.744	0.930	0.764	0.941	0.749	0.924
	3	0.998	0.999	0.998	0.999	0.999	0.999	0.998	0.999
	4	1	1	1	1	1	1	1	1
25	2	0.834	0.963	0.879	0.976	0.887	0.981	0.842	0.963
	3	0.999	1	0.999	0.999	0.999	1	0.999	1
	4	1	1	1	1	1	1	1	1

Table 4
Power comparison for the tests T_n, W^2, S^*, and KLC_{mn} when the alternative distribution is Gamma, at the significance levels $\alpha = 0.01$ and $\alpha = 0.05$ and sample sizes are $n = 5, 10, 15, 20, 25$

		T_n		W^2		S^*		KLC_{mn}	
n	β	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$
5	5	0.038	0.163	0.024	0.129	0.025	0.133	0.044	0.179
	6	0.081	0.291	0.055	0.248	0.058	0.255	0.089	0.325
	7	0.131	0.409	0.095	0.364	0.098	0.370	0.148	0.459
10	5	0.086	0.264	0.067	0.243	0.071	0.249	0.112	0.325
	6	0.235	0.515	0.213	0.539	0.217	0.544	0.305	0.627
	7	0.402	0.710	0.401	0.762	0.404	0.762	0.508	0.820
15	5	0.127	0.329	0.133	0.369	0.143	0.373	0.188	0.443
	6	0.373	0.658	0.448	0.763	0.459	0.761	0.516	0.798
	7	0.614	0.853	0.734	0.940	0.740	0.937	0.773	0.944
20	5	0.162	0.395	0.213	0.487	0.219	0.496	0.238	0.499
	6	0.485	0.762	0.652	0.894	0.654	0.893	0.650	0.874
	7	0.757	0.933	0.908	0.987	0.903	0.986	0.893	0.979
25	5	0.203	0.445	0.297	0.589	0.305	0.593	0.285	0.566
	6	0.593	0.835	0.803	0.955	0.801	0.953	0.755	0.930
	7	0.854	0.968	0.975	0.998	0.972	0.997	0.951	0.993

Table 5
Power comparison for the tests T_n, W^2, S^*, and KLC_{mn} when the alternative distribution is Log-Normal, at the significance levels $\alpha = 0.01$ and $\alpha = 0.05$ and sample sizes are $n = 5, 10, 15, 20, 25$

		T_n		W^2		S^*		KLC_{mn}	
n	β	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$
5	0.4	0.213	0.519	0.152	0.528	0.151	0.517	0.227	0.611
	0.5	0.358	0.712	0.282	0.745	0.277	0.725	0.391	0.811
	0.6	0.700	0.946	0.634	0.970	0.613	0.959	0.756	0.983
10	0.4	0.539	0.799	0.664	0.923	0.638	0.904	0.726	0.936
	0.5	0.802	0.950	0.913	0.994	0.889	0.989	0.925	0.993
	0.6	0.991	0.999	0.999	1	0.999	1	0.999	1
15	0.4	0.733	0.905	0.928	0.993	0.895	0.988	0.926	0.990
	0.5	0.943	0.991	0.997	0.999	0.993	0.999	0.995	0.999
	0.6	0.999	1	1	1	1	1	1	1
20	0.4	0.845	0.953	0.990	0.999	0.982	0.999	0.983	0.998
	0.5	0.985	0.998	0.998	1	0.999	1	0.999	1
	0.6	1	1	1	1	1	1	1	1
25	0.4	0.906	0.980	0.999	0.999	0.997	0.999	0.996	0.999
	0.5	0.995	0.999	1	1	1	1	1	1
	0.6	1	1	1	1	1	1	1	1

other statistics especially KLC_{mn}, thus T_n test needs less time than the other tests for simulations. It is also remarkable that the power of the all tests against any alternative shows an increasing pattern for the sample size.

4. An Illustrative Example

In this section we consider one real-life data analysis from Lawless (1982). We present an example to illustrate the use of the test T_n for testing the validity of Exponential distribution. The data are given below, it consist of failure times for 36 appliances subjected to an automatic life test.

Data set: 11, 35, 49, 170, 329, 381, 708, 958, 1062, 1167, 1594, 1925, 1990, 2223, 2327, 2400, 2451, 2471, 2551, 2565, 2568, 2694, 2702, 2761, 2831, 3034, 3059, 3112, 3214, 3478, 3504, 4329, 6367, 6976, 7846, 13403.

Table 6
Critical values, test statistics, and the p-values

Exponential dis.	Critical value	T_n	p-value
$\alpha = 0.01$	0.1474290	0.0495886	0.9996373
$\alpha = 0.05$	0.1029019	0.0495886	0.9996373

Table 6 shows critical values, test statistics and the p-values. Since the values of T_n are less than the critical values, test accepts the null hypothesis that failure times follow an exponential distribution at significance levels $\alpha = 0.01$ and $\alpha = 0.05$.

5. Concluding Remark

In this article, we construct a consistent goodness-of-fit test for exponential distribution via maximum cumulative residual entropy property under one constraint. Other life-time models such as Weibull and Pareto may be used, but we must prove that they have maximum cumulative residual entropy property under some constraints. Work in this direction is currently under progress.

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References

Ahmad, I. A., Alwasel, I. (1999). A goodness-of-fit test for exponentiality based on the memoryless property. *J. Roy. Statist. Soc., Ser. B* 61:681–689.

Alwasel, I. (2001). On goodness of fit testing for exponentiality using the memoryless property. *J. Nonparametric Statist.* 13:569–581.

Choi, B., Kim, K., Song, S. H. (2004). Goodness-of-fit test for exponentiality based on Kullback–Leibier information. *Simul. Computat.* 33(2):525–536.

Ebrahimi, N., Habibullah, M. (1992). Testing exponentiality based on Kullback–Leibler information. *J. Roy. Statist. Soc. Ser. B* 54:739–748.

Ebrahimi, N., Kirmani, S. N. U. A. (1996). A measure of discrimination between two residual lifetime distributions and its applications. *AIMS* 48:257–265.

Epstein, B. (1954). Truncated life tests in the exponential case. *J. Amer. Statist. Assoc.* 25:555–564.

Epstein, B. (1960). Testing for the validity of the assumption that the underlying distribution of life is exponential. *Technometrics* 2:83–101.

Epstein, B., Sobel, M. (1953). Life testing. *J. Amer. Statist. Assoc.* 48:486–502.

Epstein, B., Sobel, M. (1954). Some theorems relevant to life testing form an exponential distribution. *Ann. Math. Statist.* 25:373–381.

Epstein, B., Sobel, M. (1955). Sequential life tests in the exponential case. *Ann. Math. Statist.* 26:82–93.

Finkelstein, J., Schafer, R. E. (1971). Improved goodness of fit tests. *Biometrika* 58:641–645.

Grzegorzewski, P., Wieczorkowski, R. (1999). Entropy-based goodness-of-fit test for exponentiality. *Commun. Statist. Theor. Meth.* 28:1183–1202.

Kullback, S., Leibler, R. A. (1951). On information and sufficiency. *Ann. Math. Statist.* 22:79–86.

Lawless, J. F. (1982). *Statistical Models and Methods for Life-Time Data.* New York: Wiley.

Lillifors, H. W. (1969). On the Kolmogorov–Smirnov test for the exponential distribution with mean unknown. *J. Amer. Statist. Assoc.* 64:387–389.

Park, S., Park, D. (2003). Correcting moments for goodness of fit tests based on two entropy estimates. *JSCS* 73(9):685–694.

Rao, M. (2005). More on a new concept of entropy and information. *J. Theoret. Probab.* 18:967–981.

Rao, M., Chen, Y., Vemuri, B. C., Wang, F. (2004). Cumulative residual entropy: A new measure of information. *IEEE Trans. Inform. Theor.* 50(6):1220–1228.

Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Tec. J.* 27:379–432.

Sukhatme, P. V. (1937). Test of significance for samples of the χ^2-population with two degrees of freedom. *Ann. Eugenics* 8:52–56.

Van-Soest, F. (1969). Some goodness of fit tests for the exponential distribution. *Statistica Neerlandia* 23:41–51.