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Time decay probability distribution of the neutral meson system and *CP*-violation

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Abstract
In this paper, we use the time-super-operator formalism and the two-level Friedrichs model to obtain a phenomenological model of mesons decay. Our approach provides a fairly good estimation of the *CP*-symmetry violation parameter in the case of K, B and D mesons.

1. Introduction

The description of the decay processes of unstable particles is a long-standing problem [1] where it is generally admitted that the process follows an exponential law associated with the lifetime of the particle. It is worth noting that since Gamow, a huge work has been realized in mathematical physics in order to properly address the decay of unstable quantum particles; for recent works, see e.g.[33, 2, 34] and references therein.

Most elementary particles are unstable, so a quantum-mechanical description is of great importance. However, there is a fundamental problem related to the description of the probability distribution of time occurrence of microscopic events like the particle decay, when the system is described by a given wavefunction or a density matrix. The time-operator formalism allows us to compute the expression of the probability distribution $p_\rho(t)$ that a system, prepared at time 0 in some unstable state $\rho(0)$, is found undecayed throughout the interval $[0, t[$. The lack of such quantity in quantum theory is an issue that has been discussed for the arrival time observable of a free particle moving in one-dimensional space from a source to a detector [3]. This probability should not be confused with the standard quantity $|\langle\psi(0)|\psi(t)\rangle|^2$ which is interpreted as the probability *at the instant t* for finding the system undecayed when it is initially in the unstable wavefunction state $\psi(0)$. While $p_\rho(t)$ should be a monotonically decreasing function of t, $|\langle\psi(0)|\psi(t)\rangle|^2$ has not in general such a property [12]. This is related to the lack of a time operator in quantum mechanics. The absence of a time operator goes back to a celebrated argument of Pauli concerning the nonexistence of a canonically conjugated operator T to the Hamiltonian H:

$$[H, T] = \mathrm{i}I \tag{1}$$

on account of the lower semi-boundedness of the spectrum of H. However, time super-operator was constructed in the framework of the Liouville–von Neumann space, modeling the density matrix states [15, 16, 20]. In this space, the time-evolution operator is the Liouville–von Neuman operator, i.e. $L\rho = [H, \rho]$, which is self-adjoint and may have a spectrum extended over the real line. The time super-operator T is a self-adjoint super-operator on the Liouville–von Neumann space conjugated to L, i.e. $[T, L] = \mathrm{i}I$. This definition is equivalent to the Weyl relation: $\mathrm{e}^{\mathrm{i}tL}T\mathrm{e}^{-\mathrm{i}tL} = T + tI$.

In experimental situations, the exponential decay law is observed and the time operator provides it too. However, this approach provides a more complete description, since it allows the computation of decay time probability distribution which could be relevant in small effects like *CP*-violation effect.

L being unbounded by below and above when H is unbounded by above, Pauli's objection does not apply to the time-super-operator formalism. In order to examine the time-super-operator formalism in this setting, we consider a Hamiltonian model of the neutral kaon decay. Several theoretical and experimental works on this system have been carried out (see, e.g., the collection of papers edited in [5]) and the question is partially open today. All the existing theoretical treatments using Hamiltonian models of two-level state coupled to a continuum of degrees of freedom simulating the phenomenology of neutral kaons were based on the computation of the quantity $|\langle\psi(0)|\psi(t)\rangle|^2$ and the Wigner–Weisskopf approximate solutions to the Schrödinger equation. These solutions allow us to estimate the *CP* violation, but only qualitatively [13]. Using a time-operator approach in this approximation, we show how the usual decay intensity formula should be renormalized in order to estimate the experimental *CP*-violation parameter provided by the Christenson *et al* experiment. Our time-super-operator approach to the *CP* violation in such a model is based on the computation of $p_\rho(t)$. It is a test for the formalism of the time super-operator, and we shall show that this approach to meson systems provides quite satisfying quantitative predictions. Let us firstly recall some developments of the Wigner–Weisskopf approach to the kaons phenomenology and time-operator approach, and then we shall develop the time-super-operator approach.

It is well known [6] that kaons appear in pair K^0 and $\overline{\mathrm{K}}^0$ conjugated to each other. The decay processes of K^0 and $\overline{\mathrm{K}}^0$ correspond to two orthogonal decaying modes K_1 and K_2, that are distinguished by their lifetime and *CP*-eigenvalues. The discovery of the small *CP*-violation effect was also accompanied by the non-orthogonality of the short- and long-lived decay modes, denoted K_S and K_L, slightly different from K_1 and K_2 and depending on a *CP*-violation parameter ϵ. Lee, Oehme and Yang (LOY) [7] proposed an extension of the Wigner–Weisskopf theory [8] in order to account for the exponential decay of kaons. Later on, Khalfin [9] has pointed out that, for a quantum system with the energy spectrum bounded from below, the decay should not be exponential for large times (see also [10–13]). Khalfin also corrected the parameter ϵ at the lowest order of perturbation. His estimation has been presented and re-examined in [13] and applied to other mesons. For kaons, it leads to a numerical value that is 30 times larger than the experimental data.

An alternative time-operator approach is possible in the treatment of unstable systems in the framework of the Wigner–Weisskopf approximation, since this theory supposes an extension of the spectrum of H on the entire real axis. In this approximation, one can construct a time operator in the Hilbert space of wavefunctions. A decay time operator T' is canonically conjugated to H, that is, in the energy representation, H is given by

$$H\psi(\omega) = \omega\psi(\omega) \tag{2}$$

and T' is given by

$$T'\psi(\omega) = -\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\omega}\psi(\omega) \tag{3}$$

so that T' satisfies to the commutation relation $[H, T'] = \mathrm{i}I$. The T'-representation is therefore obtained by a Fourier transform

$$\widehat{\psi}(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\tau\omega}\, \psi(\omega)\, \mathrm{d}\omega, \tag{4}$$

and the unstable states are prepared such that the decay occurs in the future, that is, $\widehat{\psi}(\tau) = 0$ for $\tau < 0$. [17]. Any state of the form $\psi_{\mathrm{un}}(\omega) = A/(\omega - z_0)$, $(z_0 = m - \frac{\mathrm{i}}{2}\Gamma)$, belongs to this space, since by computing its Fourier transform we have

$$\widehat{\psi}_{\mathrm{un}}(\tau) = \begin{cases} \mathrm{i}A\sqrt{2\pi}\, \mathrm{e}^{-\mathrm{i}\tau z_0} & \tau \geqslant 0, \\ 0 & \tau < 0. \end{cases} \tag{5}$$

It is clear that for these states the decay probability density is defined by

$$|\widehat{\psi}_{\mathrm{un}}(\tau)|^2 = 2\pi |A|^2\, \mathrm{e}^{-\Gamma\tau}. \tag{6}$$

This is an exponential probability density of decay time which is very common in particle physics. The states with exponential distribution of decay time in the time-operator representation correspond to the so-called resonance states. But this is not the single type of unstable states.

In this paper, we go beyond the Wigner–Weisskopf approximation and consider a more physical situation in which the Hamiltonian has been bounded by the below spectrum.

Rigorously speaking, it is then forbidden to define a time operator that satisfies the commutation relation $[H, T'] = \mathrm{i}I$. In order to escape this contradiction, we go to the space of density matrices. The time super-operator T is a self-adjoint operator acting on density matrices that verifies the Weyl relation: $\mathrm{e}^{\mathrm{i}tL} T\, \mathrm{e}^{-\mathrm{i}tL} = T + tI$. The interpretation of this super-operator is that for an unstable initial state ρ, the time of decay occurrence is a random event which fluctuates and we speak of the probability of its occurrence in a time interval $I =]t_1, t_2]$. The average time of decay in the state ρ is given by

$$\langle T \rangle_\rho = \langle \rho, T\rho \rangle. \tag{7}$$

The observable $T' = -T$ is associated with the decay event. In fact, for a system prepared in the initial state ρ_0 the dynamics shifts the average time of occurrence (or lifetime) in the state ρ_0, $\langle T' \rangle_{\rho_0}$, by the time parameter t so that the average time of decay in the state ρ_t is given by

$$\langle T' \rangle_{\rho_t} = \langle T' \rangle_{\rho_0} - t. \tag{8}$$

This equation follows from the Weyl relation and the definition of the average of T. Let $\mathcal{P}_\tau$ denote the family of spectral-projection operators of T:

$$T = \int_{\mathbb{R}} \tau\, \mathrm{d}\mathcal{P}_\tau, \tag{9}$$

and let $\mathcal{Q}_\tau$ be the family of spectral projections of T'; then, in the state ρ, the probability of occurrence of the event in a time interval I is given [17], analogously with the usual quantum formulations, by

$$\mathcal{P}(I, \rho) = \|\mathcal{Q}_{t_2}\rho\|^2 - \|\mathcal{Q}_{t_1}\rho\|^2 = \|(\mathcal{Q}_{t_2} - \mathcal{Q}_{t_1})\rho\|^2 := \|\mathcal{Q}(I)\rho\|^2. \tag{10}$$

The unstable 'undecayed' states observed at $t_0 = 0$ are the states ρ such that $\mathcal{P}(I, \rho) = 0$ for any negative time interval I, that is,

$$\|\mathcal{Q}_\tau \rho\|^2 = 0, \quad \forall \tau \leqslant 0. \tag{11}$$

In other words, these are the states verifying $\mathcal{Q}_0\rho = 0$. It is straightforwardly checked that the spectral projections $\mathcal{Q}_\tau$ are related to the spectral projections $\mathcal{P}_\tau$ by the following relation:

$$\mathcal{Q}_\tau = 1 - \mathcal{P}_{-\tau}. \tag{12}$$

Let $\mathfrak{F}_\tau$ be the subspace on which $\mathcal{P}_\tau$ projects. Thus, the unstable undecayed states are those states satisfying $\rho = \mathcal{P}_0\rho$ and they coincide with the subspace $\mathfrak{F}_0$[4]. For these states, the probability that a system prepared in the undecayed state ρ is found to decay some time during the interval $I =]0, t]$ is $\|\mathcal{Q}_t\rho\|^2 = 1 - \|\mathcal{P}_{-t}\rho\|^2$ a monotonically nondecreasing quantity which converges to 1 as $t \to \infty$, while $\|\mathcal{P}_{-t}\rho\|^2$ decreases monotonically to zero. The quantity $\|\mathcal{P}_{-t}\rho\|^2$ corresponds to a *genuine survival probability* and should not be confused with the usual 'survival probability of an unstable state χ at time t' defined by $|\langle\chi, \mathrm{e}^{-\mathrm{i}tH}\chi\rangle|^2$, where χ is an eigenstate of the free Hamiltonian.

Considered so, the time-operator approach is non-standard. Actually, the key, non-standard, assumption that underlies the time-super-operator formalism is the following.

In the Liouville space, given any initial state ρ, its survival probability in the unstable space is given by

$$p_\rho(t) = \|\mathcal{P}_{-t}\rho\|^2. \tag{13}$$

This is the probability that, for a system initially in the state ρ, no decay is found during $[0, t]$. For any initial state ρ, this survival probability in the unstable space was given [15] by the expression

$$\begin{aligned} p_\rho(t) &= \|\mathcal{P}_{-t}\rho\|^2 \\ &= \|U_{-t}\mathcal{P}_0U_t\rho\|^2 \\ &= \|\mathcal{P}_0\,\mathrm{e}^{-\mathrm{i}tL}\rho\|^2, \end{aligned} \tag{14}$$

where we used the following relation: $\mathcal{P}_{-t} = U_{-t}\mathcal{P}_0U_t$. *Then, the survival probability is monotonically decreasing to* 0 *as* $t \to \infty$. This survival probability and the probability of finding the system to decay some time during the interval $I =]0, t]$, $q_\rho(t) = \|\mathcal{Q}_\rho(t)\|^2$ are related by

$$q_\rho(t) = 1 - p_\rho(t), \tag{15}$$

and $q_\rho(t) \to 1$ when $t \to +\infty$. The time derivative of this quantity is a genuine probability density function (pdf) and will be used to define the decay intensity in section 5.

The paper is organized as follows. In section 2, we present the basic phenomenology of kaons. In section 3, we introduce the Friedrichs-type Hamiltonian for kaons, where the states K_1 and K_2 are eigenfunctions of the free Hamiltonian that interact with a continuum representing the decay products. Here is the first difference with [13] where K^0 and $\overline{\mathrm{K}}^0$ are eigenfunctions of the free Hamiltonian. As a solvable model, we compute the energy spectral representation of the undecayed states in the Wigner–Weisskopf and weak coupling approximation. In section 4, we apply the time-operator formalism in the Wigner–Weisskopf approximation as introduced above. We compute the decay probability of the kaons into two pions (resp. three pions), from which we derive a formula of the *CP*-violation in terms of lifetimes and energy difference of the short and long kaon states. Our formula is different from that of [13] and improves the estimation to 0.6 times the experimental value. The main contribution of the paper in section 5 concerns the super-operator formalism which we use for computing the eigenprojections of T and the survival probability of the decaying states. This approach allows us to compute the *CP*-violation parameters for K, D and B mesons. For all of them we obtain good agreement with the experimental value. We put several lengthy calculations in the appendices.

[4] Therefore, a subspace $\mathfrak{F}_{t_0}$ is a set of decaying states prepared at time t_0. We call it an unstable subspace of T.

2. Phenomenology of kaons

For the main properties of kaons we refer to [6, 21]. A summarized presentation may be found in [30]. We recall the main notations (used later) of +1 and −1 *CP*-eigenstates, respectively,

$$|\mathrm{K}_1\rangle = \frac{1}{\sqrt{2}}(|\mathrm{K}^0\rangle - |\overline{\mathrm{K}}^0\rangle), \qquad |\mathrm{K}_2\rangle = \frac{1}{\sqrt{2}}(|\mathrm{K}^0\rangle + |\overline{\mathrm{K}}^0\rangle). \tag{16}$$

After *CP-violation* was discovered by Christenson *et al* [23], the exact eigenstates characterized by the short-lived state (K_S) and long-lived state (K_L) are expressed as coherent superpositions of the K_1 and K_2 eigenstates through

$$|\mathrm{K}_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}[\epsilon|\mathrm{K}_1\rangle + |\mathrm{K}_2\rangle], \qquad |\mathrm{K}_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}[|\mathrm{K}_1\rangle + \epsilon|\mathrm{K}_2\rangle]. \tag{17}$$

Recall that the weak disintegration process distinguishes the K_1 states which decay only into '2π', while the K_2 states decay into '$3\pi, \pi e\nu, \ldots$' [22]. The lifetime of the K_1 kaon is short ($\tau_S \approx 8.92 \times 10^{-11}$ s), while the lifetime of the K_2 kaon is longer ($\tau_L \approx 5.17 \times 10^{-8}$ s). We need also to recall the following that will be used later: K_L and K_S are the eigenstates of the Hamiltonian for the mass-decay matrix [21, 22] which has the following form in the basis $|\mathrm{K}^0\rangle$ and $|\overline{\mathrm{K}}^0\rangle$:

$$H = M - \frac{\mathrm{i}}{2}\Gamma \equiv \begin{pmatrix} M_{11} - \frac{\mathrm{i}}{2}\Gamma_{11} & M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12} \\ M_{21} - \frac{\mathrm{i}}{2}\Gamma_{21} & M_{22} - \frac{\mathrm{i}}{2}\Gamma_{22} \end{pmatrix}, \tag{18}$$

where M and Γ are individually Hermitian since they correspond to observables (mass and lifetime). The corresponding eigenvalues of the mass-decay matrix are equal to

$$m_L - \frac{\mathrm{i}}{2}\Gamma_L, \qquad m_S - \frac{\mathrm{i}}{2}\Gamma_S. \tag{19}$$

It follows from (17) that the transition amplitude of the K_L beam is given by

$$\psi(t) = A\big(\epsilon^{\exp}\,\mathrm{e}^{-\mathrm{i}(m_S - \frac{\mathrm{i}}{2}\Gamma_S)t} + \mathrm{e}^{-\mathrm{i}(m_L - \frac{\mathrm{i}}{2}\Gamma_L)t}\big) \tag{20}$$

with A being a global proportionality factor that remains constant in time. Then the intensity $I(t) = |\psi(t)|^2$ is given by

$$I(t) = I_0\big(\mathrm{e}^{-\Gamma_L t} + |\epsilon^{\exp}|^2\,\mathrm{e}^{-\Gamma_S t} + 2|\epsilon^{\exp}|\,\mathrm{e}^{-(\frac{\Gamma_S+\Gamma_L}{2})t)}\cos(\triangle m t + \arg(\epsilon^{\exp}))\big). \tag{21}$$

This leads to an experimental estimation of $\epsilon^{\exp}$ [24]:

$$|\epsilon^{\exp}| = (2.232 \pm 0.007) \times 10^{-3}, \qquad \arg(\epsilon^{\exp}) = (43.5 + 0.7)^\circ. \tag{22}$$

3. The two-level Friedrichs model

We recall here our main definitions [30] of the Friedrichs [4] interaction Hamiltonian between the two discrete modes and the continuous degree of freedom given by the operator H on the Hilbert space of the wavefunctions of the form $|\psi\rangle = \{f_1, f_2, g(\mu)\}$, $f_1, f_2 \in \mathbb{C}$, $g \in L^2(\mathbb{R}^+)$:

$$H = H_0 + \lambda_1 V_1 + \lambda_2 V_2, \tag{23}$$

where λ_1 and λ_2 are the complex coupling constants, and

$$H_0|\psi\rangle = \{\omega_1 f_1, \omega_2 f_2, \mu g(\mu)\}, \ (\omega_1 \text{ and } \omega_2 > 0). \tag{24}$$

The operators $V_i (i = 1, 2)$ are given by

$$\begin{aligned} V_1\{f_1, f_2, g(\mu)\} &= \{\langle v(\mu), g(\mu)\rangle, 0, f_1.v(\mu)\} \\ V_2\{f_1, f_2, g(\mu)\} &= \{0, \langle v(\mu), g(\mu)\rangle, f_2.v(\mu)\}, \end{aligned} \tag{25}$$

where

$$\langle v(\mu), g(\mu)\rangle = \int \mathrm{d}\mu v^*(\mu) g(\mu), \tag{26}$$

is the inner product. Thus H can be represented as a matrix :

$$H_{\text{Friedrichs}} = \begin{pmatrix} \omega_1 & 0 & \lambda_1^* v^*(\mu) \\ 0 & \omega_2 & \lambda_2^* v^*(\mu) \\ \lambda_1 v(\mu) & \lambda_2 v(\mu) & \mu \end{pmatrix}, \tag{27}$$

where $\omega_{1,2}$ represent the energies of the discrete levels and the factors $\lambda_i v(\mu)(i = 1, 2)$ represent the couplings to the continuous degree of freedom. The energies μ of the different modes of the continuum range from $-\infty$ to $+\infty$ when $v(\mu) = 1$, but we are free to tune the coupling $v(\mu)$ in order to introduce a selective cutoff to extreme energy modes. We now have to solve the eigenfunctions problem and find the energy representation of K_1 and K_2. Thus, f_1 and f_2 are given by appendix A as

$$f_1(\omega) \simeq \frac{\lambda_1^* v^*(\omega)}{\eta_1^+(\omega)}, \tag{28}$$

and the same formula for f_2 as

$$f_2(\omega) \simeq \frac{\lambda_2^* v^*(\omega)}{\eta_2^+(\omega)}. \tag{29}$$

Here, $\eta_i^{\pm}(\omega)$, $(i = 1, 2)$ are complex conjugates of each other defined by

$$\eta_i^{\pm}(\omega) = \omega - \omega_i + |\lambda_i|^2 \mathsf{P} \int_0^{\infty} \frac{|v(\omega')|^2}{\omega' - \omega} \mathrm{d}\omega' \pm \mathrm{i}\pi |\lambda_i|^2 |v(\omega)|^2, \tag{30}$$

where P indicates the 'principal value' and we used the following identity in equation (30):

$$\lim_{\varepsilon \to 0^+} \frac{1}{x - x_0 \pm \mathrm{i}\varepsilon} = \mathsf{P} \frac{1}{x - x_0} \mp \mathrm{i}\pi\delta(x - x_0). \tag{31}$$

Let $|\chi\rangle = |\epsilon_1 f_1 + \epsilon_2 f_2\rangle$, where ϵ_i, $(i = 1, 2)$ is a constant complex number. The physical meaning of such a state is that it corresponds to a coherent superposition of two exponential decay processes. In the following sections we shall compute the projection of $|\chi\rangle\langle\chi|$ on the unstable space of the time super-operator and then the survival probability $p_\rho(t)$ introduced in the section 2. We compute its expression in terms of the lifetimes and rest masses of the (mesonic) resonances.

Weak coupling conditions. Admitting that $\eta_i^+(\omega)$ in (30) has one zero in the lower half-plane which approaches ω_i for decreasing coupling, we can write

$$\eta_i^+(\omega) = \omega - z_i, \tag{32}$$

where $z_i = \widetilde{\omega}_i - \mathrm{i}b_i$, where $\widetilde{\omega}_i = \omega_i + O(|\lambda|^2)$ and $b_i = \pi|\lambda_i|^2$ is a real positive constant [20]. In this paper, we suppose that $\widetilde{\omega}_1 < \widetilde{\omega}_2$.

4. Time-operator formalism for the *CP*-violation in the Wigner–Weisskopf approximation

Let us present the fundamental ideas of the theory of spontaneous emission of an atom interacting with the electromagnetic field, given by Wigner and Weisskopf. This treatment aims at obtaining an exponential time dependence for decaying states by integrating over the continuum energy. That is, we assume that the modes of the fields are closely spaced. Then, we have to assume that the variation of $v(\mu)$ over μ is negligible with $|\mu| \lesssim$ 'uncertainty of the original state energy', i.e. $v(\mu) \approx v$ independent of μ or in the simple case it is taken to obey $v(\mu) = 1$. Also, another assumption is that the lower limit of integration over ω is replaced by $-\infty$.

The Schrödinger equation of the two-level Friedrichs model in the Wigner–Weisskopf regime becomes

$$\begin{pmatrix} \omega_1 & 0 & \lambda_1^* \\ 0 & \omega_2 & \lambda_2^* \\ \lambda_1 & \lambda_2 & \mu \end{pmatrix} \begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\mu,t) \end{pmatrix} = \mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\mu,t) \end{pmatrix} \tag{33}$$

which means

$$\omega_1 f_1(t) + \lambda_1^* \int_{-\infty}^{\infty} \mathrm{d}\mu g(\mu,t) = \mathrm{i}\frac{\partial f_1(t)}{\partial t}, \tag{34}$$

$$\omega_2 f_2(t) + \lambda_2^* \int_{-\infty}^{\infty} \mathrm{d}\mu g(\mu,t) = \mathrm{i}\frac{\partial f_2(t)}{\partial t} \tag{35}$$

and

$$\lambda_1 f_1(t) + \lambda_2 f_2(t) + \mu g(\mu,t) = \mathrm{i}\frac{\partial g(\mu,t)}{\partial t}. \tag{36}$$

Let us now solve the Schrödinger equation and trace out the continuum in order to derive the Master equation for the two-level system. From equation (36) we can obtain $g(\mu,t)$, taking $g(\mu,0) = 0$, as

$$g(\mu,t) = -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\mu t}\int_0^t \mathrm{d}\tau[\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)]\,\mathrm{e}^{\mathrm{i}\mu\tau}, \tag{37}$$

where $t > 0$. Then, we substitute $g(\mu,t)$ in equation (34) and we obtain

$$\mathrm{i}\frac{\partial f_1(t)}{\partial t} = \omega_1 f_1(t) - \mathrm{i}\lambda_1^* \int_{-\infty}^{\infty} \mathrm{d}\mu\,\mathrm{e}^{-\mathrm{i}\mu t}\int_0^t \mathrm{d}\tau[\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)]\,\mathrm{e}^{\mathrm{i}\mu\tau}; \tag{38}$$

we also obtain the same relation for $f_2(t)$ from equation (35):

$$\mathrm{i}\frac{\partial f_2(t)}{\partial t} = \omega_2 f_2(t) - \mathrm{i}\lambda_2^* \int_{-\infty}^{\infty} \mathrm{d}\mu\,\mathrm{e}^{-\mathrm{i}\mu t}\int_0^t \mathrm{d}\tau[\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)]\,\mathrm{e}^{\mathrm{i}\mu\tau}. \tag{39}$$

Finally, we obtain a Master equation with a non-Hermitian effective Hamiltonian as [25]

$$H_{\text{eff}} = \begin{pmatrix} \omega_1 - \mathrm{i}\pi|\lambda_1|^2 & -\mathrm{i}\pi\lambda_1^*\lambda_2 \\ -\mathrm{i}\pi\lambda_1\lambda_2^* & \omega_2 - \mathrm{i}\pi|\lambda_2|^2 \end{pmatrix}. \tag{40}$$

The eigenvalues of the above effective Hamiltonian under the weak coupling constant approximation are

$$\omega_+ = \omega_1 - \mathrm{i}\pi|\lambda_1|^2 + O(\lambda^4), \qquad \omega_- = \omega_2 - \mathrm{i}\pi|\lambda_2|^2 + O(\lambda^4). \tag{41}$$

In a first and very rough approximation, the eigenvectors of the effective Hamiltonian are the same as the postulated kaons states:

$$|f_+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |K_1\rangle \text{ and } |f_-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |K_2\rangle. \tag{42}$$

Phenomenology imposes that the complex Friedrichs energies $\omega_\pm$ coincide with the observed complex energies. The Friedrichs energies depend on the choice of the four parameters ω_1, ω_2, λ_1 and λ_2 and the observed complex energies are directly derived from the experimental determination of four other parameters, the masses m_S and m_L and the lifetimes τ_S and τ_L. We must thus adjust the theoretical parameters in order that they fit the experimental data. This can be done by comparing the eigenvalue of the effective matrix with the eigenvalue of the mass-decay matrix which is taken in expression (19). Finally, we have

$$\begin{aligned} \omega_1 &= m_S, \qquad 2\pi|\lambda_1|^2 = \Gamma_S, \\ \omega_2 &= m_L, \qquad 2\pi|\lambda_2|^2 = \Gamma_L. \end{aligned} \tag{43}$$

The above identities yield

$$\lambda_1 = \sqrt{\frac{\Gamma_S}{2\pi}}\,\mathrm{e}^{\mathrm{i}\theta_S}, \qquad \lambda_2 = \sqrt{\frac{\Gamma_L}{2\pi}}\,\mathrm{e}^{\mathrm{i}\theta_L}, \tag{44}$$

where θ_S and θ_L are real constants. Appendix B gives the *CPT*-invariant effective Hamiltonian as follows:

$$H_{\mathrm{eff}} = \begin{pmatrix} m_S - \frac{\mathrm{i}}{2}\Gamma_S & -\frac{1}{2}\sqrt{\Gamma_S\Gamma_L} \\ \frac{1}{2}\sqrt{\Gamma_S\Gamma_L} & m_L - \frac{\mathrm{i}}{2}\Gamma_L \end{pmatrix}. \tag{45}$$

The effective Hamiltonian (45) acts on the $|\mathrm{K}_L\rangle$ vector state as an eigenstate corresponding to the eigenvalue $\omega_- = m_L - \mathrm{i}\frac{\Gamma_L}{2}$, so that we must impose that

$$H_{\mathrm{eff}}|\mathrm{K}_L\rangle = A\,H_{\mathrm{eff}}\begin{pmatrix}\alpha\\1\end{pmatrix} = A\,\omega_-\begin{pmatrix}\alpha\\1\end{pmatrix} \tag{46}$$

from which, after straightforward calculations, we obtain

$$\alpha = \sqrt{\frac{\Gamma_L}{\Gamma_S}}\,\frac{\frac{1}{2}}{\frac{\Delta m}{\Gamma_S} - \mathrm{i}\,\frac{\Delta\Gamma}{2\Gamma_S}}, \tag{47}$$

where $\triangle m = m_S - m_L$ and $\triangle\Gamma = \Gamma_S - \Gamma_L$. Similarly, the effective Hamiltonian (45) acts on the $|\mathrm{K}_S\rangle$ vector state as an eigenstate corresponding to the eigenvalue $\omega_+ = m_S - \mathrm{i}\frac{\Gamma_S}{2}$, so that we have

$$H_{\mathrm{eff}}|\mathrm{K}_S\rangle = A\,H_{\mathrm{eff}}\begin{pmatrix}1\\\alpha\end{pmatrix} = A\,\omega_+\begin{pmatrix}1\\\alpha\end{pmatrix}. \tag{48}$$

In order to get a reasonable value for the *CP*-violation parameter we shall use in a first step the time-operator formalism as explained in the introduction. To estimate the decay rate in the *CP* sectors to 2π $(CP = 1)$ and 3π $(CP = -1)$ when the source produces neutral kaon beams, we compute the energy representation of the initial state as a superposition of these modes. By considering relation (32) and taking into account that in the present approximation $v(\omega) = 1$, we can write equations (28) and (29) as

$$f_i(\omega) = \frac{\sqrt{b_i}\,\mathrm{e}^{-\mathrm{i}\theta_i}}{\omega - \widetilde{\omega}_i + \mathrm{i}b_i}, \qquad (i = 1, 2), \tag{49}$$

where θ_i is the phase of the possibly complex coefficients λ_i and $b_i = \pi|\lambda_i|^2$. Using the Fourier transforms, equation (4), for the above equation, (49), we obtain for $(i = 1, 2)$

$$\hat{f}_i(\tau) = \begin{cases} N\sqrt{2\pi\,b_i}\,\mathrm{e}^{-(\mathrm{i}\widetilde{\omega}_i + b_i)t - \mathrm{i}\theta_i}, & t \geqslant 0 \\ 0, & t < 0, \end{cases} \tag{50}$$

where N is the normalization constant. For $s = -t < 0$, we have

$$\hat{f}_i(s) = \begin{cases} \sqrt{2\pi b_i}\,\mathrm{e}^{(\mathrm{i}\widetilde{\omega}_i + b_i)s - \mathrm{i}\theta_i}, & s \leqslant 0 \\ 0, & s > 0. \end{cases} \tag{51}$$

Finally, the normalization relation, i.e.

$$\int_{-\infty}^{+\infty} \mathrm{d}s\,|\hat{f}_i(s)|^2 = 1, \qquad (i = 1, 2), \tag{52}$$

yields $N = 1/\sqrt{\pi}$ and then we have

$$\hat{f}_i(s) = \begin{cases} \sqrt{2b_i}\, e^{(i\widetilde{\omega}_i + b_i)s - i\theta_i}, & s \leqslant 0 \\ 0, & s > 0. \end{cases} \tag{53}$$

Here, $|\hat{f}_i(s)|^2, i = 1, 2,$ has the form of the probability density of the short and long decay modes; thus we shall denote the two resonances $f_1(\omega)$ and $f_2(\omega)$ as $f_S(\omega)$ and $f_L(\omega)$. The 2π and 3π modes are respectively represented by $C_1(f_S(\omega) + \alpha f_L(\omega))$ and $C_2(\alpha f_S(\omega) + f_L(\omega))$, where $C_{1(2)}$ are normalization constants. That is, at time $t = -s$, the time-operator decay probability density associated with the first mode predicts the 2π decay intensity $I_1(s)$ as

$$\begin{aligned} I_1(s) &= |C_1(\hat{f}_S(s) + \alpha \hat{f}_L(s))|^2 \\ &= I_0^1 \left(e^{2b_1 s} + |\epsilon^{\text{th}}|^2 e^{2b_2 s} + |\epsilon^{\text{th}}|\, e^{(b_1+b_2)s} \cos((\widetilde{\omega}_1 - \widetilde{\omega}_2)s + \arg(\epsilon^{\text{th}})) \right), \end{aligned} \tag{54}$$

where choosing $\triangle\theta = \theta_L - \theta_S = \frac{\pi}{2}$ as explained in Appendix B, we put

$$\epsilon^{\text{th}} := i\sqrt{\frac{b_2}{b_1}}\, \alpha. \tag{55}$$

This equation is to be compared with equation (21). Then, from the above equation and equation (55), we give the CP-violation parameter, ϵ^{th}, as follows:

$$\epsilon^{\text{th}} = \frac{\Gamma_L}{\Gamma_S} \frac{\frac{i}{2}}{\frac{\triangle m}{\Gamma_S} - i\frac{\triangle\Gamma}{2\Gamma_S}} = 0.6\, \epsilon^{\text{exp}}. \tag{56}$$

4.1. Fit with Christenson et al [23] experimental data

Essentially, the Christenson *et al* experiment [23] consisted of measuring the ratio R between the number of charged pion pairs (π^+, π^-) and triplets at a (proper) time quite longer than τ_S. During the experiment, 45 pairs were observed and 22 700 decays occurred so that $R = \frac{45}{22\,700} = (2.0 \pm 0.4) \times 10^{-3}$. According to the authors [23], the relation between this ratio and $|\epsilon^{\text{exp}}|$ is $|\epsilon^{\text{exp}}|^2 = R_T \frac{\tau_1}{\tau_2}$, where $R_T = \frac{3}{2}R$, while $\frac{\tau_1}{\tau_2}$ is equal (in the notation of 1964) to the ratio between the short and long lifetimes. The correction factor $\frac{3}{2}$ is explained to be due to the fact that decay in the $CP = +1$ sector branches to charged pion pairs with probability 2/3 (the remaining 1/3 corresponding to neutral pion pairs that were not detected). Substituting the measured value of $R = 2 \times 10^{-3}$ into the expression, we obtain $|\epsilon^{\text{exp}}|^2 = \frac{3}{2}R\frac{\tau_1}{\tau_2} = \frac{3}{2}\left(2.0 \times 10^{-3}\right)\left(\frac{8.92\times10^{-11}}{5.17\times10^{-8}}\right)$, so that finally we find $|\epsilon|^2 = 5.2 \times 10^{-6}$ which corresponds to the CP-violation parameter $\epsilon^{\text{exp}} = 2.3 \times 10^{-3}$ reported in [23], in agreement with the commonly accepted value mentioned in equation (22).

In order to fit our data with Christenson *et al* [23] results, we firstly evaluate the production rates by unit of time in the time-operator approach. This can be done, making use of the time-operator decay probability density associated with the first mode 2π decay I_1 (equation (54)) and a similar equation for the second mode 3π decay I_2:

$$\begin{aligned} I_{1(2)}(s) &= \left|C_{1(2)}\left(\epsilon_S^{1(2)} \hat{f}_S(t) + \epsilon_L^{1(2)} \hat{f}_L(t)\right)\right|^2 \\ &= I_0^{1(2)}(b_1|\epsilon_S^{1(2)}|^2 e^{-2b_1 t} + b_2|\epsilon_L^{1(2)}|^2 e^{-2b_2 t} \\ &\quad + \sqrt{b_1 b_2}|\epsilon_S^{1(2)}\epsilon_L^{1(2)}|\, e^{-(b_1+b_2)t} \cos\left((\widetilde{\omega}_1 - \widetilde{\omega}_2)t + \arg\left(\epsilon_S^{1(2)}\right) - \arg\left(\epsilon_L^{1(2)}\right)\right), \end{aligned} \tag{57}$$

where in accordance with (46) and (48)

$$\begin{aligned} \epsilon_S^1 &= 1, & \epsilon_L^1 &= \alpha, \\ \epsilon_S^2 &= \alpha, & \epsilon_L^2 &= 1. \end{aligned} \tag{58}$$

Therefore, expressed in the time-operator approach, the production rates by unit of time of pion pairs and triplets are respectively equal far enough from the source to $I_0 b_2 |\epsilon_L^1|^2 \Gamma_L \, \mathrm{e}^{-\Gamma_L t}$ and $I_0 b_2 |\epsilon_L^2|^2 \Gamma_L \, \mathrm{e}^{-\Gamma_L t}$. Their ratio is thus equal to $R^{\mathrm{th}} = \frac{|\epsilon_L^1|^2}{|\epsilon_L^2|^2} = |\alpha|^2 = \frac{\Gamma_L}{2\Gamma_S} = 0.9 \times 10^{-3}$. Then, using the relation between the branching ratio and CP-violation parameter described previously in this section, we have $|\epsilon^{\mathrm{th}}|^2 = R^{\mathrm{th}} \frac{\tau_1}{\tau_2} = \frac{1}{3}|\epsilon^{\exp}|^2$ which gives $|\epsilon^{\mathrm{th}}| \simeq 0.6|\epsilon^{\exp}|$. Making use of the expression for α from above, the estimated phase of the CP-violation parameter ϵ^{th} is correct (close to 45°). Finally, we see that the above discussion gives the same result as (56).

In the following section, we shall use the time-super-operator (T) formalism as a non-Wigner–Weisskopf approximation method to obtain an improvement of the CP-violation parameter.

5. Survival probability in the time-super-operator formalism

In this section, we will compute the survival probability and obtain the theoretical CP-violation parameters for the mesons K, B and D. Then, we compare our results to the experimental CP-violation parameters. We shall see that our theoretical results provide a reasonably good estimation of the experimentally measured quantities. Moreover, a fine structure appears in the case of kaons, which brought us to conceive an experimental test of the time-super-operator approach, that we shall discuss in the conclusion.

Here, we consider, as before, the pure state as a coherent superposition of two resonances denoted simply by $|\chi\rangle = |(\epsilon_1 f_1 + \epsilon_2 f_2)\rangle$, where ϵ_i, $(i = 1, 2)$, is a constant complex number including the normalization constants, i.e. $(|\epsilon_1|^2 + |\epsilon_2|^2) = 1$. We identify this state with the element $\rho = |\chi\rangle\langle\chi|$ of the Liouville space, that is, the kernel operator:

$$\rho = \sum_{i=1}^{2}\sum_{j=1}^{2} \rho_{ij}(\omega, \omega') = \sum_{i=1}^{2}\sum_{j=1}^{2} \epsilon_i \epsilon_j^* f_i(\omega)\overline{f_j(\omega')} = \sum_{i=1}^{2}\sum_{j=1}^{2} \epsilon_i \epsilon_j^* \mathfrak{F}_{ij}. \tag{59}$$

We shall compute the survival probability $\|\mathcal{P}_{-s}\rho\|^2$ of the state ρ and show how it reaches the following limit:

$$\lim_{s\to\infty} \|\mathcal{P}_{-s}\rho\|^2 \to 0. \tag{60}$$

As explained in appendix C, the Liouville operator, L, is given by equation (C.2) and the spectral representation of L after the following change of variables [16, 17]:

$$\nu = \omega - \omega', \tag{61}$$

and

$$E = \min(\omega, \omega') \tag{62}$$

is given by

$$\mathfrak{F}_{ij}(\nu, E) := f_i(\omega)\overline{f_j(\omega')} = \begin{cases} \lambda_i \lambda_j^* \frac{v(E)}{\eta_i^-(E)} \frac{v^*(E+\nu)}{\eta_j^+(E+\nu)} & \nu > 0 \\ \lambda_i^* \lambda_j \frac{v^*(E)}{\eta_j^+(E)} \frac{v(E-\nu)}{\eta_i^-(E-\nu)} & \nu < 0, \end{cases} \tag{63}$$

where $i, j = 1, 2$. Considering $v(\omega)$ as a real constant test function, we obtain $\mathfrak{F}_{ji}(\nu, E)$ in the following form:

$$\mathfrak{F}_{ji}(\nu, E) = \begin{cases} \frac{\lambda_j \lambda_i^*}{\nu_j^*(\nu+\nu_i)} & \nu > 0 \\ \frac{\lambda_j^* \lambda_i}{\nu_i(\nu_j^*-\nu)} & \nu < 0, \end{cases} \tag{64}$$

where $i, j = 1, 2$ and

$$\nu_i := a_i + \mathrm{i}b_i := (E - \widetilde{\omega}_i) + \mathrm{i}\pi|\lambda_i|^2, \qquad \left(\lambda_i = \sqrt{\frac{b_i}{\pi}}\,\mathrm{e}^{\mathrm{i}\theta_i}\right). \tag{65}$$

The spectral projection of $\mathfrak{F}_{ij}(\nu, E)$, i.e. $\mathcal{P}_s\mathfrak{F}_{ij}(\nu, E)(s < 0)$, is given in appendix C and we shall use formula (C.12). Then, the survival probability is defined as follows:

$$p_\rho(s) = \|\mathfrak{P}_\rho(s)\|^2 = \|\,|\epsilon_1|^2\mathcal{P}_s\mathfrak{F}_{11}(\nu, E) + \epsilon_1\epsilon_2^*\mathcal{P}_s\mathfrak{F}_{12}(\nu, E) + \epsilon_2\epsilon_1^*\mathcal{P}_s\mathfrak{F}_{21}(\nu, E) + |\epsilon_2|^2\mathcal{P}_s\mathfrak{F}_{22}(\nu, E)\|^2, \tag{66}$$

where $\|\cdot\|^2 = \int_0^\infty \mathrm{d}E \int_{-\infty}^\infty \mathrm{d}\nu\,|\cdot|^2$. Appendix D gives the survival probability as

$$p_\rho(s) = \mathfrak{I}_1\left(|\epsilon_1|^2\,\mathrm{e}^{2b_1s} + \left(\frac{\mathrm{i}\epsilon_1^*\epsilon_2\sqrt{b_1b_2}\,\mathrm{e}^{\mathrm{i}(\theta_2-\theta_1)}\,\mathrm{e}^{(b_1+b_2)s}\,\mathrm{e}^{-\mathrm{i}(\widetilde{\omega}_2-\widetilde{\omega}_1)s}}{(\widetilde{\omega}_2 - \widetilde{\omega}_1) + \mathrm{i}(b_1 + b_2)} + \mathrm{C.C.}\right)\right) + (\mathfrak{I}_1 + \mathfrak{I}_2)\,|\epsilon_2|^2\,\mathrm{e}^{2b_2s}, \tag{67}$$

where

$$\begin{aligned}\pi\mathfrak{I}_1 &= \int_0^{\widetilde{\omega}_1}\mathrm{d}E\left|\frac{\epsilon_1\lambda_1}{\nu_1} + \frac{\epsilon_2\lambda_2}{\nu_2}\right|^2 = |\epsilon_1|^2\arctan\frac{\widetilde{\omega}_1}{b_1} + |\epsilon_2|^2\left(\arctan\frac{\widetilde{\omega}_2-\widetilde{\omega}_1}{b_2} + \arctan\frac{\widetilde{\omega}_1}{b_2}\right)\\ &\quad -\left[\left(\frac{\epsilon_1^*\epsilon_2\sqrt{b_1b_2}}{(\widetilde{\omega}_1-\widetilde{\omega}_2)+\mathrm{i}(b_1+b_2)}\right)\left(\mathrm{i}\left(\frac{\pi}{2} + \arctan\frac{b_1}{\widetilde{\omega}_1} + \arctan\frac{b_2}{\widetilde{\omega}_2}\right.\right.\right.\\ &\quad \left.\left.\left. + \arctan\frac{b_2}{\widetilde{\omega}_2-\widetilde{\omega}_1}\right) + \frac{1}{2}\log\frac{b_1^2(\widetilde{\omega}_2^2+b_2^2)}{(\widetilde{\omega}_1^2+b_1^2)((\widetilde{\omega}_2-\widetilde{\omega}_1)^2+b_2^2)}\right) + \mathrm{C.C.}\right],\end{aligned} \tag{68}$$

and

$$\pi\mathfrak{I}_2 = \int_{\widetilde{\omega}_1}^{\widetilde{\omega}_2}\mathrm{d}E\left|\frac{\epsilon_1\lambda_1}{\nu_1}\right|^2 = |\epsilon_1|^2\arctan\frac{\widetilde{\omega}_2-\widetilde{\omega}_1}{b_1}. \tag{69}$$

5.1. K *meson*

For the weak-coupling constants, we have $b_i \ll \widetilde{\omega}_i$, $(i = 1, 2)$, and also by supposing $\widetilde{\omega}_1 \sim \widetilde{\omega}_2$, $(\widetilde{\omega}_2 - \widetilde{\omega}_1) \sim b_1$ and $\frac{b_2}{b_1} \ll 1$, we have

$$\mathfrak{I}_1 = \frac{1}{2}\left(|\epsilon_1|^2 + 2|\epsilon_2|^2 + \left(\frac{\epsilon_1^*\epsilon_2\lambda_1^*\lambda_2}{(\widetilde{\omega}_1-\widetilde{\omega}_2)+\mathrm{i}(b_1+b_2)} + \mathrm{C.C.}\right)\right) \tag{70}$$

$$\mathfrak{I}_2 = \frac{1}{4}|\epsilon_1|^2 \tag{71}$$

$$\mathfrak{I}_1 + \mathfrak{I}_2 = \frac{1}{2}\left(\frac{3}{2}|\epsilon_1|^2 + 2|\epsilon_2|^2 + \left(\frac{\epsilon_1^*\epsilon_2\lambda_1^*\lambda_2}{(\widetilde{\omega}_1-\widetilde{\omega}_2)+\mathrm{i}(b_1+b_2)} + \mathrm{C.C.}\right)\right). \tag{72}$$

We now use the normalization relation, i.e. $(|\epsilon_1|^2 + |\epsilon_2|^2) = 1$ and $\epsilon_1 \simeq 1$. Then, we obtain $\mathfrak{I}_1$ and $(\mathfrak{I}_1 + \mathfrak{I}_2)$ as follows:

$$\mathfrak{I}_1 = \tfrac{1}{2}(1 + |\epsilon_2|^2 + \cdots) \simeq \tfrac{1}{2} \tag{73}$$

$$\mathfrak{I}_1 + \mathfrak{I}_2 = \tfrac{1}{2}\left(\tfrac{3}{2} + \tfrac{1}{2}|\epsilon_2|^2 + \cdots\right) \simeq \tfrac{1}{2}\left(P\tfrac{3}{2}\right). \tag{74}$$

Replacing the above results in equation (67) and factoring by $\left(\frac{1}{2}\right)$, we obtain

$$p_\rho(s) \simeq \frac{1}{2}\left[|\epsilon_1|^2\,\mathrm{e}^{2b_1s} + \frac{3}{2}|\epsilon_2|^2\,\mathrm{e}^{2b_2s} + \left(\frac{\mathrm{i}\epsilon_1^*\epsilon_2\sqrt{b_1b_2}\,\mathrm{e}^{\mathrm{i}(\theta_2-\theta_1)}\,\mathrm{e}^{(b_1+b_2)s}\,\mathrm{e}^{-\mathrm{i}(\widetilde{\omega}_2-\widetilde{\omega}_1)s}}{(\widetilde{\omega}_2-\widetilde{\omega}_1)+\mathrm{i}(b_1+b_2)} + \mathrm{C.C.}\right)\right]. \tag{75}$$

The derivative of equation (75) yields the time-super-operator density of the probability or intensity:

$$I_1(s) := \frac{\mathrm{d}p_\rho(s)}{\mathrm{d}s} = \frac{1}{2}\Bigg[2b_1|\epsilon_1|^2\,\mathrm{e}^{2b_1 s} + 3b_2|\epsilon_2|^2\,\mathrm{e}^{2b_2 s} + \left(\frac{\mathrm{i}\epsilon_1^*\epsilon_2\sqrt{b_1b_2}\,[(b_1+b_2)-\mathrm{i}(\widetilde{\omega}_2-\widetilde{\omega}_1)]}{(\widetilde{\omega}_2-\widetilde{\omega}_1)+\mathrm{i}(b_1+b_2)}\,\mathrm{e}^{\mathrm{i}(\theta_2-\theta_1)}\,\mathrm{e}^{(b_1+b_2)s}\,\mathrm{e}^{-\mathrm{i}(\widetilde{\omega}_1-\widetilde{\omega}_2)s} + \mathrm{C.C.}\right)\Bigg]$$
$$= \frac{|\epsilon_1|^2 b_1}{2}\left[2\,\mathrm{e}^{2b_1 s} + 3\frac{|\epsilon_2|^2}{|\epsilon_1|^2}\frac{b_2}{b_1}\,\mathrm{e}^{2b_2 s} + \left(\mathrm{i}\,\frac{\epsilon_2}{\epsilon_1}\sqrt{\frac{b_2}{b_1}}\,\mathrm{e}^{(b_1+b_2)s}\,\mathrm{e}^{-\mathrm{i}(\widetilde{\omega}_1-\widetilde{\omega}_2)s} + \mathrm{C.C.}\right)\right].$$

Taking into account (58) we have $\frac{\epsilon_2}{\epsilon_1} = \alpha$, which is defined by equations (47), equation (76) becomes

$$I_1(s) = I_0\left[2\,\mathrm{e}^{2b_1 s} + 3|\alpha|^2\frac{b_2}{b_1}\,\mathrm{e}^{2b_2 s} + \left(\mathrm{i}\,\alpha\sqrt{\frac{b_2}{b_1}}\,\mathrm{e}^{(b_1+b_2)s}\,\mathrm{e}^{-\mathrm{i}(\widetilde{\omega}_1-\widetilde{\omega}_2)s} + \mathrm{C.C.}\right)\right],$$

where $I_0 = \frac{|\epsilon_1|^2 b_1^2 C_1}{2}$ and C_1 is a constant used in (57). In equation (76) for the long time enough (as occurs for kaon decays), the dominant term becomes $\left(|\epsilon^{\mathrm{th}}|^2\,\mathrm{e}^{2b_2 s}\right)$ where $|\epsilon^{\mathrm{th}}|^2 = 3|\alpha|^2\,\frac{b_2}{b_1}$. Thus, the above equation can be written as

$$I_1(s) = I_0\left(2\,\mathrm{e}^{2b_1 s} + |\epsilon^{\mathrm{th}}|^2\,\mathrm{e}^{2b_2 s} + \frac{2}{\sqrt{3}}|\epsilon^{\mathrm{th}}|\,\mathrm{e}^{(b_1+b_2)s}\,\cos\left((\widetilde{\omega}_1-\widetilde{\omega}_2)s + \arg(\epsilon^{\mathrm{th}})\right)\right).$$

We know that $I_1(s)$ is the intensity and it does not need to be normalized to 1. Finally, ϵ^{th} is given by

$$\epsilon^{\mathrm{th}} = \mathrm{i}\alpha\sqrt{\frac{3\Gamma_L}{\Gamma_S}} = \sqrt{3}\,\frac{\Gamma_L}{\Gamma_S}\,\frac{\frac{\mathrm{i}}{2}}{\frac{\triangle m}{\Gamma_S} - \mathrm{i}\,\frac{\triangle\Gamma}{2\Gamma_S}}, \tag{76}$$

where $\triangle m = (m_L - m_S)$ and $\triangle\Gamma = (\Gamma_L - \Gamma_S)$. Then, replacing the experimental data we have

$$\epsilon^{\mathrm{th}} = (2.29\times 10^{-3})\times \mathrm{e}^{\mathrm{i}(43.5)^\circ} \simeq \epsilon^{\mathrm{exp}}. \tag{77}$$

Finally, using the above time-super-operator intensity, the new ratio of pair and triplet rates is, far from the source, equal to $|\alpha'|^2 = \frac{|\epsilon_L^1|^2}{|\epsilon_L^2|^2} = \frac{3\Gamma_L}{2\Gamma_S} = 2.7\times 10^{-3}$. Repeating the discussion in the end of section 4.1, we obtain $\epsilon^{\mathrm{th}} \simeq \epsilon^{\mathrm{exp}}$ which is another proof of relation (77).

Khalfin discussed a power-law decay for K_S by the time when K_L is almost depleted. A power law in the long-time behavior of the probability decay has been computed in the one-level Friedrichs model [20]. We did not repeat here those evaluations for the two-level Friedrichs model. In appendix D, where we compute the probability decay, we only considered the exponential contributions. For kaons, the power-law decay will be studied in a separate paper.

Remark 1. Comparing the demonstration of this section's result with the time-operator formalism given in section 4, we see that we replaced α in equation (58) by $\alpha' := \sqrt{3}\alpha$ so that the time-super-operator method gives an exact *CP*-violation parameter. Consequently, effective Hamiltonian (equation (45)) must be changed because the eigenvectors $|\mathrm{K}_S\rangle$ and $|\mathrm{K}_L\rangle$ (equations (46) and (48)) are changed. This change in effective Hamiltonian means that we have to add an additional Hamiltonian term. As shown in appendix A and especially in equation (D.7), this additional Hamiltonian term depends on different masses of kaons. This correction in the *CP*-violation parameter is equal to the effect of kaon decay to $\pi^0\pi^0$ (see the computation of epsilon by Christenson *et al* [23] in section 4.1).

5.2. D *meson*

The other example is the *CP*-violation in the decay of D meson. The experimental values for *CP*-violation of $D^0 \to K_S^0 \pi^+ \pi^-$ as reported by Belle in [29] are as follows:

$$\frac{\Delta\Gamma}{2\Gamma} = \left(0.37 \pm 0.25^{+0.07+0.07}_{-0.13-0.08}\right), \tag{78}$$

$$\frac{\Delta m}{\Gamma} = \left(0.81 \pm 0.30^{+0.10+0.09}_{-0.07-0.16}\right), \tag{79}$$

where $1/\Gamma = \tau$, $(\hbar = 1)$, is the mean lifetime:

$$\frac{1}{\Gamma} = \tau = \frac{\tau_{\overline{D}^0} + \tau_{D^0}}{2} = (410.1 \pm 1.5) \times 10^{-3}\,\text{ps}. \tag{80}$$

The *CP*-violation parameters are experimentally denoted by $\left(\frac{q}{p}\right)$ and given by

$$\left|\frac{q}{p}\right|^{\text{exp}} = \left|\frac{1-\epsilon^{\text{exp}}}{1+\epsilon^{\text{exp}}}\right| = \left(0.86^{+0.30+0.06}_{-0.29-0.03}\right) \tag{81}$$

and

$$\phi^{\text{exp}} = \arg\left(\frac{q}{p}\right)^{\text{exp}} = \arg\left(\frac{1-\epsilon^{\text{exp}}}{1+\epsilon^{\text{exp}}}\right) = \left(-14^{+16+5+2}_{-18-3-4}\right)^{\circ}. \tag{82}$$

From (69), we see that here $\mathfrak{I}_2 = \frac{1}{\pi}\arctan(0.81)\,|\epsilon_1|^2 \simeq 0.43\,\left(\frac{1}{2}\right)\,|\epsilon_1|^2$; then, $\mathfrak{I}_1 + \mathfrak{I}_2 = 1.43$. Thus, we have

$$\epsilon^{\text{th}} = \sqrt{2.86}\,\frac{\Gamma_L}{\Gamma_S}\,\frac{\frac{\mathrm{i}}{2}}{\frac{\Delta m}{\Gamma_S} - \mathrm{i}\,\frac{\Delta\Gamma}{2\Gamma_S}}. \tag{83}$$

Replacing the experimental values in the above expression, we obtain

$$\epsilon^{\text{th}} = (-0.059 + 0.130\,\mathrm{i}). \tag{84}$$

Consequently,

$$\left|\frac{q}{p}\right|^{\text{th}} = 1.123, \qquad \phi^{\text{th}} = -14.758^{\circ}, \tag{85}$$

which is once again in fairly good agreement with the experimental value.

5.3. B_s^0 *meson*

The experimental values of *CP*-violation in the decay of B_s^0 and $\overline{B}_s^0$ are [27]

$$\frac{\Delta\Gamma_s}{2\Gamma_s} = 0.069^{+0.058}_{-0.062}, \qquad \frac{1}{\Gamma_s} = 1.470^{+0.026}_{-0.027}\,\text{ps}, \tag{86}$$

or equivalently $(\Gamma_{L,H} = \Gamma_s \pm \Delta\Gamma_s/2)$,

$$\frac{1}{\Gamma_L} = 1.419^{+0.039}_{-0.038}\,\text{ps}, \qquad \frac{1}{\Gamma_H} = 1.525^{+0.062}_{-0.063}\,\text{ps}, \tag{87}$$

and the difference of masses is

$$\Delta m = 17.7^{+6.4}_{-2.1}\,\text{ps}^{-1}, \qquad \frac{\Delta m}{\Gamma_s} = 26.1 \pm 0.5, \tag{88}$$

and the experimental *CP*-violation parameter of the B_s^0 meson is [27, 28]

$$\mathcal{A}_{SL}^{\text{exp}} \simeq 4\mathcal{R}e\left(\epsilon_B^{\text{exp}}\right) = (-0.4 \pm 5.6) \times 10^{-3} \Rightarrow \left|\frac{q}{p}\right|^{\text{exp}} = 1.0002 \pm 0.0051, \tag{89}$$

where $\frac{\mathcal{A}_{SL}^{\text{exp}}}{2} \approx 1 - \left|\frac{q}{p}\right|^{\text{exp}}$.

From (69) we see that here $\mathfrak{I}_2 = \frac{1}{\pi}\arctan(26.1)\,|\epsilon_1|^2 \simeq 1.57\,\left(\frac{1}{2}\right)\,|\epsilon_1|^2$; then, $\mathfrak{I}_1+\mathfrak{I}_2 = 2$. Thus, we have

$$\epsilon_B^{\rm th} = 2\,\frac{\Gamma_L}{\Gamma_H}\,\frac{\frac{\rm i}{2}}{\frac{\Delta m}{\Gamma_s} - {\rm i}\frac{\Delta\Gamma_s}{2\Gamma_s}}. \tag{90}$$

Replacing the experimental values in the above equation we obtain

$$\epsilon_B^{\rm th} = -0.1\times 10^{-3} + 0.038\,{\rm i}. \tag{91}$$

Thus, our theoretical $\left|\frac{q}{p}\right|^{\rm th}$ prediction is

$$\left|\frac{q}{p}\right|^{\rm th} = \left|\frac{1-\epsilon^{\rm th}}{1+\epsilon^{\rm th}}\right| = 1.0002 \tag{92}$$

which is in fairly good agreement with the experimental value.

5.4. B_d^0 *meson*

The experimental values of *CP*-violation in the decay of B_d^0 and $\overline{B}_d^0$ are [27]

$$\frac{\Delta\Gamma_s}{2\Gamma_s} = 0.009 \pm 0.037 \tag{93}$$

and the difference of masses is

$$\frac{\Delta m}{\Gamma_s} = 0.776 \pm 0.008, \tag{94}$$

and the experimental *CP*-violation parameter of the B_s^0 meson is [27, 28]

$$\left|\frac{q}{p}\right|^{\rm exp} = 1.0002 \pm 0.0028. \tag{95}$$

From (69) we see that here $\mathfrak{I}_2 = \frac{1}{\pi}\arctan(0.776)\,|\epsilon_1|^2 \simeq 0.42\,\left(\frac{1}{2}\right)\,|\epsilon_1|^2$; then, $\mathfrak{I}_1+\mathfrak{I}_2 = 1.42$. Thus, we have

$$\epsilon_B^{\rm th} = \sqrt{2.84}\,\frac{\Gamma_L}{\Gamma_H}\,\frac{\frac{\rm i}{2}}{\frac{\Delta m}{\Gamma_s} - {\rm i}\frac{\Delta\Gamma_s}{2\Gamma_s}}. \tag{96}$$

Replacing the experimental values in equation (90), we obtain

$$\epsilon_B^{\rm th} = -0.013 + 1.0186\,{\rm i}. \tag{97}$$

Thus, our theoretical $\left|\frac{q}{p}\right|^{\rm th}$ prediction is

$$\left|\frac{q}{p}\right|^{\rm th} = \left|\frac{1-\epsilon^{\rm th}}{1+\epsilon^{\rm th}}\right| = 1.012 \tag{98}$$

which is in fairly good agreement with the experimental value.

6. Concluding remarks

The formalism of the mass-decay matrix for the kaon decay was first introduced by LOY [7]. Then several other authors [11, 9, 25] improved this model. The LOY model requires the Wigner–Weisskopf approximation, i.e. it requires to assume that the energy interval varies from $-\infty$ to $+\infty$ and also that the coupling between discrete and continuous modes is not restricted by a factor form or a cutoff.

In [25], we used the two-level Friedrichs model and the Wigner–Weisskopf approach to obtain a mass-decay matrix. This approach was improved by using a new concept of probability decay density for mesons (see also [14]). Beyond the Wigner–Weisskopf approximation, we used the Friedrichs model with a cutoff that amounts to bound from below the energy spectrum of the Hamiltonian [30]. In this paper, we derived the decay probability density in the formalism of the time-super-operator, that also goes beyond the Wigner–Weisskopf approximation. The main difference between our model and the standard model is described as follows. The CKM matrix is aimed at describing *CP*-violation via interactions of the quarks with the Higgs field and Yukawa coupling. In this approach, quarks acquire mass through spontaneous symmetry breaking. Diagonalizing the mass matrices yields mass eigenstates by rotating quark fields with a unitary complex matrix, the CKM matrix. The CKM matrix elements describe processes at the fundamental quark level. Our method does not refer to the quark structure; it describes the decay process at the phenomenological level of mesons, using only energies and lifetimes given by experimental results. The Wigner–Weisskopf model used here (studied as a spectral mathematical problem by Friedrichs) is a phenomenological model of interaction of two discrete states encompassing only the two decay modes of mesons interacting with a continuum representing the decay products. Our complex coupling constants with the continuum λ_1 and λ_2 involve only energies and lifetimes of decaying modes of mesons. In our approach these modes are related to the two-pion and three-pion channels but our model is too crude to incorporate more precise description of the decay products. Despite the obvious oversimplifications made in our model, a comparison with the data shows that there is a room for such an approach. The reason is that, in the standard model, it is not possible to estimate exactly the experimental *CP*-violation parameters. The best that has been done so far was to show that the CKM matrix is compatible with experimental data, but *a priori* the CKM phases are free parameters. At a less fundamental level than the CKM model it is justified to make use of such models.

It would be too difficult to tackle the question of the relevance of the time-super-operator formalism in the framework of the standard model, although the probability decay can be treated thanks to the two-level Friedrich model, which is the main novel result derived in our paper.

One could ask if our model has something to do with the so-called superweak model for *CP*-violation. Roughly speaking, the prediction of the superweak model is that *CP*-violation could be explained in terms of the presence of an imaginary off-diagonal component in the Hermitian part of the mass-decay matrix expressed in the neutral kaon basis. In our case, the value of these imaginary components is exactly equal to 0 (cf equations (B.4– 6)), in agreement with the standard explanation for the *CP*-violation (in terms of direct violation) [31] and in agreement with experimental data [32] that connect the magnitude of the superweak violation to the estimation of the parameter $\epsilon'/\epsilon = (1.72 \pm 0.018).10^{-3}$. Actually, the ϵ' parameter is a free parameter in our approach because in our description we do not provide an accurate description of the decay products; for instance, we do not establish a distinction between charged pairs of pions and neutral ones, simply because in the Friedrichs model these parameters do not appear. We checked at the level of our mass-decay matrix that our model does not belong to the class of the superweak models and that $\Im(\Gamma_{12})$ is not zero, a condition given by [21] as equivalent to direct *CP*-violation.

The proper treatment of the statistical distribution of decay times is a long-standing problem that stimulated fascinating research in the past and is still a subject of fundamental interest. The time-super-operator approach is one among several possible approaches, thatwould be too long to describe here (see for instance [11, 33–36] and references therein).

Appendix A. Spectral eigenfunctions of the two-level Friedrichs model

The two-level Friedrics model Schrödinger equation with $\hbar = 1$ is formally written as

$$\begin{pmatrix} \omega_1 & 0 & \lambda_1^* v^*(\mu) \\ 0 & \omega_2 & \lambda_2^* v^*(\mu) \\ \lambda_1 v(\mu) & \lambda_2 v(\mu) & \mu \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ g(\mu) \end{pmatrix} = \omega \begin{pmatrix} f_1 \\ f_2 \\ g(\mu) \end{pmatrix}. \tag{A.1}$$

That is to say

$$\omega_1 f_1(\omega) + \lambda_1^* \int \mathrm{d}\mu v^*(\mu) g(\mu) = \omega f_1(\omega), \tag{A.2}$$

$$\omega_2 f_2(\omega) + \lambda_2^* \int \mathrm{d}\mu v^*(\mu) g(\mu) = \omega f_2(\omega), \tag{A.3}$$

and

$$\lambda_1 v(\omega) f_1(\omega) + \lambda_2 v(\omega) f_2(\omega) + \mu g(\omega) = \omega g(\omega). \tag{A.4}$$

The solution of (A.4), for 'outgoing' wave, is

$$g(\mu) = \delta(\mu - \omega) - \lim_{\epsilon \to 0} \frac{\lambda_1 v(\mu) f_1 + \lambda_2 v(\mu) f_2}{\omega - \mu - \mathrm{i}\epsilon}; \tag{A.5}$$

inserting the above equation in equation (A.2) yields

$$f_1(\omega) = \frac{\lambda_1^* v^*(\omega)}{\eta_1^+(\omega)} - \left(\lambda_1^* \lambda_2 \lim_{\epsilon \to 0} \int \mathrm{d}\mu \frac{|v(\mu)|^2}{\mu - \omega - \mathrm{i}\epsilon} \right) f_2(\omega), \tag{A.6}$$

where

$$\eta_1^+(\omega) = \omega - \omega_1 + |\lambda_1|^2 \lim_{\epsilon \to 0} \int \mathrm{d}\mu \frac{|v(\mu)|^2}{\mu - (\omega + \mathrm{i}\epsilon)}. \tag{A.7}$$

We can also obtain the similar relations for f_2 by changing the indices 1 with 2 and vice versa as

$$f_2(\omega) = \frac{\lambda_2^* v^*(\omega)}{\eta_2^+(\omega)} - \left(\lambda_1 \lambda_2^* \lim_{\epsilon \to 0} \int \mathrm{d}\mu \frac{|v(\mu)|^2}{\mu - \omega - \mathrm{i}\epsilon} \right) f_1(\omega). \tag{A.8}$$

By substituting $f_2(\omega)$ from the above equation in equation (A.6), we obtain

$$\begin{aligned} f_1(\omega) &= \frac{1}{1 - \left(\lambda_1^* \lambda_2 \int \mathrm{d}\mu \frac{|v(\mu)|^2}{\mu - \omega - \mathrm{i}0} \right)^2} \left(\frac{\lambda_1^* v^*(\omega)}{\eta_1^+(\omega)} - \frac{\lambda_1^* |\lambda_2|^2}{\eta_2^+(\omega)} \int \mathrm{d}\mu \frac{|v(\mu)|^2}{\mu - \omega - \mathrm{i}0} \right) \\ &= \frac{1}{1 - O(|\lambda|^4)} \left(\frac{\lambda_1^* v^*(\omega)}{\eta_1^+(\omega)} - O(\lambda_1^* |\lambda_2|^2) \right). \end{aligned} \tag{A.9}$$

Thus, to the order two approximation we have formulas (28) and (29). The above formulae may be obtained in different ways using [19].

Appendix B. Computation of *CPT*-invariant effective Hamiltonian

Let us now discuss the *CPT*-invariance in our model. As mentioned in the textbooks like [21, 22], the *CPT*-invariance imposes some conditions on the mass-decay matrix, i.e.

$$M_{11} = M_{22}, \qquad \Gamma_{11} = \Gamma_{22}, \qquad M_{12} = M_{21}^* \quad \text{and} \quad \Gamma_{12} = \Gamma_{21}^* \tag{B.1}$$

in the K^0 and $\overline{K}^0$ bases. But we note that our effective Hamiltonian is written in the K_1 and K_2 bases. Thus, we have to rewrite in the K^0 and $\overline{K}^0$ bases. Thus, the transformation matrix T from the K_1 and K_2 bases to the K^0 and $\overline{K}^0$ bases is obtained as

$$T = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = T^{-1}. \tag{B.2}$$

Then, the effective Hamiltonian in the K^0 and $\overline{K}^0$ bases, $H_{\text{eff}}^{0\bar{0}}$, is obtained as

$$H_{\text{eff}}^{0\bar{0}} = TH_{\text{eff}}T^{-1} = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\omega_1 - \mathrm{i}\pi|\lambda_1|^2 & -\mathrm{i}\pi\lambda_1^*\lambda_2\\ -\mathrm{i}\pi\lambda_1\lambda_2^* & \omega_2 - \mathrm{i}\pi|\lambda_2|^2\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}. \tag{B.3}$$

Replacing the corresponding experimental values for $(\lambda_1, \lambda_2, \omega_1, \omega_2)$, we have $H_{\text{eff}}^{0\bar{0}} =$

$$\begin{pmatrix}(m_S+m_L) - \frac{\mathrm{i}}{2}(\Gamma_S+\Gamma_L+2\sqrt{\Gamma_S\Gamma_L}\cos\Delta\theta), & (m_S-m_L) - \frac{\mathrm{i}}{2}(\Gamma_S-\Gamma_L+2\mathrm{i}\sqrt{\Gamma_S\Gamma_L}\sin\Delta\theta)\\ (m_S-m_L) - \frac{\mathrm{i}}{2}(\Gamma_S-\Gamma_L-2\mathrm{i}\sqrt{\Gamma_S\Gamma_L}\sin\Delta\theta), & (m_S+m_L) - \frac{\mathrm{i}}{2}(\Gamma_S+\Gamma_L-2\sqrt{\Gamma_S\Gamma_L}\cos\Delta\theta)\end{pmatrix}, \tag{B.4}$$

where $\Delta\theta = \theta_L - \theta_S$. CPT-invariance conditions in (B.1) impose that

$$\Delta\theta = k\pi + \frac{\pi}{2}, \qquad (k = \ldots, -1, 0, 1, \ldots). \tag{B.5}$$

Here, we choose $k = 0$, consequently, $\Delta\theta = \frac{\pi}{2}$. Then, we have

$$\begin{aligned} M_{11} &= M_{22} = (m_S + m_L), \quad & \Gamma_{11} &= \Gamma_{22} = \Gamma_S + \Gamma_L,\\ M_{12} &= M_{21}^* = (m_S - m_L), \quad & \Gamma_{12} &= \Gamma_{21}^* = \Gamma_S - \Gamma_L + 2\mathrm{i}\sqrt{\Gamma_S\Gamma_L}. \end{aligned} \tag{B.6}$$

Thus, the effective Hamiltonian in the K_1 and K_2 base becomes as given in equation (45).

Appendix C. Expression of the eigenprojections for time-super-operator

The expression of the time operator is given in a *spectral representation of* H, that is, in the representation in which H is diagonal. As shown in [16], H should have an unbounded absolutely continuous spectrum. In the simplest case, we shall suppose that H is represented as the multiplication operator on $\mathcal{H} = L^2(\mathbb{R}^+)$:

$$H\psi(\omega) = \omega\psi(\omega). \tag{C.1}$$

The Hilbert–Schmidt operators on $L^2(\mathbb{R}^+)$ correspond to the square-integrable functions $\rho(\omega, \omega') \in L^2(\mathbb{R}^+ \times \mathbb{R}^+)$ and the Liouville–von Neumann operator L is given by

$$L\rho(\omega, \omega') = (\omega - \omega')\rho(\omega, \omega'). \tag{C.2}$$

Then we obtain a spectral representation of L via the change of variables in (61), and (62) gives a spectral representation of L:

$$L\rho(\nu, E) = \nu\rho(\nu, E), \tag{C.3}$$

where $\rho(\nu, E) \in L^2(\mathbb{R} \times \mathbb{R}^+)$. In this representation, $T\rho(\nu, E) = \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\nu}\rho(\nu, E)$, so that the spectral representation of T is obtained by the inverse Fourier transform:

$$\hat{\rho}(\tau, E) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \mathrm{e}^{\mathrm{i}\tau\nu}\rho(\nu, E)\,\mathrm{d}\nu = (\mathcal{F}^*\rho)(\tau, E) \tag{C.4}$$

and

$$T\hat{\rho}(\tau, E) = \tau\hat{\rho}(\tau, E). \tag{C.5}$$

The spectral-projection operators $\mathcal{P}_s$ of T are given in the (τ, E)-representation by

$$\mathcal{P}_s\hat{\rho}(\tau, E) = \chi_{]-\infty,s]}(\tau)\hat{\rho}(\tau, E), \tag{C.6}$$

where $\chi_{]-\infty,s]}$ is the characteristic function of $]-\infty, s]$. So, to obtain in the (ν, E) representation the expression of these spectral-projection operators, we use the Fourier transform

$$\mathcal{P}_s\rho(\nu, E) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{s} \mathrm{e}^{-\mathrm{i}\nu\tau}\hat{\rho}(\tau, E)\,\mathrm{d}\tau \\ = \mathrm{e}^{-\mathrm{i}\nu s}\int_{-\infty}^{0} \mathrm{e}^{-\mathrm{i}\nu\tau}\hat{\rho}(\tau + s, E)\,\mathrm{d}\tau. \tag{C.7}$$

Let $g \in L^2(\mathbb{R})$ and denote its Fourier transform by $\mathcal{F}g(\nu) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}\nu\tau}g(\tau)\,\mathrm{d}\tau$. Using the Hilbert transformation:

$$\mathbf{H}g(x) = \frac{1}{\pi}\mathsf{P}\int_{-\infty}^{\infty}\frac{g(t)}{t-x}\,\mathrm{d}t. \tag{C.8}$$

We have [18] the following formula:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}\mathrm{e}^{-\mathrm{i}\nu\tau}g(\tau)\,\mathrm{d}\tau = \frac{1}{2}(\mathcal{F}(g) - \mathrm{i}\mathbf{H}\mathcal{F}(g)). \tag{C.9}$$

Finally, using the well-known property of the translated Fourier transform: $\sigma_s g(\tau) = g(\tau + s)$,

$$\mathcal{F}(\sigma_s g)(\nu) = \mathrm{e}^{\mathrm{i}\nu s}\mathcal{F}.g(\nu). \tag{C.10}$$

Equations (C.7) and (C.9) yield

$$\mathcal{P}_s\rho(\nu, E) = \tfrac{1}{2}\,\mathrm{e}^{-\mathrm{i}\nu s}[\mathrm{e}^{\mathrm{i}\nu s}\rho(\nu, E) - \mathrm{i}\mathbf{H}(\mathrm{e}^{\mathrm{i}\nu s}\rho(\nu, E))]. \tag{C.11}$$

Thus

$$\mathcal{P}_s\rho(\nu, E) = \tfrac{1}{2}\,[\rho(\nu, E) - \mathrm{i}\,\mathrm{e}^{-\mathrm{i}\nu s}\mathbf{H}(\mathrm{e}^{\mathrm{i}\nu s}\rho(\nu, E))]. \tag{C.12}$$

It is to be noted that $\mathcal{P}_s\rho(\nu, E)$ is in the Hardy class $\mathbb{H}^+$ (i.e. it is the limit as $y \to 0^+$ of an analytic function $\Phi(\nu + \mathrm{i}y)$ such that $\int_{-\infty}^{\infty} \mid \Phi(\nu + \mathrm{i}y) \mid^2\ \mathrm{d}y < \infty$) [15].

Appendix D. Computation of survival probability

In this appendix, we compute $\mathcal{P}_s\mathfrak{F}_{ij}(\nu, E)(s < 0)$ and survival probability $p_\rho(s) = \|\mathfrak{P}_\rho(s)\|$. Using formula (C.12) could give $\mathcal{P}_s\mathfrak{F}_{ij}(\nu, E)(s < 0)$. At first we compute

$$G_{ji}(\nu, E) = \mathbf{H}(\mathrm{e}^{\mathrm{i}s\nu}\mathfrak{F}_{ji})(\nu, E) = \frac{1}{\pi}\mathsf{P}\int_{-\infty}^{\infty}\frac{\mathrm{e}^{\mathrm{i}sx}\mathfrak{F}_{ji}(x, E)}{x-\nu}\,\mathrm{d}x. \tag{D.1}$$

Now, we substitute (64) into (D.1), so we have

$$G_{ji}(\nu, E) = \frac{1}{\pi}\mathsf{P}\left[\lambda_i\lambda_j^*\int_{-\infty}^{0}\frac{\mathrm{e}^{\mathrm{i}sx}}{\nu_i(x-\nu)(\nu_j^*-x)}\,\mathrm{d}x + \lambda_i^*\lambda_j\int_{0}^{+\infty}\frac{\mathrm{e}^{\mathrm{i}sx}}{\nu_j^*(x-\nu)(\nu_i+x)}\,\mathrm{d}x\right] \tag{D.2}$$

which for the $\nu > 0$ has the following form:

$$G_{ji}(\nu, E) = \frac{1}{\pi}\left[\lambda_i\lambda_j^*\int_{-\infty}^{0}\frac{\mathrm{e}^{\mathrm{i}sx}}{\nu_i(x-\nu)(\nu_j^*-x)}\,\mathrm{d}x + \lambda_i^*\lambda_j\mathsf{P}\int_{0}^{+\infty}\frac{\mathrm{e}^{\mathrm{i}sx}}{\nu_j^*(x-\nu)(\nu_i+x)}\,\mathrm{d}x\right]. \tag{D.3}$$

A complete computation of the $G_{ii}(\nu, E)$ is shown in [20]. Finally, $\mathcal{P}_s\mathfrak{F}_{ij}(\nu, E)$ is obtained as, for $i = j$,

$$\begin{aligned}\mathcal{P}_s\mathfrak{F}_{ii}(\nu, E) = {} & \mathrm{i}|\lambda_i|^2\,\mathrm{e}^{-\mathrm{i}s\nu}\left[\frac{-1}{2\pi\nu_i(\nu_i^* - \nu)}\left(\int_{-\infty}^{0}\frac{\mathrm{e}^{-sy}}{y + \mathrm{i}\nu_i^*}\mathrm{d}y - \int_{-\infty}^{0}\frac{\mathrm{e}^{-sy}}{y + \mathrm{i}\nu}\,\mathrm{d}y\right)\right.\\ & \left. + \frac{1}{2\pi\nu_i^*(\nu + \nu_i)}\left(\int_{-\infty}^{0}\frac{\mathrm{e}^{-sy}}{y - \mathrm{i}\nu_i}\,\mathrm{d}y - \int_{-\infty}^{0}\frac{\mathrm{e}^{-sy}}{y + \mathrm{i}\nu}\,\mathrm{d}y\right)\right]\\ & + \begin{cases}|\lambda_i|^2\,\mathrm{e}^{-\mathrm{i}s\nu}\left[\dfrac{\mathrm{e}^{\mathrm{i}s\nu_i^*}}{\nu_i(\nu_i^* - \nu)} - \dfrac{\mathrm{e}^{-\mathrm{i}s\nu_i}}{\nu_i^*(\nu_i + \nu)}\right], & E < \widetilde{\omega}_1\\ 0, & E > \widetilde{\omega}_1.\end{cases}\end{aligned} \tag{D.4}$$

and by considering $\widetilde{\omega}_i < \widetilde{\omega}_j$, $\mathfrak{F}_{ij}$, for $i \neq j$, have the following form:

$$\begin{aligned}\mathcal{P}_s\mathfrak{F}_{ji}(\nu, E) = {} & \mathrm{i}\,\mathrm{e}^{-\mathrm{i}s\nu}\left[\frac{-\lambda_i\lambda_j^*}{2\pi\nu_i(\nu_j^* - \nu)}\left(\int_{-\infty}^{0}\frac{\mathrm{e}^{-sy}}{y + \mathrm{i}\nu_j^*}\,\mathrm{d}y - \int_{-\infty}^{0}\frac{\mathrm{e}^{-sy}}{y + \mathrm{i}\nu}\,\mathrm{d}y\right)\right.\\ & \left. + \frac{\lambda_i^*\lambda_j}{2\pi\nu_j^*(\nu + \nu_i)}\left(\int_{-\infty}^{0}\frac{\mathrm{e}^{-sy}}{y - \mathrm{i}\nu_i}\,\mathrm{d}y - \int_{-\infty}^{0}\frac{\mathrm{e}^{-sy}}{y + \mathrm{i}\nu}\,\mathrm{d}y\right)\right]\\ & + \begin{cases}\mathrm{e}^{-\mathrm{i}s\nu}\left[\dfrac{\lambda_i\lambda_j^*\mathrm{e}^{\mathrm{i}s\nu_j^*}}{\nu_i(\nu_j^* - \nu)} - \dfrac{\lambda_i^*\lambda_j\,\mathrm{e}^{-\mathrm{i}s\nu_i}}{\nu_j^*(\nu_i + \nu)}\right], & E < \widetilde{\omega}_i\\ \lambda_i\lambda_j^*\,\mathrm{e}^{-\mathrm{i}s\nu}\,\dfrac{\mathrm{e}^{\mathrm{i}s\nu_i^*}}{\nu_i(\nu_j^* - \nu)}, & \widetilde{\omega}_i < E < \widetilde{\omega}_j\\ 0, & E > \widetilde{\omega}_j.\end{cases}\end{aligned} \tag{D.5}$$

In equations (D.4) and (D.5), the non-integral terms yield the poles and lead to the resonance, and the integral terms yield an algebraical behavior analogue to the background in the Hamiltonian theories [26]. We can also compute the same result for the case $\nu < 0$. We will neglect the background (the integrals terms). Then, the above equation is rewritten as

$$\mathcal{P}_s\mathfrak{F}_{11}(\nu, E) \simeq \begin{cases}|\lambda_1|^2\,\mathrm{e}^{-\mathrm{i}s\nu}\left[\dfrac{\mathrm{e}^{\mathrm{i}s\nu_1^*}}{\nu_1(\nu_1^* - \nu)} - \dfrac{\mathrm{e}^{-\mathrm{i}s\nu_1}}{\nu_1^*(\nu_1 + \nu)}\right], & E \leqslant \widetilde{\omega}_1\\ 0, & E > \widetilde{\omega}_1\end{cases} \tag{D.6}$$

and

$$\mathcal{P}_s\mathfrak{F}_{12}(\nu, E) \simeq \begin{cases}\mathrm{e}^{-\mathrm{i}s\nu}\left[\dfrac{\lambda_1\lambda_2^*\mathrm{e}^{\mathrm{i}s\nu_2^*}}{\nu_1(\nu_2^* - \nu)} - \dfrac{\lambda_1^*\lambda_2\,\mathrm{e}^{-\mathrm{i}s\nu_1}}{\nu_2^*(\nu_1 + \nu)}\right], & E \leqslant \widetilde{\omega}_1\\ \lambda_1\lambda_2^*\,\mathrm{e}^{-\mathrm{i}s\nu}\,\dfrac{\mathrm{e}^{\mathrm{i}s\nu_2^*}}{\nu_1(\nu_2^* - \nu)}, & \widetilde{\omega}_1 < E \leqslant \widetilde{\omega}_2\\ 0, & E > \widetilde{\omega}_2.\end{cases} \tag{D.7}$$

It is easy to see that $\mathcal{P}_s\mathfrak{F}_{21}(\nu, E) = [\mathcal{P}_s\mathfrak{F}_{12}(-\nu, E)]^*$. The second line in the above equation depends on the mass difference of the particle and anti-particle. If this difference of mass is not negligible, it adds a correction term to the *CP*-violation parameter which depends on mass difference and lifetimes.

Now, the survival probability is defined as follows:

$$\begin{aligned}p_\rho(s) = \|\mathfrak{P}_\rho(s)\|^2 = {} & \|\,|\epsilon_1|^2\mathcal{P}_s\mathfrak{F}_{11}(\nu, E) + \epsilon_1\epsilon_2^*\mathcal{P}_s\mathfrak{F}_{12}(\nu, E) + \epsilon_2\epsilon_1^*\mathcal{P}_s\mathfrak{F}_{21}(\nu, E)\\ & + |\epsilon_2|^2\mathcal{P}_s\mathfrak{F}_{22}(\nu, E)\|^2,\end{aligned} \tag{D.8}$$

where $\|\cdot\|^2 = \int_0^\infty \mathrm{d}E \int_{-\infty}^\infty \mathrm{d}\nu\, |\cdot|^2$. We see that $\mathfrak{P}_\rho(s)$ can be written as

$$\mathfrak{P}_\rho(s) \simeq \begin{cases} \mathrm{e}^{-\mathrm{i}s\nu}\Bigg[\left(\frac{\epsilon_1^*\lambda_1}{\nu_1}+\frac{\epsilon_2^*\lambda_2}{\nu_2}\right)\left(\frac{\epsilon_1\lambda_1^*\,\mathrm{e}^{\mathrm{i}s\nu_1^*}}{\nu_1^*-\nu}+\frac{\epsilon_2\lambda_2^*\,\mathrm{e}^{\mathrm{i}s\nu_2^*}}{\nu_2^*-\nu}\right) \\ \quad -\left(\frac{\epsilon_1\lambda_1}{\nu_1^*}+\frac{\epsilon_2\lambda_2}{\nu_2^*}\right)\left(\frac{\epsilon_1^*\lambda_1^*\,\mathrm{e}^{-\mathrm{i}s\nu_1}}{\nu_1+\nu}+\frac{\epsilon_2^*\lambda_2^*\,\mathrm{e}^{-\mathrm{i}s\nu_2}}{\nu_2+\nu}\right)\Bigg] & E\leqslant\widetilde{\omega}_1, \\ \mathrm{e}^{-\mathrm{i}s\nu}\left[\frac{\epsilon_1^*\lambda_1}{\nu_1}\frac{\epsilon_2\lambda_2^*\,\mathrm{e}^{\mathrm{i}s\nu_2^*}}{(\nu_2^*-\nu)}-\frac{\epsilon_1\lambda_1^*}{\nu_1^*}\frac{\epsilon_2^*\lambda_2\,\mathrm{e}^{-\mathrm{i}s\nu_2}}{(\nu_2+\nu)}\right], & \widetilde{\omega}_1<E\leqslant\widetilde{\omega}_2 \\ 0, & E>\widetilde{\omega}_2 \end{cases} \tag{D.9}$$

Now, by remembering that $b_i = |\lambda_i|^2$, $(i = 1, 2)$, the square norm of $\mathfrak{P}_\rho(s)$ is obtained as

$$|\mathfrak{P}_\rho(s)|^2 \simeq \begin{cases} \left|\frac{\epsilon_1\lambda_1}{\nu_1}+\frac{\epsilon_2\lambda_2}{\nu_2}\right|^2\Bigg[\frac{|\epsilon_1|^2|\lambda_1|^2\,\mathrm{e}^{2b_1s}}{|\nu_1^*-\nu|^2}+\frac{|\epsilon_2|^2|\lambda_2|^2\,\mathrm{e}^{2b_2s}}{|\nu_2^*-\nu|^2} \\ +\frac{|\epsilon_1|^2|\lambda_1|^2\,\mathrm{e}^{2b_1s}}{|\nu_1+\nu|^2}+\frac{|\epsilon_2|^2|\lambda_2|^2\,\mathrm{e}^{2b_2s}}{|\nu_2+\nu|^2} \\ +\mathrm{e}^{(b_1+b_2)s}\left(\frac{\epsilon_1\epsilon_2^*\lambda_1^*\lambda_2\mathrm{e}^{\mathrm{i}(a_1-a_2)s}}{(\nu_1^*-\nu)(\nu_2-\nu)}+\frac{\epsilon_1\epsilon_2^*\lambda_1^*\lambda_2\,\mathrm{e}^{\mathrm{i}(a_1-a_2)s}}{(\nu_1^*+\nu)(\nu_2+\nu)}+\mathrm{C.C.}\right)\Bigg], & E\leqslant\widetilde{\omega}_1 \\ \left|\frac{\epsilon_1\lambda_1}{\nu_1}\right|^2\left[\frac{|\epsilon_2|^2|\lambda_2|^2\,\mathrm{e}^{2b_2s}}{|\nu_2^*-\nu|^2}+\frac{|\epsilon_2|^2|\lambda_2|^2\,\mathrm{e}^{2b_2s}}{|\nu_2+\nu|^2}\right], & \widetilde{\omega}_1<E\leqslant\widetilde{\omega}_2 \\ 0, & E>\widetilde{\omega}_2 \end{cases} \tag{D.10}$$

where the terms that oscillate with a frequency equal to the difference of the two masses, i.e. $(\widetilde{\omega}_2 - \widetilde{\omega}_1)$ are kept, while the other decay terms oscillating with the frequency of one of the masses only are neglected since we have the weak coupling and the high-mass regime.

The integral over ν leads to

$$\int_{-\infty}^{\infty}\mathrm{d}\nu\,|\mathfrak{P}_\rho(s)|^2 \simeq \begin{cases} 2\pi\left|\frac{\epsilon_1\lambda_1}{\nu_1}+\frac{\epsilon_2\lambda_2}{\nu_2}\right|^2\Bigg[|\epsilon_1|^2\,\mathrm{e}^{2b_1s}+|\epsilon_2|^2\,\mathrm{e}^{2b_2s} \\ +\left(\frac{2\mathrm{i}\epsilon_1^*\epsilon_2\lambda_1^*\lambda_2\,\mathrm{e}^{(b_1+b_2)s}\,\mathrm{e}^{-\mathrm{i}(\widetilde{\omega}_1-\widetilde{\omega}_2)s}}{(\widetilde{\omega}_2-\widetilde{\omega}_1)+\mathrm{i}(b_1+b_2)}+\mathrm{C.C.}\right)\Bigg], & E\leqslant\widetilde{\omega}_1 \\ 2\pi\left|\frac{\epsilon_1\lambda_1}{\nu_1}\right|^2|\epsilon_2|^2\,\mathrm{e}^{2b_2s}, & \widetilde{\omega}_1<E\leqslant\widetilde{\omega}_2 \\ 0, & E>\widetilde{\omega}_2. \end{cases} \tag{D.11}$$

Only the terms of the square norm are dependent on E and we have

$$\left|\frac{\epsilon_1\lambda_1}{\nu_1}+\frac{\epsilon_2\lambda_2}{\nu_2}\right|^2 = \left|\frac{\epsilon_1\lambda_1}{E-\widetilde{\omega}_1+\mathrm{i}b_1}\right|^2+\left|\frac{\epsilon_2\lambda_2}{E-\widetilde{\omega}_2+\mathrm{i}b_2}\right|^2 + \left(\frac{\epsilon_1^*\lambda_1^*}{(E-\widetilde{\omega}_1+\mathrm{i}b_1)}\frac{\epsilon_2\lambda_2}{(E-\widetilde{\omega}_2-\mathrm{i}b_2)}+\mathrm{C.C.}\right). \tag{D.12}$$

The integral over E of the above expression is like the following integrals:

$$\int \mathrm{d}E\left|\frac{\sqrt{b_i}}{(E-\widetilde{\omega}_i)+\mathrm{i}b_i}\right|^2 = \arctan\left(\frac{E-\widetilde{\omega}_i}{b_i}\right) \tag{D.13}$$

and

$$\int \mathrm{d}E \frac{\lambda_1^*\lambda_2}{(x-a_1+\mathrm{i}b_1)(E-a_2-\mathrm{i}b_2)} = \frac{-\lambda_1^*\lambda_2}{(\widetilde{\omega}_2-\widetilde{\omega}_1)+\mathrm{i}(b_1+b_2)}\left(\mathrm{i}\arctan\frac{b_1}{E-\widetilde{\omega}_1}\right.$$
$$\left.+\,\mathrm{i}\arctan\frac{b_2}{E-\widetilde{\omega}_2}+\log\sqrt{(E-\widetilde{\omega}_1)^2+b_1^2}-\log\sqrt{(E-\widetilde{\omega}_2)^2+b_2^2}\right). \quad \text{(D.14)}$$

Now, we integrate equation (D.12) over E from 0 to ∞ in order to obtain (67). Firstly, for the interval $E \in [0, \widetilde{\omega}_1]$, we have $\mathfrak{I}_1$ (equation (68)) and for $E \in]\widetilde{\omega}_1, \widetilde{\omega}_2]$ we have $\mathfrak{I}_2$ (equation (69)).

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