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# Two-level Friedrichs model and kaonic phenomenology

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**Abstract**

In the present Letter, we study in the framework of the Friedrichs model the evolution of a two-level system coupled to a continuum. This unitary evolution possesses a non-unitary component with a non-Hermitian effective Hamiltonian. We show that this model is well adapted in order to describe kaon phenomenology (oscillation, regeneration) and leads to a $CP$ violation, although in this case the prediction is not quantitatively quite satisfying.

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## 1. Introduction

We shall show that the two-level Friedrichs system [1] makes it possible to describe a class of systems that exhibit rich and complex behaviors: oscillations, regenerations and so on, and provides a relatively exact phenomenological model of kaons physics. There have been several approaches to $CP$-violations in kaons using gauge theory [2] or renormalization theory [3]. We do not consider these aspects here, also because the question is still partially open today. Our treatment is based on the description of decaying systems similarly to the generalization of the Weisskopf–Wigner approach, formulated by Lee, Oehme and Yang (LOY) [4] in the case of kaonic decay. Later on, Chiu and Sudarshan [5] used a Lee model in order to obtain a correction to the LOY theory for short times (Zeno effect). Our new approach is based on the derivation of a Master equation from a Hamiltonian description for $K_1$ and $K_2$ decaying modes weakly coupled to the decay product and not for $(K^0, \bar{K}^0)$ modes as done in LOY theory. In this Letter, we use a simple version of the model with a constant factor form. This leads to a Markovian master equation which allows us to simulate the kaonic lifetimes as well as kaonic oscillations and regeneration. It even predicts a $CP$ symmetry breaking. Unfortunately this last prediction is not very accurate quantitatively, which, in a sense, is not astonishing for such a simplified approach. In any case, our computations show that it is possible with a very simple model such as the two-level Friedrichs model to capture essential features of the very rich kaon phenomenology, and of their non-trivial temporal survival distributions.

In Section 2, we recall the main features of kaon phenomenology. In the third section we show how to simulate them thanks to the Friedrichs model. We show that the fit with phenomenological data about $CP$-violation is satisfying since we recover the experimental data for the phase, but not quantitatively (our estimation of the modulus effect is fourteen times too strong in comparison to experimental data). At the end of the Letter, we make some remarks on an improvement of the model.

## 2. Main features of kaon phenomenology

Kaons are bosons that were discovered in the forties during the study of cosmic rays. They are produced by collision processes in nuclear reactions during which the strong interac-

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tions dominate. They appear in pairs $K^0$, $\bar{K}^0$ [6,7]. It is possible to produce preferentially the $K^0$ particle essentially due to the fact that the $\bar{K}^0$ kaon is less probable kinematically and that the threshold pion energy for its production is higher.

The K mesons are eigenstates of the parity operator $P$: $P|K^0\rangle = -|K^0\rangle$, and $P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$. $K^0$ and $\bar{K}^0$ are charge conjugate to each other $C|K^0\rangle = |\bar{K}^0\rangle$, and $C|\bar{K}^0\rangle = |K^0\rangle$. We get thus

$$CP|K^0\rangle = -|\bar{K}^0\rangle, \qquad CP|\bar{K}^0\rangle = -|K^0\rangle. \tag{2.1}$$

Clearly $|K^0\rangle$ and $|\bar{K}^0\rangle$ are not $CP$-eigenstates, but the following combinations

$$|K_1\rangle = \frac{1}{\sqrt{2}}\big(|K^0\rangle + |\bar{K}^0\rangle\big), \qquad |K_2\rangle = \frac{1}{\sqrt{2}}\big(|K^0\rangle - |\bar{K}^0\rangle\big), \tag{2.2}$$

are $CP$-eigenstates.

$$CP|K_1\rangle = -|K_1\rangle, \qquad CP|K_2\rangle = +|K_2\rangle. \tag{2.3}$$

In the absence of matter, kaons disintegrate through weak interactions [7]. Actually, $K^0$ and $\bar{K}^0$ are distinguished by their mode of *production*, $K_1$ and $K_2$ are distinguished by their mode of *decay*. In first approximation we can neglect $CP$-violation so that the weak Hamiltonian commutes with $CP$. In this regime, the weak disintegration process distinguishes the $K_1$ states which decay only into "$2\pi$" while the $K_2$ states decay into "$3\pi, \pi e\nu, \ldots$" [8]. The lifetime of the $K_1$ kaon is short ($\tau_S \approx 8.92 \times 10^{-11}$ s), while the lifetime of the $K_2$ kaon is quite longer ($\tau_L \approx 5.17 \times 10^{-8}$ s). The difference of mass of the 1 and 2 kaons is quite small in comparison to their mass ($\frac{m_L - m_S}{m_S + m_L} \approx 0.35 \times 10^{-14}$, with $(m_L - m_S)c^2 \approx 3.52 \times 10^{-6}$ eV). The amplitude of the mode $K_1$ at time $t$ can be written as

$$a_1(t) = a_1(0)e^{-\frac{iE_S}{\hbar}t}e^{-\frac{\Gamma_S}{2\hbar}t}, \tag{2.4}$$

where $E_S$ is the total energy of particle and $\Gamma_S = \frac{\hbar}{\tau_S}$ is the width of the state. We can write the amplitude of the mode $K_2$ in a similar fashion for the long lifetime. The intensity is

$$\begin{aligned} I_1(t) &= a_1(t)a_1^*(t) = a_1(0)a_1^*(0)e^{-\frac{\Gamma_S}{\hbar}t} \\ &= I_1(0)e^{-\frac{t}{\tau_S}}. \end{aligned} \tag{2.5}$$

Setting $\hbar = c = 1$ and considering a situation during which kaons are at rest we get that $\tau_S$ is the proper lifetime and $E_S = m_S$, the rest mass of the $K_1$ particle. Its amplitude is then

$$a_1(t) = a_1(0)e^{-(\mathrm{i}m_S + \frac{\Gamma_S}{2})t}. \tag{2.6}$$

Similarly, for $K_2$,

$$a_2(t) = a_2(0)e^{-(\mathrm{i}m_L + \frac{\Gamma_L}{2})t}. \tag{2.7}$$

From Eq. (2.2) we can write [6] the corresponding amplitudes of $K^0$ and $\bar{K}^0$ as

$$a_0(t) = \frac{1}{\sqrt{2}}\big(a_1(t) + a_2(t)\big), \qquad \bar{a}_0(t) = \frac{1}{\sqrt{2}}\big(a_1(t) - a_2(t)\big) \tag{2.8}$$

and the intensities are equal to

$$I_0(t) = \frac{I_0(0)}{4}\big(e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos(\Delta m t)\big) \tag{2.9}$$

and

$$\bar{I}_0(t) = \frac{\bar{I}_0(0)}{4}\big(e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos(\Delta m t)\big). \tag{2.10}$$

Here $\Delta m = |m_L - m_S| = 3.52 \times 10^{-6}$ and $\Delta m \tau_S \approx 0.47$, so that $K^0$- and $\bar{K}^0$-intensities *oscillate* with the frequency $|\Delta m|$.

This corresponds to the process called kaonic *oscillation*. We can explain it intuitively as follows: in the vacuum the disintegration of kaons is due to weak interactions, and the weak Hamiltonian controls and dominates the evolution. Therefore, the eigenstates of the "free" (weak) Hamiltonian in vacuum are (in first approximation) the $K_1$ and $K_2$ kaons. In the presence of matter, strong interactions are present during the collisions between kaons and nuclei. They dominate the decay process and therefore $K^0$ and $\bar{K}^0$ kaons are observed, and it is also possible to distinguish them experimentally because they possess different disintegration channels. Because the preparation and measurement bases differ from the eigenbasis of the Hamiltonian that controls the free evolution, interference effects are likely to occur. This is the essence of kaonic oscillations. What is interesting is that if we compare their difference of mass (in convenient units) to the inverse of the lifetime of the $K_1$ kaon, we get a comparable result: $(m_L - m_S)\tau_S \approx 0.47$. Thanks to this relation and due to the fact that it was possible experimentally to carry out observations occurring during a time comparable to the lifetime of the $K_1$ kaon, which is relatively long in comparison to other elementary particles, it was possible to observe kaonic oscillations experimentally.

*Generation* and *regeneration* are similar phenomena. If we produce (in matter, in the strong regime) $K^0$ particles, no $\bar{K}^0$ particle is present, but if we wait (in absence of matter) during a time long relatively to $\tau_S$ the lifetime of the $K_1$ kaon, the $K_2$ particle only has survived and the probability to find a $\bar{K}^0$ particle is 0.5, so that $\bar{K}^0$ particles were *generated*.

*Regeneration* is due to the fact that in the presence of matter, the $\bar{K}^0$ particle disintegrates more quickly than the $K^0$ one. Henceforth their respective amplitudes are not equal in modulus with as a consequence that $a_1(t) = \frac{1}{\sqrt{2}}(a_0(t) + \bar{a}_0(t))$ differs from zero. Consequently, even if we wait (in the absence of matter, in the weak regime) a time longer than the lifetime of the $K_1$ kaon, and that only the $K_2$ particle is present, the $K_1$ component is re-generated in the presence of matter.

*$CP$-violation* is another interesting feature of the kaons phenomenology. It was discovered by Christenson et al. [9]. $CP$-violation means that the long-lived kaon can also decay to "$2\pi$" then, the $CP$ symmetry is slightly violated (by a factor of $10^{-3}$) by weak interactions so that the $CP$ eigenstates $K_1$ and $K_2$ are not exact eigenstates of the decay interaction. Let us consider that $K_S$ ($S$ = short-lived) and $K_L$ ($L$ = long-lived) are the eigenstates of the decay interaction; they can be expressed as a superpositions of the $K_1$ and $K_2$ eigenstates. Then

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\big[\epsilon|K_1\rangle + |K_2\rangle\big] = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\big[(1+\epsilon)\big|K^0\big\rangle - (1-\epsilon)\big|\bar{K}^0\big\rangle\big], \tag{2.11}$$

and

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\big[|K_1\rangle + \epsilon|K_2\rangle\big] = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\big[(1+\epsilon)\big|K^0\big\rangle + (1-\epsilon)\big|\bar{K}^0\big\rangle\big], \tag{2.12}$$

where $\epsilon$ is a $CP$-violation parameter, $|\epsilon| \ll 1$ where $\epsilon$ does not have to be real. $K_L$ and $K_S$ are the eigenstates of the Hamiltonian for the mass-decay matrix [7,8], i.e.

$$H = M - \frac{i}{2}\Gamma \equiv \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}, \tag{2.13}$$

where $M$ and $\Gamma$ are individually hermitian since they correspond to observable (mass and lifetime). The corresponding eigenvalues of the mass-decay matrix are equal to

$$m_L - \frac{i}{2}\Gamma_L, \qquad m_S - \frac{i}{2}\Gamma_S. \tag{2.14}$$

The $CP$-violation was established by the observation that $K_L$ decays not only via three-pion, which has natural $CP$ parity, but also via the two-pion mode with a $|\epsilon|$ of order $10^{-3}$, which is truly unexpected. The experimental value of $\epsilon$ is

$$|\epsilon| = (2.27 \pm 0.02) \times 10^{-3}, \qquad \arg(\epsilon) = 43.37 \tag{2.15}$$

## 3. Friedrichs's model and kaon phenomenology

### *3.1. The two-levels Friedrichs model*

The Friedrichs interaction Hamiltonian between the two modes and the continuous degree of freedom is the following [1,10–12]:

$$H_{\text{Friedrichs}} = \begin{pmatrix} \omega_1 & 0 & \lambda_1 \\ 0 & \omega_2 & \lambda_2 \\ \lambda_1 & \lambda_2 & \omega \end{pmatrix}. \tag{3.1}$$

The masses $\omega_{1,2}$ represent the energies of the discrete levels, and the factors $\lambda_{1,2}$ represent the couplings to the continuum of decay product. In this model, the energies $\omega$ of the different modes of the continuum range from $-\infty$ to $+\infty$. The two-level Friedrichs model Schrödinger equation is

$$\begin{pmatrix} \omega_1 & 0 & \lambda_1 \\ 0 & \omega_2 & \lambda_2 \\ \lambda_1 & \lambda_2 & \omega \end{pmatrix}\begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\omega,t) \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\omega,t) \end{pmatrix} \tag{3.2}$$

which means:

$$\omega_1 f_1(t) + \lambda_1 \int_{-\infty}^{\infty} d\omega\, g(\omega,t) = i\frac{\partial f_1(t)}{\partial t}, \tag{3.3}$$

$$\omega_2 f_2(t) + \lambda_2 \int_{-\infty}^{\infty} d\omega\, g(\omega,t) = i\frac{\partial f_2(t)}{\partial t}, \tag{3.4}$$

and

$$\lambda_1 f_1(t) + \lambda_2 f_2(t) + \omega g(\omega,t) = i\frac{\partial g(\omega,t)}{\partial t}. \tag{3.5}$$

$\omega$ is coupled here through uniform factor forms $(\lambda_1, \lambda_2)$; this constitutes a very rough approximation which allows an integration of the equation of motion and an illustration of the application to $CP$-violation in kaons. More physical cutoffs that can improve our estimation will be studied in a future publication, as well as the analogy between our model and models used in quantum optics in order to simulate certain spontaneous radiative processes. Let us now solve the Schrödinger equation and trace out the continuum in order to derive the master equation for the two-level system. From Eq. (3.5) we can obtain $g(\omega,t)$, taking $g(\omega,0) = 0$, as

$$g(\omega,t) = -ie^{-i\omega t}\int_0^t d\tau\,\big[\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)\big]e^{i\omega\tau}, \tag{3.6}$$

where $t > 0$. Then, we substitute $g(\omega,t)$ in Eq. (3.3) we obtain

$$i\frac{\partial f_1(t)}{\partial t} = \omega_1 f_1(t) - i\lambda_1 \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \times \int_0^t d\tau\,\big[\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)\big]e^{i\omega\tau}, \tag{3.7}$$

we also obtain the same relation for $f_2(t)$ from Eq. (3.4):

$$i\frac{\partial f_2(t)}{\partial t} = \omega_2 f_2(t) - i\lambda_2 \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \times \int_0^t d\tau\,\big[\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)\big]e^{i\omega\tau}. \tag{3.8}$$

### *3.2. The two-levels Friedrichs model and kaonic behavior*

In this subsection, we shall make use of the Friedrichs model in order to simulate interesting properties of the kaonic systems. In order to do so, we shall identify the discrete modes of the Friedrichs model with the $K_1$ and $K_2$ states and $\omega_1$ and $\omega_2$ with their masses, respectively. This is our basic postulate according to which we can now make use of the Friedrichs model in order to establish a phenomenology for the kaonic behavior. More precisely, we shall assume that

$$|K_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |K_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{3.9}$$

The continuum mode aims at representing the decay products as explained in Section 2. Let us consider the solution of two-level Friedrichs model Schrödinger Eq. (3.2). According to this equation, the state is at time $t$ superposition of two components that correspond to the two (complex) eigenvalues of the effective Hamiltonian. In order to avoid confusion, we shall use different parameters when we deal with the "real" kaons that are associated with experimental data and when we deal with the "theoretic" ones in the framework of the Friedrichs model.

It is worth noting that the use of simple two-level models to explain kaon oscillations goes back to Gell-Mann and Pais (see Feynman lectures vol. III, pp. 11–16). What is new in our Letter is that we introduce a continuous degree of freedom (a scalar field) in a simple and exactly solvable model to describe kaon decay, in which $|K_1\rangle$ and $|K_2\rangle$ particles communicate via the decay channel (and not $|K^0\rangle$ and $|\bar{K}^0\rangle$ as in the LOY theory).

- The masses $m_S$ and $m_L$ and the lifetimes $\tau_S$ and $\tau_L$ will remain attributed to the real objects.
- The parameters $\omega_1$, $\omega_2$, $\lambda_1$, $\lambda_2$, $\omega_+$ and $\omega_-$ will refer to the theoretic quantities.

To solve Eqs. (3.7) and (3.8) we shall compute the integral part of Eq. (3.7) $f_1$ and $f_2$ being supposed integrable functions on $[0,\infty[$. We consider a test function as $e^{-\alpha^2\omega^2}$, then we can rewrite it as follows

$$\int_0^t d\tau\,\big(\lambda_1 f_1(\tau)+\lambda_2 f_2(\tau)\big)\int_{-\infty}^{\infty} d\omega\, e^{-\mathrm{i}\omega(t-\tau)}e^{-\alpha^2\omega^2}, \tag{3.10}$$

with the limit $\alpha \to 0$. After integration on $\omega$ of in the above equation we obtain

$$\frac{\sqrt{\pi}}{\alpha}\int_0^t d\tau\,\big[\lambda_1 f_1(\tau)+\lambda_2 f_2(\tau)\big]e^{-\frac{(t-\tau)^2}{4\alpha^2}} \tag{3.11}$$

or

$$\frac{\sqrt{\pi}}{\alpha}\big[\lambda_1 f_1(t)+\lambda_2 f_2(t)\big]*e^{-\frac{t^2}{4\alpha^2}} \tag{3.12}$$

where we used the convolution definition, i.e.

$$\int_0^t k(t-u)y(u)\,du = k(t)*y(t). \tag{3.13}$$

Laplace transformation of Eq. (3.12) yields

$$\pi\big[\lambda_1 F_1(s)+\lambda_2 F_2(s)\big]e^{\alpha^2 s^2}\,\mathrm{Erfc}(\alpha s), \tag{3.14}$$

where

$$\mathrm{Erfc}(x) = 1-\mathrm{Erf}(x) = 1-\frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}\,dy. \tag{3.15}$$

Taking the limit $\alpha \to 0$ we obtain

$$\pi\big[\lambda_1 F_1(s)+\lambda_2 F_2(s)\big], \tag{3.16}$$

and taking and the inverse Laplace transformation yields $\pi[\lambda_1 f_1(t)+\lambda_2 f_2(t)]$. So we proved

$$\int_{-\infty}^{\infty} d\omega\, e^{-\mathrm{i}\omega t}\int_0^t d\tau\,\big[\lambda_1 f_1(\tau)+\lambda_2 f_2(\tau)\big]e^{\mathrm{i}\omega\tau} = \pi\big[\lambda_1 f_1(t)+\lambda_2 f_2(t)\big]. \tag{3.17}$$

Now, we substitute the above result in Eqs. (3.7) and (3.8). Thus, we obtain

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} f_1(t)\\ f_2(t)\end{pmatrix} = \begin{pmatrix}\omega_1-\mathrm{i}\pi\lambda_1^2 & -\mathrm{i}\pi\lambda_1\lambda_2\\ -\mathrm{i}\pi\lambda_1\lambda_2 & \omega_2-\mathrm{i}\pi\lambda_2^2\end{pmatrix}\begin{pmatrix} f_1(t)\\ f_2(t)\end{pmatrix}. \tag{3.18}$$

Thus, we obtain an effective non-Hermitian Hamiltonian evolution, $H_{\mathrm{eff}} = M - \mathrm{i}\frac{\Gamma}{2}$. The eigenvalues of the system are

$$\begin{aligned}\omega_\pm = \frac{1}{2}\Big\{&(\omega_1+\omega_2)-\mathrm{i}\pi\big(\lambda_1^2+\lambda_2^2\big)\\ &\pm\Big[\big((\omega_1+\omega_2)-\mathrm{i}\pi\big(\lambda_1^2+\lambda_2^2\big)\big)^2\\ &-4\big(\omega_1\omega_2-\mathrm{i}\pi\big(\lambda_1^2\omega_2+\lambda_2^2\omega_1\big)\big)\Big]^{1/2}\Big\},\end{aligned} \tag{3.19}$$

and under the weak coupling constant approximation, they become

$$\begin{aligned}\omega_+ &= \omega_1-\mathrm{i}\pi\lambda_1^2+O\big(\lambda^4\big),\\ \omega_- &= \omega_2-\mathrm{i}\pi\lambda_2^2+O\big(\lambda^4\big).\end{aligned} \tag{3.20}$$

In a first and very rough approximation, the eigenvectors of the effective Hamiltonian are the same as the postulated kaons states.

$$|f_+\rangle = \begin{pmatrix}1\\0\end{pmatrix} = |K_1\rangle \quad\text{and}\quad |f_-\rangle = \begin{pmatrix}0\\1\end{pmatrix} = |K_2\rangle, \tag{3.21}$$

and the solutions of Schrödinger equations are superpositions of these two states with amplitudes

$$f_1(t) = e^{-\mathrm{i}\omega_+ t},\qquad f_2(t) = e^{-\mathrm{i}\omega_- t}. \tag{3.22}$$

Phenomenology imposes that the complex Friedrichs energies $\omega_\pm$ coincide with the observed complex energies. The Friedrichs energies depend on the choice of the four parameters $\omega_1$, $\omega_2$, $\lambda_1$ and $\lambda_2$ and the observed complex energies are directly derived from the experimental determination of four other parameters, the masses $m_S$ and $m_L$ and the lifetimes $\tau_S$ and $\tau_L$. We must thus adjust the theoretical parameters in order that they fit the experimental data. This can be done by comparing the normalized intensities $\frac{4I_0(t)}{I_0(0)}$ and $\frac{4\bar{I}_0(t)}{\bar{I}_0(0)}$ of Eqs. (2.9) and (2.10) with the theoretical prediction for the $K^0$ and $\bar{K}^0$ intensities:

$$\big|f_1(t)\pm f_2(t)\big|^2 = \big(e^{-2\pi\lambda_1^2 t}+e^{-2\pi\lambda_2^2 t}\pm 2e^{-\pi(\lambda_1^2+\lambda_2^2)t}\cos(\Delta\omega t)\big), \tag{3.23}$$

where $\Delta\omega = |\omega_1-\omega_2|$. From this comparison of experimental and theoretical results we obtain (see Eqs. (2.9) and (2.10)).

$$\begin{aligned}\omega_1 &= m_S, & 2\pi\lambda_1^2 &= \Gamma_S,\\ \omega_2 &= m_L, & 2\pi\lambda_2^2 &= \Gamma_L.\end{aligned} \tag{3.24}$$

*CP-violation*: Let us study in this case the $CP$-violation. The Friedrichs model allows us to estimate the value of $\epsilon$. For this purpose, the effective Hamiltonian (Eq. (3.18)) acts on the $|K_S\rangle$

vector states (Eq. (2.11)) as an eigenstate corresponding to the eigenvalue $\omega_+ = \omega_1 - i\pi\lambda_1^2 = m_S - i\frac{\Gamma_S}{2}$, so that we must impose that $H_{\text{eff}}\binom{1}{\epsilon} = \omega_-\binom{1}{\epsilon}$, from which we obtain after straightforward calculations that

$$\epsilon = \frac{i\pi\lambda_1\lambda_2}{(\omega_2-\omega_1) - i\pi(\lambda_2^2-\lambda_1^2)} \tag{3.25}$$

and if we replace $\lambda$'s and $\omega$'s by corresponding values in Eq. (3.24) we have

$$\epsilon = \frac{\frac{i}{2}\sqrt{\Gamma_L\Gamma_S}}{(m_L-m_S) - \frac{i}{2}(\Gamma_L-\Gamma_S)}. \tag{3.26}$$

By using the above experimental values of $\Gamma_L$, $\Gamma_S$, $m_L$, $m_S$ and the ratio $\frac{(m_L-m_S)}{-(\Gamma_L-\Gamma_S)} \approx \Delta m\tau_S \approx 0.47$ we obtain the following estimated value for $\epsilon$:

$$\epsilon = \sqrt{(1.82\times 10^{-3})/2}e^{i(43.37)^\circ} \tag{3.27}$$

which shows that our estimation of the modulus of $\epsilon$ is $\sim 14$ times greater than its experimental modulus value while the estimated phase is correct.

Although this last prediction is not very accurate quantitatively, which in a sense is not astonishing for such a simplified approach, we think that a better fit is possible provided we finely tune the cutoff between the discrete levels and the continuous modes, especially when negative energy modes are decoupled from the two-level system. In any case, our computations show that it is possible with a very simple model such as the two level Friedrichs model to capture essential features of the very rich kaon phenomenology, and of their non-trivial temporal survival distributions.

## 4. Conclusions

We have shown that the framework of the Friedrichs model is relevant in order to grasp, despite of its simplicity, essential features of kaons decay. This model allows us to describe complex temporal evolutions (such as kaonic oscillations, generation and regeneration) and to simulate at least qualitatively $CP$-violation. We also recover the experimental value of the phase, 43.37°, as a result of Eqs. (3.26)–(3.27).

We have to notice that ever since LOY paper, new problems and effects have been studied in $CP$-violation among then we notice the paper of Khalfin [16] which could not be considered in the scope of this Letter.

It is also out of the scope of the present Letter but it would be very interesting to study the properties of the Friedrichs model and of kaonic oscillations in terms of the time operator approach. This can be done for the one-level Friedrichs model [13,14] but higher level systems present more subtle and involved temporal behavior [15] so that it is worth studying the time-operator in this context.

One should consider situations during which the spectrum of the continuous degree of freedom is cutoff, because an unbounded spectrum in energy is not very sound from a physical point of view. This will be studied in a future publication as well as the analogy between our model and models used in quantum optics in order to simulate certain spontaneous radiative processes.

Finally, it is worth comparing our results with those obtained in Refs. [17,18] where it is shown that in the framework of the rigged Hilbert space approach another effect of $CP$-violation is also predicted despite of the fact that the Hamiltonian respects the $CP$ symmetry. In our case the Friedrichs Hamiltonian does not respect this symmetry, that is, if we consider all the degrees of freedom of the system, the commutator between the Hamiltonian and the $CP$ operator is different from zero. The commutator is equal to zero when we consider the free Hamiltonian (corresponding to $\lambda_1 = \lambda_2 = 0$). As we noted before, this result can be understood as follows: in our approach, the continuous degree of freedom mediates an effective weak-like interaction of order two in the coupling constants, which "explains" why the $CP$-violation is small.

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