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Cotton Yarn Engineering Via Fuzzy Least Squares Regression

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Abstract: Modeling of yarn and fiber properties has been a popular topic in the field of textile engineering in recent decades. The common method for fitting models has been to use classical regression analysis, based on the assumptions of data crispness and deterministic relations among variables. However, in modeling practical systems such as cotton spinning, the above assumptions may not hold true. Prediction is influential and we should therefore attempt to analyze the behavior and structure of such systems more realistically. In the present research, we investigate a procedure to provide a soft regression method for modeling the relationships between fiber properties, roving properties, and yarn count as independent variables and yarn properties as dependent (response) variable. We first selected the effective variables by multivariate test (mtest) and then considered fuzzy least squares regression for evaluating relationship between cotton yarn properties such as tensile, hairiness, unevenness and fiber properties that were measured by HVI system. We also used mean of capability index (MCI) to evaluate the goodness of fit of the fuzzy regression models. The results showed that the equations were significant at very good MCI levels.

Keywords: Multivariate test, Fuzzy least squares regression, Mean of capability index (MCI), Cotton yarns, Ring spinning, Yarn quality properties

Introduction

The main purpose of many textile studies in the past century was to predict the yarn's important characteristic such as tensile, unevenness, and hairiness of yarn from fiber properties. Two main approaches used in these studies are statistical and mathematical approaches. One of the most common statistical approaches is the multiple regression method. Many researchers used linear multiple regression method for the estimation of yarn quality characteristics [1-5].

In spite of classical regression models in many fields, however, in practical studies the following problems arise in using statistical regression modeling: 1) low sample size, 2) imprecise observation, 3) vagueness in the relationships between variables (which do not follow the random error patterns). For instance, in quality studies of fine and expensive fabrics, the number of data available may be few. Also in Nano-fiber electro spinning, there are very low productions and complex conditions. Such situations, in which the data is few or relationships between variables are not precise (have been reported in literature [6,7]). We need, therefore, to investigate some alternative soft procedures to deal with the above situations. Fuzzy set theory provides appropriate alternative procedures for modeling the variables of interest when only few data are available and/or the data are reported as imprecise quantities and/or the relationship between variables is defined vaguely.

Over the past decades, several approaches to fuzzy regression have been developed by authors. From a general perspective, there are two main approaches to regression modeling using the fuzzy set theory. The first one, which is introduced by Tanaka *et al*. [8] is called possibilistic regression. In possibilistic approach, the coefficients of the model are assumed to be fuzzy numbers. This approach is essentially based on transforming the problem of fitting a fuzzy model to a linear (non-linear) programming problem. This approach has been developed by some authors [9-11].

Another approach to fuzzy regression is the fuzzy least squares approach which is an extension of the ordinary least squares regression and is proposed by Celmins and Diamond [12,13]. In fuzzy least squares approach, the optimal model is derived based on a distance between the observed (fuzzy) values and the estimated fuzzy values of the response variable. This approach is also investigated by some authors [13-16]. A review of the literature on the topic is provided in Taheri [17].

In spite of several works in theory and applications of the fuzzy regression models, as far as the authors know, there have been a few works in applications of fuzzy regression analysis in textile researches. In this regard, we could indicate to a work by Tavanai *et al.* [18]. They investigated and applied fuzzy possibilistic regression for modeling the colour yield in polyester high temperature dying as a function of disperse dyes concentration, temperature, and time.

In the present work, we investigate a fuzzy regression method for modeling the relationships between fiber properties as independent variables and yarn properties as dependent variables. In fact, after selecting effective variables by using the common statistical methods, a fuzzy least squares regression was used to predict the cotton yarn's important

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properties from fiber properties. In this work, observations of the dependent (response) variables are considered as non-precise (fuzzy) data, and the final model is considered as a fuzzy model. This approach is based on the notion of distance between the predicted fuzzy outputs and the observed fuzzy inputs. We used a goodness of fit index to reliability analysis in obtained fuzzy models. The mean of capability index (*MCI*) for the obtained models are very good, indicating that predictive ability of the models is satisfactory. This is notable that the proposed approach can be extended to other cases in textile engineering where the data are non-precise (fuzzy) or the number of data is few.

Background

In this section, we review some basic concepts in multivariate statistical regression and the related statistical tests, as well as some preliminaries from fuzzy arithmetic. In addition, a method of fuzzy least squares regression, for crisp input-fuzzy output data, is investigated.

Multivariate Tests

The multivariate testes are used to test hypotheses in multivariate regression models, where there are several dependent variables to fit to the same regressors. The hypotheses that can be tested, are of the form

$$(L\beta - cj)M = 0$$

where L is a linear function on the regressor side, β is a matrix of parameters, c is a column vector of constant, j is a row vector of ones, and M is a linear function on dependent side. The special case where the constants are zero is $L\beta M=0$.

To test these hypotheses, two matrices called H and E are constructed that correspond to the numerator and denominator of a univariate F test:

$$H = M'(L\beta - cj)'(L(X'X)^{-}L')^{-1}(L\beta - cj)M$$

$$E = M'(YY' - \beta'(X'X)\beta)M$$

Four tests statistics based on the eigenvalues of $E^{-1}H$ or $(E+H)^{-1}H$ are formed. These are Wilks' Lambda, Pillai's Trace, the Hotelling-Lawley Trace, and Roy's Maximum Root. As an example, m test Y_1, Y_2, Y_3/X_1 tests the hypotheses that the X_1 parameter is the same for all three dependent variables (Y_1, Y_2, Y_3) [19].

Fuzzy Sets and Fuzzy Numbers

A fuzzy set $\tilde{A}$ of the universal set X is defined by its membership function $\mu_{\tilde{A}}: X \rightarrow [0, 1]$, with the set $\mathrm{supp}(\tilde{A}) = \{x \in X: \mu_{\tilde{A}}(x) > 0\}$, the support of $\tilde{A}$. In this work, we consider R (the real line) as the universal set. We denote by $\tilde{A}_\alpha$ the α-cut of the fuzzy set $\tilde{A}$ of R, defined for every $\alpha \in (0, 1]$, by $\tilde{A}_\alpha = \{x \in R: \mu_{\tilde{A}} \geq \alpha\}$, and $\tilde{A}_0$ is the closure of supp ($\tilde{A}$).

The fuzzy set $\tilde{A}$ of R is called a fuzzy number if for every $\alpha \in (0, 1]$, the set $\tilde{A}_\alpha$ is a non-empty compact interval. Such an interval will be denoted by $\tilde{A}_\alpha = [\tilde{A}^L_\alpha, \tilde{A}^U_\alpha]$, where $\tilde{A}^L_\alpha = \inf\{x: x \in \tilde{A}_\alpha\}$ and $\tilde{A}^U_\alpha = \sup\{x: x \in \tilde{A}_\alpha\}$.

Since the model presented in this study is essentially based on the *LR*-type fuzzy numbers, we recall a few definitions and results concerning such type fuzzy numbers [20].

Definition 1. A fuzzy number $\tilde{A}$ is said to be a triangular fuzzy number if its membership function can be expressed as

$$\tilde{A}(x) = \begin{cases} \dfrac{x-(a-S^l_a)}{S^l_a} & a-S^l_a \leq x \leq a \\ \dfrac{(a+S^R_a)-x}{S^R_a} & a \leq x \leq a+S^R_a \end{cases}$$

We denote such fuzzy numbers by $\tilde{A} = (a, S^l_a, S^R_a)$, where a is the mean value of $\tilde{A}$, and S^l_a and S^R_a are called left and right spreads, respectively. In the special case, if, $S^l_a = S^R_a = S_a$ then $\tilde{A}$ is called symmetric triangular fuzzy number and we write $\tilde{A} = (a, S_a)$.

Proposition 1. Let $\tilde{A} = (a, S^l_a, S^R_a)$ and $\tilde{B} = (b, S^l_b, S^R_b)$, be two triangular fuzzy numbers. Then,

1a) $\lambda \otimes \tilde{A} = (\lambda a, \lambda s^l_a, \lambda S^R_a) \qquad \lambda > 0,$

1b) $\lambda \otimes \tilde{A} = (\lambda a, -\lambda S^R_a, -\lambda s^l_a) \qquad \lambda < 0,$

2) $\tilde{A} \oplus \tilde{B} = (a+b, S^L_a + S^L_b, S^R_a + S^R_b)$.

In order to find an optimal fuzzy regression model, defining a distance between two fuzzy numbers is necessary. We use the following distance, which is introduced by Xu and Li [21], to establish the multivariate least squares fitting fuzzy model.

Definition 2. Suppose that $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers. The distance between $\tilde{A}$ and $\tilde{B}$ according to function $f(\alpha)$ is defined as

$$d(\tilde{A}, \tilde{B}) = \left[\int_0^1 f(\alpha) d^2(\tilde{A}_\alpha, \tilde{B}_\alpha) d\alpha\right]^{1/2} \tag{1}$$

In which, $d^2(\tilde{A}_\alpha, \tilde{B}_\alpha) = [a_1(\alpha) - b_1(\alpha)]^2 + [a_2(\alpha) - b_2(\alpha)]^2$ and $\tilde{A}_\alpha = [a_1(\alpha), a_2(\alpha)]$, $\tilde{B}_\alpha = [b_1(\alpha), b_2(\alpha)]$, are α-cuts of $\tilde{A}$ and $\tilde{B}$, respectively, and $f(\alpha)$ is an increasing function on [0, 1], for which $f(0) = 0$ and $\int_0^1 f(\alpha) d\alpha = 0.5$. Note that $d(\tilde{A}_\alpha, \tilde{B}_\alpha)$ defines a distance between the α level sets of fuzzy numbers $\tilde{A}$ and $\tilde{B}$. Function $f(\alpha)$ can be regarded as a weighting factor for $d^2(\tilde{A}_\alpha, \tilde{B}_\alpha)$. Monotonic increasing behavior of $f(\alpha)$ leads to placing more importance on higher membership degrees to determine distance between two fuzzy numbers. The conditions $f(0) = 0$ and $\int_0^1 f(\alpha) d\alpha = 0.5$ ensure that the above mentioned distance will be only a generalization of a conventional distance in R. In the following, $f(\alpha) = \alpha$ was used as the weighting function.

In this paper, we use the above distance to establish a method of multiple least squares model fitting. Using distance d, we can draw the following result about the distance of symmetric triangular fuzzy numbers.

Proposition 2. If $\tilde{A}=(a,S_a)$ and $\tilde{B}=(b,S_b)$ are two symmetric triangular fuzzy numbers, then:

$$d^2(\tilde{A},\tilde{B})=(a-b)^2+\frac{1}{6}(S_a-S_b)^2 \tag{2}$$

Formulation of Fuzzy Least Squares Regression

The fuzzy regression analysis studied and applied in this paper, can be stated as follows. Given the set of observed data $(\tilde{y}_i,x_{i1},\ldots,x_{in})$, $i=1,\ldots,m$, and $x_{ij}\in R$, $j=1,\ldots,n$, find an optimal model with fuzzy parameters such as

$$\tilde{Y}=\tilde{A}_0+\tilde{A}_1x_1+\ldots,\tilde{A}_nx_n \qquad j=1,\ldots,n \tag{3}$$

Note that, here it is assumed that data on dependent variable, i.e. $\tilde{y}_i$, $i=1,\ldots,m$ and the coefficient $\tilde{A}$, $j=1,\ldots,n$ are symmetric triangular fuzzy numbers, and independent variables are assumed to be crisp.

Estimation of Model Parameters

In either cases of fuzzy regression, a criterion needs to be selected so as to obtain an optimal model of the form of (3). We do this by a least squares error method, using distance d in (1), as a measure of distance between the estimated value $\tilde{Y}_i$ and the observed value $\tilde{y}_i$. This can be achieved by minimizing the sum of squared errors between $\tilde{Y}_i$ and $\tilde{y}_i$, $i=1,\ldots,m$, i.e.

$$SSE(\tilde{A}_0,\tilde{A}_1,\ldots,\tilde{A}_n)=\sum_{i=1}^{m}d^2(\tilde{Y}_i,\tilde{y}_i)$$

$$=\sum_{i=1}^{m}d^2(\tilde{A}_0,\tilde{A}_1x_{i1},\ldots,\tilde{A}_nx_{in},\tilde{y}_i)$$

Since, based on Proposition 1,

$$\tilde{A}_0,\tilde{A}_1x_{i1},\ldots,\tilde{A}_nx_{in}=(a_0,a_1x_{i1},\ldots,a_nx_{in},\sigma_0+\sigma_1x_{i1},\ldots,\sigma_nx_{in})$$

therefore, by Eq. (2), the minimization problem can be rewritten as

$$\text{Minimize SSE}(\tilde{A}_0,\tilde{A}_1,\ldots,\tilde{A}_n)=\sum_{i=1}^{m}(a_0+a_1x_{i1}+\ldots+a_nx_{in}-y_i)^2$$

$$+\frac{1}{6}\sum_{i=1}^{m}(\sigma_0+\sigma_1x_{i1}+\ldots+\sigma_nx_{in}-S_i)^2$$

Setting the partial derivatives of the SSE with respect to a_j and σ_j to zero, leads to the following equations:

$$a_0\sum_{i=1}^{m}x_{i0}x_{ij}+a_1\sum_{i=1}^{m}x_{i1}x_{ij}+\ldots+a_n\sum_{i=1}^{m}x_{in}x_{ij}=\sum_{i=1}^{m}y_ix_{ij}$$
$$j=0,1,\ldots,n$$

$$\sigma_0\sum_{i=1}^{m}x_{i0}x_{ij}+\sigma_1\sum_{i=1}^{m}x_{i1}x_{ij}+\ldots+\sigma_n\sum_{i=1}^{m}x_{in}x_{ij}=\sum_{i=1}^{m}s_ix_{ij}$$
$$j=0,1,\ldots,n$$

where $x_{i0}=1$, $i=1,\ldots,m$.

These equations can be represented in matrix form:

$$Aa=y,\ A\sigma=S \tag{4}$$

where

$$A=X'X,\quad X=\begin{bmatrix}1 & x_{11} & \ldots & x_{1n}\\ 1 & x_{21} & \ldots & x_{2n}\\ \ldots & \ldots & \ldots & \ldots\\ 1 & x_{m1} & \ldots & x_{mn}\end{bmatrix}_{m\times(n+1)}$$

$$a=(a_0,a_1,\ldots,a_n)^T,\ y=\left(\sum_{i=1}^{m}y_ix_{i0},\sum_{i=1}^{m}y_ix_{i1},\ldots,\sum_{i=1}^{m}y_ix_{in}\right)^T$$

$$\sigma=(\sigma_0,\sigma_1,\ldots,\sigma_n)^T,\ S=\left(\sum_{i=1}^{m}S_ix_{i0},\sum_{i=1}^{m}S_ix_{i1},\ldots,\sum_{i=1}^{m}S_ix_{in}\right)^T$$

The following results about the existence and uniqueness of solution for system (4), hold [21].

Proposition 3. If Rank $(X)=n+1$, then matrix A is positive definite, and so the system of equation has unique solution as:

$$a=A^{-1}y,\quad \sigma=A^{-1}S \tag{5}$$

Proposition 4. If Rank $(X)=n+1$ and $A^{-1}S\geq 0$, then the minimization problem has a unique solution which can be achieved by (5).

The case in which $A^{-1}S<0$

It is possible to encounter conditions that $A^{-1}S<0$. Under this circumstance, Proposition 4 can not be used. Thus, it seems to be necessary to consider the case in which $A^{-1}S<0$ when it occurs. We suggest the following procedure to remove this difficulty. If we encounter negative values for one or more σ_i, spread of parameters, we may use the model but by considering such parameters as crisp, i.e. the spread of the related fuzzy coefficient is set to zero.

Reliability Analysis

In order to evaluate the goodness of fit of the model, we use the following goodness of fit index introduced in Taheri and Kelkinnama [22] which is reasonable and has desired properties.

Definition 3. Suppose that A and B are two fuzzy numbers. Then the capability index between A and B is defined by

$$I_{UI}=\frac{Card(A\cap B)}{Card(A\cup B)},\ \text{where } Card(\tilde{A})=\int_R\tilde{A}(x)dx$$

We use "min" operator for intersection of two fuzzy sets and "max" operator for union of them.

Definition 4. For a fuzzy linear regression model, the mean of capability index *(MCI)* is defined by:

$$MCI = \frac{1}{n}\sum_{i=1}^{m} I_{UI}(y_i, Y_i) \tag{6}$$

We have $0 \leq MCI \leq 1$, so that larger *MCI* is corresponding to better goodness of fit.

Data Collection

In this study, a total of forty different rovings obtained from different short-staple cotton (for increasing of variations limits of fiber properties) were selected from various spinning mills. The spinning operations can affect the fiber properties in different ways, depending on the machinery line and adjustments, etc. Thus, for the minimum of the random errors and the elimination of these effects, fiber properties were measured from rovings (50 to 60 grams of rovings were untwisted carefully) by using the Premier *HVI* testing system (*HFT* 9000 *V*2). All samples (108 Yarn samples) were spun into yarns on a *SKF* lab spinner machine under standard condition at yarn counts of 16, 20, 24, 28, and 32 Ne (2 or 3 yarn counts from each roving according to roving count). Each yarn count was spun at optimum twist factor that was calculated by formula of, $T.F._{op} = 50 - S.L._{50\%}$ (mm) $+ F(\mu g/in)/9$ and they are tested on practical cases again [23]. The appropriate settings were adjusted on the ring spinning machine for each sample. Other spinning conditions were kept constant. Orbit rings (42 mm diameter) and travelers (suitable weights were selected for each yarn count) were used. The tensile properties of the yarns were measured on a (*SDL*) tensile testing machine. Unevenness and hairiness tests of yarns and unevenness tests of rovings were measured on the premier evenness tester (7000 *V*3). The measurements of the main properties are shown in Table 1.

Table 1. Summary of the statistical results for fiber and yarn properties

Properties	Min	Max	Mean	S.D.	Index
Fiber length (UHML) (mm)	28.52	30.67	29.64	0.52	X_1
Mean length (ML) (mm)	23.24	26.65	25.29	0.74	X_2
Uniformity index (U.I) (%)	81.50	87.20	85.30	1.29	X_3
Fiber bundle tenacity (g/tex)	24.70	39.80	34	3.81	X_4
Fiber elongation (%)	7	7.30	7.12	0.06	X_5
Short fiber index (S.F.I) (%)	3	8.70	3.97	1.29	X_6
Micronaire (μg/in)	3.90	4.80	4.39	0.25	X_7
Roving unevenness (CV%)	3.30	14.70	7.38	2.93	X_8
Roving count (Ne)	0.70	1.40	1.02	0.17	X_9
Yarn count (Ne)	15.80	32.10	-	-	X_{10}
Fiber maturity (M.R) (%)	81	88	84.62	1.50	X_{11}
Yarn tenacity (g/tex)	6.81	12.77	10.28	1.19	y_1
Yarn elongation (%)	2.75	4.87	3.79	0.49	y_2
Yarn unevenness (CV%)	10.26	24.39	16.40	3.75	y_3
Yarn hairiness (H)	3.85	6.49	4.99	0.67	y_4

Statistical Analysis

Selecting Appropriate Variables by Using the Mtest

We used multivariate test (mtest) for selecting suitable variables that involve several dependent variables [19]. The results about all of the independent variables are shown in Table 2.

The four multivariate test statistics are all highly significant for five independent variables (X_4, X_6, X_7, X_8, and X_{10}) giving strong evidence (P<0.0001) that the coefficients of these independent variables are not zero for all of dependent variables.

Fuzzy Regression (Estimation of Model Parameters)

First, we recalled the obtained effective variables as X_1, X_2, X_3, X_4, and X_5 that are fiber bundle tenacity, short fiber index, fiber's micronaire, roving unevenness, and Yarn count, respectively.

Then, values of dependent variable, y_i, were fuzzified using a symmetric triangular fuzzy number with right and left spreads proportional to y_i. For all dependent variables fuzzication was performed using $0.10y_i$. This value for the amount of vagueness in observation is preferred as the acceptable level of uncertainty [24]. After fuzzication of observation, $S_i = 0.10y_i$, the following matrix system was developed:

$$X = \begin{bmatrix} 1 & X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & X_{m1} & X_{m2} & X_{m3} & X_{m4} & X_{m5} \end{bmatrix} = \begin{bmatrix} 1 & 32.5 & 3.4 & 4.45 & 6.15 & 15.9 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 36.9 & 3.4 & 4.5 & 6 & 24 \end{bmatrix}_{6\times 6}$$

$$A = X'X = \begin{bmatrix} 108 & 3673 & 428.30 & 474.70 & 797.40 & 2365.75 \\ 3673 & 126470.64 & 14213.24 & 16213.29 & 26368.03 & 80312.29 \\ 428.30 & 14213.24 & 1877.99 & 1867.58 & 3402.06 & 9342.80 \\ 474.70 & 16213.29 & 1867.58 & 2093.42 & 3449.37 & 10389.36 \\ 797.40 & 26368.03 & 3402.06 & 3449.37 & 6809.72 & 17781.64 \\ 2365.75 & 80312.29 & 9342.80 & 10389.36 & 17781.64 & 53985.74 \end{bmatrix}_{6\times 6}$$

We then derived the optimal model for each dependent variable that is yarn tenacity, yarn elongation, yarn unevenness, and hairiness of yarn, respectively, as follow.

Yarn Tenacity

The system of equations becomes:

$$y = \left(\sum_{i=1}^{108} y_i x_{i0}, \ldots, \sum_{i=1}^{108} y_i x_{i5}\right)$$

$$y = (1107.96, 38051.39, 4292.62, 4892.69, 7909.65, 23962.57)$$

Table 2. Multivariate statistics and F approximations for independent variables

Independent variables	Wilk's lambda	Pillai's trace	Hotelling-Lawly- trace	Roy's greatest root	F value	Num DF	Den DF	P-value
X_1	0.86	0.14	0.16	0.16	3.73	4	93	0.0073
X_2	0.86	0.14	0.16	0.16	3.68	4	93	0.0080
X_3	0.88	0.12	0.13	0.13	3.15	4	93	0.0178
X_4	0.72	0.27	0.38	0.38	8.84	4	93	0.0001
X_5	0.97	0.03	0.03	0.03	0.68	4	93	0.6056
X_6	0.79	0.21	0.27	0.27	6.34	4	93	0.0001
X_7	0.55	0.45	0.83	0.83	19.34	4	93	0.0001
X_8	0.73	0.27	0.37	0.37	8.63	4	93	0.0001
X_9	0.90	0.10	0.12	0.12	2.71	4	93	0.0349
X_{10}	0.52	0.48	0.91	0.91	21.17	4	93	0.0001
X_{11}	0.93	0.07	0.07	0.07	1.76	4	93	0.1434

$$S = \left(\sum_{i=1}^{108} S_i x_{i0}, \ldots, \sum_{i=1}^{108} S_i x_{i5}\right)$$

$$S = (110.80, 3805.14, 429.26, 489.27, 790.96, 2396.26)$$

Since Rank $(X) = 6$, according to Proposition 3, the unique solution was found for system of $Aa = y$, and $A\sigma = S$ as follows:

$$a = A^{-1}y = (4.53, 0.09, -0.27, 1.53, -0.01, -0.13)$$

$$\sigma = A^{-1}S = (0.45, 0.01, -0.03, 0.15, -0.0010, -0.01)$$

Considering σ_2, σ_4, and σ_5, the condition of $A^{-1}S \geq 0$ is violated. Thus, Proposition 4 could not be used anymore. We can, therefore, use this model by considering such parameters as crisp, i.e., the spread of fuzzy coefficient of these parameters are set to zero. Consequently, the optimal model, is obtained as

$$\tilde{y}_1 = \tilde{A}_0 + \tilde{A}_1 X_1 + \tilde{A}_2 X_2 + \tilde{A}_3 X_3 + \tilde{A}_4 X_4 + \tilde{A}_5 X_5$$

$$\tilde{y}_1 = (4.53, 0.45) + (0.09, 0.01)X_1 - 0.27X_2 + (1.53, 0.15)X_3 - 0.01X_4 - 0.13X_5$$

In this case, there are three parameters of fuzzy (β_0, β_1, β_3) and the rest of them crisp.

We then used *MCI* to evaluate the goodness of fit of the fuzzy regression model. The result evaluating the goodness of fit between the observed values and the estimated values obtained by the fuzzy regression model are shown in Table 3.

As we see, the *MCI* for the obtained model is 0.66 which is a good value, indicating a good fitting of the model. In practice, we may apply the obtained model to predict the amount of yarn strength. For instance, if X_1=32.5, X_2=3.4, X_3=4.45, X_4=6.15, and X_5=15.9, then the predicted value of yarn strength will be: $\tilde{y}_1$ =(11.17, 1.43). It means that, the predicted value of yarn strength is almost 11.17 with possible minimum and maximum values 9.74 and 12.60, respectively.

Table 3. Capability indices for yarn tenacity (y_1)

No.	y_1	$\hat{y}_1$	I_{UI}
1	(10.59, 1.06)	(11.17, 1.43)	0.42
2	(10.22, 1.02)	(10.67, 1.42)	0.50
⋮	⋮	⋮	⋮
108	(10.15, 1.01)	(10.57, 1.47)	0.52
Mean	-	-	0.66

Yarn Elongation

The following equations for y_2 were developed:

$$y = \left(\sum_{i=1}^{108} y_i x_{i0}, \ldots, \sum_{i=1}^{108} y_i x_{i5}\right)$$

$$y = (409.06, 14010.91, 1600.97, 1804.41, 2938, 8777.28)$$

$$S = \left(\sum_{i=1}^{108} S_i x_{i0}, \ldots, \sum_{i=1}^{108} S_i x_{i5}\right)$$

$$S = (40.91, 1401.09, 160.10, 180.44, 293.80, 877.73)$$

$$a = A^{-1}y = (2.89, 0.02, -0.05, 0.48, -0.0004, -0.08)$$

$$\sigma = A^{-1}S = (0.29, 0.0020, -0.0050, 0.05, -0.0001, -0.0080)$$

The spread of coefficient of negative parameters are assumed to be zero. Thus, the optimal model is obtained as follow.

$$\tilde{y}_2 = (2.89, 0.29) + (0.02, 0.0020)X_1 - 0.05X_2 + (0.48, 0.05)X_3 - 0.0004X_4 - 0.08X_5$$

Table 4. Capability indices for yarn elongation (y_2)

No.	y_2	$\hat{y}_2$	I_{UI}
1	(4.12, 0.41)	(4.3, 0.57)	0.50
2	(3.89, 0.39)	(3.99, 0.58)	0.68
⋮	⋮	⋮	⋮
108	(13.68, 0.37)	(3.76, 0.59)	0.71
Mean	-	-	0.57

According to this equation, the parameters of β_0, β_1 and β_3 are fuzzy. Table 4 shows the value of *MCI* for this model ($MCI = 0.57$) that is acceptable.

Yarn Unevenness

The system of equations of yarn unevenness is as following

$$y = \left(\sum_{i=1}^{108} y_i x_{i0}, \ldots, \sum_{i=1}^{108} y_i x_{i5}\right)$$

$$y = (1771.42, 59212.30, 7334.26, 7705.60, 14086.32, 39450.90)$$

$$S = \left(\sum_{i=1}^{108} S_i x_{i0}, \ldots, \sum_{i=1}^{108} S_i x_{i5}\right)$$

$$S = (177.14, 5921.23, 733.43, 770.56, 1408.63, 3945.09)$$

$$a = A^{-1}y = (32.02.\ 0.0040.\ 0.60.\ -6.03.\ 0.50.\ 0.21)$$

$$\sigma = A^{-1}S = (3.20, 0.0004, 0.06, -0.60, 0.05, 0.02)$$

Therefore, the optimal model is as follow:

$$\tilde{y}_3 = (32.02, 3.20) + (0.0040, 0.0004)X_1 + (0.60, 0.06)X_2 - 6.03X_3 + (0.50, 0.05)X_4 + (0.21, 0.02)X_5$$

In the model, only the spread of coefficient of β_3 parameter is zero and this parameter is assumed to be crisp while the rest of parameters are fuzzy.

The value of *MCI* for goodness of fit in this model is shown in Table 5. This value ($MCI = 0.62$) is satisfactory.

Yarn Hairiness

The following equations for hairiness of yarn were developed.

Table 5. Capability indices for yarn unevenness (y_3)

No.	y_3	$\hat{y}_3$	I_{UI}
1	(14.93, 1.49)	(13.83, 3.96)	0.48
2	(16.12, 1.61)	(14.64, 3.94)	0.38
⋮	⋮	⋮	⋮
108	(15.94, 1.59)	(15.18, 3.92)	0.61
Mean	-	-	0.62

Table 6. Capability indices for yarn hairiness (y_4)

No.	y_4	$\hat{y}_4$	I_{UI}
1	(5.90, 0.59)	(5.38, 1.21)	0.34
2	(4.88, 0.49)	(5.05, 1.21)	0.68
⋮	⋮	⋮	⋮
108	(4.55, 0.45)	(4.53, 1.22)	0.95
Mean	-	-	0.66

$$y = \left(\sum_{i=1}^{108} y_i x_{i0}, \ldots, \sum_{i=1}^{108} y_i x_{i5}\right)$$

$$y = (537.59, 18128.42, 2187.09, 2352.002, 4063.22, 11601.88)$$

$$S = \left(\sum_{i=1}^{108} S_i x_{i0}, \ldots, \sum_{i=1}^{108} S_i x_{i5}\right)$$

$$S = (53.76, 1812.84, 218.71, 235.20, 406.32, 1160.19)$$

$$a = A^{-1}y = (11.60, -0.02, 0.14, -1.05, 0.01, 0.08)$$

$$\sigma = A^{-1}S = (1.16, -0.0020, 0.01, -0.10, 0.0010, -0.0090)$$

Thus, the optimal model is as follow:

$$\tilde{y}_4 = (11.60, 1.16) - 0.02X_1 + (0.14, 0.01)X_2 - 1.05X_3 + (0.01, 0.0010)X_4 - 0.08X_5$$

According to the equation, the parameters of β_0, β_2, and β_4 are fuzzy. Table 6 shows the value of *MCI* ($MCI = 0.66$) that is very good.

Conclusion

In this study, we tried to predict the most important yarn properties of ring spun cotton yarns with minimum random errors, maximum accuracy and actual values of fiber properties into yarn. To this end, we used untwisted rovings for measuring fiber properties. Also, we produced the cotton yarns with optimum twist factor because of maximum performance of length and fineness of fibers in yarn. It might be that the observed/obtained data and relation among variables are not crisp. Thus, the fuzzy least squares regression was used for the estimation of yarn properties. The results indicated that all of yarn properties (strength, elongation, unevenness, and hairiness of yarn) are influenced by fiber tenacity, short fiber index, fineness of fibers, roving unevenness, and yarn count. Most of parameters in the equations are obtained as fuzzy numbers. We also explain the obtained optimal equation, for each dependent variable as follows:

The optimal equation of tenacity shows that the some parameters of the model (the coefficient of constant value, fiber tenacity, and fiber micronaire) are fuzzy and they have positive effect. The rest of parameters are crisp and have negative effect. The optimal equation of elongation and strength are similar. But, the Mean of Capability Index (*MCI*) for the strength model is 0.66 which is better than 0.57 i.e. the *MCI* for the elongation model. In other words, the *MCI* values show the strength model gives better prediction results than the model obtained from elongation. The optimal equation of yarn unevenness showed that all parameters except coefficient of fiber micronaire are fuzzy and they have positive effect on yarn unevenness. This only parameter is crisp and has a negative effect. The *MCI* value for this model is 0.62 that it is satisfactory. This value shows that the model gives good prediction result. The model of yarn hairiness indicates that increasing short fiber index and roving unevenness, will increase yarn hairiness (positive effect). Also increasing fiber strength, fiber micronaire and yarn count (Ne) will reduce yarn hairiness (negative effect). The parameters of short fiber index and roving unevenness are fuzzy and the other parameters are crisp. The value of *MCI* is 0.66 that it is very good. It means that the predictive ability of our model is satisfactory [22].

This is notable that the proposed approach can be extended to other cases in textile engineering where the data available are non-precise (fuzzy) or the number of data is few.

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