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Position Control of Servomotors Using Neural Dynamic Sliding Mode

In this paper, position control of servomotors is addressed. A radial basis function neural network is employed to identify the unknown nonlinear function of the plant model, and then a robust adaptive law is developed to train the parameters of the neural network, which does not require any preliminary off-line weight learning. Moreover, base on the identified model, we propose a new dynamic sliding mode control (DSMC) for a general class of nonaffine nonlinear systems by defining a new adaptive proportional-integral sliding surface and employing a linear state feedback. The main property of proposed controller is that it does not need an upper bound for the uncertainty and identified model; moreover, the switching gain increases and decreases according to the system circumstance by employing an adaptive procedure. Then, chattering is removed completely by using the DSMC with a small switching gain. [DOI: 10.1115/1.4004782]

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1 Introduction

During the last three decades, considerable effort has been devoted to the control of nonlinear systems using sliding mode control (SMC) methodology [1]. It has been shown that the SMC, because of its invariance property, is a powerful tool in facing structured or unstructured uncertainties, disturbances, and noises that always produce difficulties in the realization of designed controller for real systems [2–4]. Note that invariance is stronger than robustness [4]. The greatest shortcoming of SMC is chattering, the high (but finite) frequency oscillations with small amplitude that produce heat losses in electrical power circuits and wear mechanical parts [5]. Chattering often results in the excitation of high frequency unmodeled (ignored) dynamics (sensors, actuators, and plant) [3,6,7]. Excitation of these dynamics is due to two causes: high controller gain and high frequency switching of input control signal [1].

Four design methodologies have been proposed to overcome this problem: boundary layer, adaptive boundary layer, dynamic sliding mode control (DSMC), and higher order sliding mode control (HOSMC). Boundary layer and adaptive boundary layer methods cannot preserve the invariance property of SMC. Nevertheless, these two methods are adopted [8], because they can reduce or suppress switching of input control signal by employing a high-gain control inside the boundary layer [2,9]. Use of high-gain control causes instability inside the boundary layer leading to chattering [1,7]. In DSMC an integrator (or any other strictly low-pass filter) is placed before the input control signal of the plant. Then, switching is removed from the input control signal since the integrator filters the high frequency switching which is, due to the use of sliding mode control [10]. However, in DSMC the augmented system is one dimension bigger than the actual system, and then the plant model should be completely known when one needs to use SMC to control the augmented system states [6,10]. HOSMC is proposed to reliably prevent chattering [11,12]. In higher order SMC, the effect of switching is totally eliminated by moving the switching to the higher order derivatives of desired output [3,11,12]. Many algorithms are proposed for implementation of second or higher order SMC [12,13]. However, the main drawback is that the control methods generally require the knowledge of higher order derivative of surface. As far, when the relative degree is 2, the usually nonmeasurable surface derivative must be estimated by means of some observer, for example, high-gain observer [14] or sliding differentiator [15]. Moreover, chattering cannot be suppressed only by removing the switching. For example, it has been shown [16,17], that in HOSMC chattering may happen in power-fractional algorithm, proposed in Ref. [18], and in super-twisting algorithm, proposed in Ref. [12]. Both of these algorithms utilize a continuous nonlinear function with infinite gain. Therefore, the other concept, which should be considered for chattering suppression, is reducing the switching gain. One way to reduce switching gain is to use adaptive switching gain.

In this paper, a method is presented for implementation of DSMC via adaptive switching gain. The proposed method, therefore, will alleviate the two reasons that can excite unmodeled dynamics. To overcome the drawback of DSMC a radial basis function neural network (RBFNN) [19] is employed to identify the plant model. To guarantee the robustness of the neural network identification procedure, a new robust adaptive law is developed.

Servomotors are used in many automatic systems, including drives for printers, tape records, robot manipulators, etc. Recently, field-oriented methods have been used in the design of induction servomotors for high performance applications [20,21]. With these control approaches, the dynamic behavior of the induction motor is similar to that of a separately excited dc motor. Furthermore, in practical applications, the control performance of the induction motor is still influenced by the uncertainties of the plant, such as mechanical parameter uncertainty, external load disturbance, and unmodeled dynamics. These uncertainties make the design of the controller difficult and a robust controller with a good performance is needed [22]. The invariance property of SMC is our motivation to position control of induction motors.

The remainder of this paper is organized as follows: in Sec. 2, we provide the preliminary background about the problem. In Sec. 3, the neural identification method is proposed. Section 4 is devoted to designing DSMC methodology. In Sec. 5, an adaptive procedure is proposed for adjusting the switching gain. Finally, in

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Sec. 6 we discuss simulation results to verify theoretical concepts presented in Sec. 5. The conclusion is given in Sec. 7.

2 Problem Formulation

SMC consist of three phases: reaching phase (the time needed for hitting the sliding surface), sliding phase (sliding on a stable manifold), and steady state phase. To preserve the invariance property during sliding and steady state phases and guarantee reaching the sliding surface in finite time, one should use the reaching law [4]

$$\dot{s} = -\eta \, sign(s) \tag{1}$$

in which η is a positive large enough constant. It is known that use of this sign function results in high frequency switching with amplitude η, called the switching gain. Therefore, chattering can be suppressed by Ref. [23]:

1. removing the effect of high frequency switching of input control signal
2. reducing the amplitude of the switching, i.e., reducing the switching gain

In this paper, a method is presented, having the above two characteristics, by employing adaptive switching gain controller and dynamic sliding mode control. Consider the class of induction servomotors in the form of n-dimensional nonlinear system

$$\begin{aligned} \dot{x}_i &= x_{i+1} : i = 1,2,\ldots,n-1 \\ \dot{x}_n &= f(X,u) \end{aligned} \tag{2}$$

with input u and state $X = [x_1, x_2 \ldots, x_n]^T$. The goal is to have the states of this system, i.e., vector X, track the states of the following desired linear system, i.e., variables $y_i : i = 1,2,\ldots,n$, which is used as a reference model

$$\begin{aligned} \dot{y}_i &= y_{i+1} : i = 1,2,\ldots,n-1 \\ \dot{y}_n &= \sum_{i=1}^{n} d_i y_i + u_d \end{aligned} \tag{3}$$

In which, $d_i(t) : i = 1,2,\ldots,n$ can be time varying coefficients. Note that the function $f(X,u)$ is unknown.

Consider the following assumptions, which are widely used [24–26]:

Assumption 1. *The desired state variables $y_i : i = 1,2,\ldots,n$ are continuous and bounded.*

Assumption 2. *X is a measured signal.*

Assumption 3. *$f(X, u)$ is a continuous function for all $(X, u) \in \Re^{n+1}$.*

Definition 1. *The solution of a dynamical system is uniformly ultimately bounded (UUB) if for a compact set $\phi \in \Re^n$ and for all $X(t_0) = X_0 \in \phi$, there exist an $E_X > 0$ and a number $T(E_X, X_0)$ such that $\|X(t)\| < E_X$ for all $t > t_0 + T$* [2].

3 Robust Neural Adaptive Identification

Equation (2) can be written as follows:

$$\dot{X} = AX + Bf(X, u) \tag{4}$$

where A and B are as follow with appropriate dimensions

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & \ldots & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

or as follows:

$$\dot{X} = A_s X + (A - A_s) X + Bf(X, u) \tag{6}$$

or

$$\dot{X} = A_s X + B\, g(X, u) \tag{7}$$

where

$$g(X, u) = f(X, u) + \sum_{i=1}^{n} a_i x_i \tag{8}$$

and

$$A_s = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & 0 & 1 \\ -a_1 & -a_2 & \ldots & -a_{n-1} & -a_n \end{bmatrix} \tag{9}$$

Assume that a_i are such that A_s is stable, i.e., for any symmetric positive definite matrix Q, there exist a symmetric positive definite matrix P satisfying the following Lyapunov equation:

$$A_s^T P + PA_s = -Q \tag{10}$$

In order to estimate the nonlinear function $g(X, u)$, we propose a RBFNN, described by the following equation:

$$\hat{g} = \hat{w}^T \xi(X, u) \tag{11}$$

where $\hat{w} \in \Re^m$ is the weight vector estimate, and $\xi(.) : \Re^{n+1} \to \Re^m$ is a radial basis function vector [26]. Then, the neural model of the plant can be written as follows:

$$\dot{\hat{X}} = A_s \hat{X} + B\, \hat{w}^T \xi(X, u) \tag{12}$$

where $\hat{X} = [\hat{x}_1, \hat{x}_2 \ldots, \hat{x}_n]^T$ is the identified model state vector. Due to the approximation capability of the RBFNNs [26], there exists an ideal weight vector w with arbitrary large enough dimension m such that the system (7) can be written as follows:

$$\dot{X} = A_s X + B\, w^T \xi(X, u) + B\varepsilon_X \tag{13}$$

where ε_X is an arbitrary small reconstruction error [27]. We also make the following two assumptions:

Assumption 4. *The RBFNN reconstruction error bound B_ε is a constant on a compact set, i.e., $|\varepsilon_X| < B_\varepsilon$.*

Assumption 5. *The ideal weight is bounded by a known positive value B_w such that $\|w\| < B_w$.*

These assumptions are common in the neural networks literature [27] and are due to universal approximation property of the RBFNNs. Notice that by choosing more proper values for the parameters of ξ, the value of B_ε is decreased. We also include a term $r(t)$ in Eq. (12) to be calculated such that the robustness of the dynamic model (12) is maintained. Thus, we have

$$\dot{\hat{X}} = A_s \hat{X} + B\, \hat{w}^T \xi(X, u) + r(t) \tag{14}$$

Subtracting Eq. (14) from Eq. (13), we obtain

$$\dot{\tilde{X}} = A_s \tilde{X} + B\, \tilde{w}^T \xi(X, u) + B\varepsilon_X - r(t) \tag{15}$$

where $\tilde{X}(t) = X(t) - \hat{X}(t)$ and $\tilde{w} = w - \hat{w}$ are the state and parameter estimation errors. The following theorem gives a robust adaptive law for the dynamic neural model.

Theorem 1. *Given the assumptions 4 and 5 for the system (13) and the estimator (14), and using the following RBFNN weight update law*

$$\dot{\hat{w}} = k_w\, \xi(X,u)\,(PB)^T\tilde{X} - 4\,k_e k_w\,\|\tilde{X}\|\,\hat{w} \tag{16}$$

with the term given by

$$r(t) = k_x\tilde{X}(t) \tag{17}$$

causes the estimation error $\tilde{X}(t)$ and $\tilde{w}(t)$ be UUB with specific bounds given by

$$B_{\tilde{X}} = \frac{\lambda_{\max}(P)\,B_\varepsilon + k_e B_w^2}{\frac{1}{2}\lambda_{\min}(Q) + k_x\,\lambda_{\min}(P)},\; B_{\tilde{w}} = \frac{1}{2}B_w + \sqrt{\frac{\lambda_{\max}(P)\,B_\varepsilon}{4k_e} + \frac{1}{4}B_w^2} \tag{18}$$

respectively, where k_w, k_e, and k_x are arbitrary positive scalar constants, $\lambda_{\min}(Q)$ and $\lambda_{\min}(P)$ are the smallest eigenvalues of positive definite matrices Q and P, respectively, which satisfy Lyapunov equation (10), and $\lambda_{\max}(P)$ is the largest eigenvalues of P.

Proof. Consider the following Lyapunov function candidate:

$$V(t) = \frac{1}{2}\tilde{X}^T P\tilde{X} + \frac{1}{2k_w}\tilde{w}^T\tilde{w} \tag{19}$$

Taking the derivative of $V(t)$ yields

$$\dot{V}(t) = \frac{1}{2}\dot{\tilde{X}}^T P\tilde{X} + \frac{1}{2}\tilde{X}^T P\dot{\tilde{X}} + \frac{1}{k_w}\tilde{w}^T\dot{\tilde{w}} \tag{20}$$

Substituting Eqs. (10) and (15) in the above equation follows that

$$\begin{aligned}\dot{V}(t) = &-\frac{1}{2}\tilde{X}^T Q\tilde{X} + \tilde{X}^T P\,B\varepsilon_X - \tilde{X}^T P\,r\\ &+\tilde{w}^T\left(\frac{1}{k_w}\,\dot{\tilde{w}} + \xi(X,u)\,(PB)^T\tilde{X}\right)\end{aligned} \tag{21}$$

since $\dot{\tilde{w}} = -\dot{\hat{w}}$, using the tuning law (16) and the term (17) in the above equation gives

$$\dot{V}(t) = -\frac{1}{2}\tilde{X}^T Q\,\tilde{X} + \tilde{X}^T P\,B\,\varepsilon_X - k_x\tilde{X}^T P\tilde{X}\; + 4\,k_e\|\tilde{X}\|\tilde{w}^T\hat{w} \tag{22}$$

Considering the properties of positive definite matrixes Q and P, and using $\hat{w} = w - \tilde{w}$, the above equation yields

$$\begin{aligned}\dot{V}(t) \le &-\left\{\frac{1}{2}\lambda_{\min}(Q) + k_x\,\lambda_{\min}(P)\right\}\|\tilde{X}\|^2\\ &+\left\{\lambda_{\max}(P)\,B_\varepsilon - 4\,k_e(\|\tilde{w}\|^2 - B_w\,\|\tilde{w}\|)\right\}\|\tilde{X}\|\end{aligned} \tag{23}$$

Using bound $B_{\tilde{X}}$ defined in Eq. (18) follows that

$$\begin{aligned}\dot{V}(t) \le &-\left\{\frac{1}{2}\,\lambda_{\min}(Q) + k_x\,\lambda_{\min}(P)\right\}\|\tilde{X}\|\left\{\|\tilde{X}\| - B_{\tilde{X}}\right\}\\ &-4\,k_e\left(\|\tilde{w}\| - \frac{1}{2}B_w\right)^2\|\tilde{X}\|\end{aligned} \tag{24}$$

or

$$\dot{V}(t) \le -\left\{\frac{1}{2}\,\lambda_{\min}(Q) + k_x\,\lambda_{\min}(P)\right\}\|\tilde{X}\|\left\{\|\tilde{X}\| - B_{\tilde{X}}\right\} \tag{25}$$

By taking $\omega(t) = \left\{\frac{1}{2}\,\lambda_{\min}(Q) + k_x\,\lambda_{\min}(P)\right\}\|\tilde{X}\|\left\{\|\tilde{X}\| - B_{\tilde{X}}\right\}$ one can write

$$\dot{V} \le -\omega(t) \tag{26}$$

and integrating the above equation from zero to t yields

$$0 \le \int_0^t \omega(\tau)d\tau \le \int_0^t \omega(\tau)d\tau + V(t) \le V(0) \tag{27}$$

when $t\to\infty$, the above integral exist and is less than or equal to $V(0)$. Since $V(0)$ is positive and finite, according to Barbalat's lemma [2,28] we obtain

$$\lim_{t\to\infty}\omega(t) = \lim_{t\to\infty}\left\{\frac{1}{2}\,\lambda_{\min}(Q) + k_x\,\lambda_{\min}(P)\right\}\|\tilde{X}\|\left\{\|\tilde{X}\| - B_{\tilde{X}}\right\} = 0 \tag{28}$$

Suppose $\|\tilde{X}\| > B_{\tilde{X}}$, since $\left\{\frac{1}{2}\,\lambda_{\min}(Q) + k_x\,\lambda_{\min}(P)\right\}$ and $\left\{\|\tilde{X}\| - B_{\tilde{X}}\right\}$ are greater than zero, Eq. (28) implies that, $\lim_{t\to\infty}\|\tilde{X}\| = 0$, which results in decreasing $\|\tilde{X}\|$ until it becomes less than $B_{\tilde{X}}$. This guarantees that $\tilde{X}(t)$ is UUB with ultimate bound $B_{\tilde{X}}$. In addition, from Eqs. (18) and (23), it is easy to show that if $\|\tilde{w}\|$ is greater than the desired ultimate bound $B_{\tilde{w}}$ then, $\|\tilde{w}\|$ decreases until it becomes less than $B_{\tilde{w}}$. Therefore, $\tilde{w}(t)$ is UUB with ultimate bound $B_{\tilde{w}}$.

Remark 1. As is clear from Eq. (18), the ultimate estimation error bound B_x can be made as small as desired by increasing k_x. Moreover, parameter k_e offers a design tradeoff between the relative ultimate bounds of $\|\tilde{X}\|$ and $\|\tilde{w}\|$. However, it should be noted that it is usually desired to keep the ultimate bound $\|\tilde{X}\|$ as small as possible.

Remark 2. The first term of adaptive law given by Eq. (16) is a continuous time version of the standard back propagation algorithm, and the second term in Eq. (16) corresponds to the ε -modification [29] used in adaptive identification schemes to guarantee bounded of the parameter estimates. Hence, this algorithm does not require persistent excitation condition.

4 Controller Design

According to Eqs. (8) and (11) we can write as follows:

$$\hat{f}(X,u) = \hat{w}^T\xi(X,u) + \Lambda A_s X \tag{29}$$

and

$$\Lambda = [0, 0\ldots, 0, 1] \in \Re^n \tag{30}$$

To apply the DSMC to system (2), we define the following augmented system:

$$\begin{aligned}\hat{x}_{n+1} &= \hat{f}(X,u)\\ \dot{x}_i &= x_{i+1} : i = 1,2,\ldots,n-1\\ \dot{x}_n &= \hat{x}_{n+1}\\ \dot{\hat{x}}_{n+1} &= \varphi + \gamma\,\dot{u}\end{aligned} \tag{31}$$

where $\hat{x}_{n+1}$ is the neural identified of unknown variable x_{n+1} and

$$\varphi(X,x_{n+1},u) = \dot{\hat{w}}^T\xi + \hat{w}^T\frac{\partial\xi}{\partial X}\dot{X} - \Lambda A_s\dot{X},\; \gamma(X,u) = \hat{w}^T\frac{\partial\xi}{\partial u} \tag{32}$$

and consequently the augmented reference model will be

$$y_{n+1} = \sum_{i=1}^{n} d_i y_i + u_d$$
$$\dot{y}_i = y_{i+1} : i = 1, 2, \ldots, n \quad (33)$$
$$\dot{y}_{n+1} = \sum_{i=1}^{n} d_i y_{i+1} + \dot{u}_d$$

Now, defining the following variables:

$$D = [0, d_1(t), d_2(t), \ldots, d_n(t)] \quad (34)$$

$$Y = [y_1, y_2, \ldots, y_{n+1}]^T \quad (35)$$

$$X_a = [x_1, x_2, \ldots, x_n, \hat{x}_{n+1}]^T \quad (36)$$

$$e_i = x_i - y_i : i = 1, 2, \ldots, n \quad (37)$$

$$e_{n+1} = \hat{x}_{n+1} - y_{n+1} \quad (38)$$

$$E = [e_1, e_2, \ldots, e_{n+1}]^T = X_a - Y \quad (39)$$

Then, augmented reference model can be written as follows:

$$y_{n+1} = \sum_{i=1}^{n} d_i y_i + u_d$$
$$\dot{y}_i = y_{i+1} : i = 1, 2, \ldots, n \quad (40)$$
$$\dot{y}_{n+1} = DY + \dot{u}_d$$

We have

$$\begin{aligned}\dot{e}_{n+1} &= \dot{\hat{x}}_{n+1} - \dot{y}_{n+1} = \varphi + \gamma\dot{u} - DY - \dot{u}_d \\ &= \phi + \gamma\dot{u} - DY - \dot{u}_d + DX_a - DX_a + v - v + \dot{u} - \dot{u} \\ &= DE + v - \dot{u}_d + (\varphi + \gamma\dot{u} - \dot{u}) + (\dot{u} - DX_a - v)\end{aligned} \quad (41)$$

Defining the following linear state feedback:

$$\dot{u} = DX_a + v \quad (42)$$

and the variable

$$W = (\varphi + \gamma\dot{u} - \dot{u}) \quad (43)$$

We obtain

$$\dot{e}_{n+1} = DE + v - \dot{u}_d + W \quad (44)$$

where v is the new input control signal to be calculated via sliding mode control and $W(X, x_{n+1}, u, \dot{u})$ is a matched uncertainty. The matched uncertainty can be cancelled out directly by the input [30].

Remark 3. Variable W is considered as uncertainty due to its dependency to unknown variable x_{n+1}.

The control problem now is finding a suitable input control signal $v(t)$ such that the states of system (31), X_a, track the states of system (33), Y, or equivalently the error dynamic (44) converge to zero in finite time. To this end, the following new proportional-integral sliding surface will be defined:

$$s(t) = \sigma e_{n+1}(t) - \int_0^t ([\sigma D(\tau) + K(\tau)]E(\tau) + v_s(\tau))d\tau \quad (45)$$

in which vector $K(t)$ and signal $v_s(t)$ are design parameters. A method for calculating the values of these parameters will be provided. Differentiating this surface equation with respect to time leads to

$$\dot{s} = \sigma\dot{e}_{n+1} - \sigma DE - KE - v_s \quad (46)$$

and by substituting from Eq. (44) one gets

$$\dot{s} = \sigma v - \sigma\dot{u}_d + \sigma W - KE - v_s \quad (47)$$

the scalar σ can be time-varying in order to provide the desired properties for the sliding surface. Nevertheless, in this paper, we have chosen σ to be a constant

$$\sigma = 1 \quad (48)$$

Then

$$\dot{s} = v - \dot{u}_d + W - KE - v_s \quad (49)$$

The input control signal $v(t)$ consists of two parts. The first part is equivalent control and will act when the error trajectories are on the sliding surface, i.e., during sliding phase. The second part consists of discontinuity to remove the effect of uncertainty W during this phase [1–4]. Therefore, the first part, v_{eq}, will obtained from $\dot{s} = 0$ when $W = 0$ as follows:

$$v_{eq} = KE + \dot{u}_d + v_s \quad (50)$$

By substituting v_{eq} into Eq. (44) (and keeping $W = 0$), the following dynamic equation will be obtained in the sliding mode:

$$\dot{e}_{n+1} = DE + KE + v_s = (D + K)E + v_s \quad (51)$$

Equation (51) shows the error dynamics during sliding phase and yields the zero dynamics of the sliding surface; therefore, vector $K(t)$ and signal $v_s(t)$ should be selected such that the linear dynamical system (51) is stable. Suppose, $v_s(t) = 0$, and choosing $K(t)$ as

$$K(t) = M - D(t) \quad (52)$$

results in the following linear constant coefficient system:

$$\dot{e}_{n+1} = ME \quad (53)$$

which may be made stable by the proper choice of the constant row vector $M = [m_1, m_2, \ldots, m_{n+1}]$. Note that in this case convergence of error to acceptable levels may be slow.

Now, letting $v_s(t) \neq 0$ and considering $v_s(t)$ as the input control signal for Eq. (51), it can be calculated such that to force the error signal to converge to zero in finite time as terminal SMC [18,31]. Consider the error dynamics (51), i.e., when $t \geq t_f$ (t_f is the reaching time to sliding surface) and the system is on the sliding surface (45), the error trajectory converges to zero in finite time $t_e + t_f$ with

$$t_e = \frac{|[C^T, 1]E(0)|}{\eta_e} \quad (54)$$

provided we choose

$$v_s(t) = -\left((D + K)E + [1, C^T]E + \eta_e sign([C^T, 1]E) - e_1\right) \quad (55)$$

with $\eta_e > 0$, and $C = [c_1, c_2, \ldots, c_n]^T$ such that $s_e = [C^T, 1]E$, i.e., the following polynomial is stable:

$$s_e = e_{n+1} + c_n e_n + \ldots + c_2 e_2 + c_1 e_1 \quad (56)$$

In this case, K may have any value, even zero. Differentiating s_e and using Eq. (51) yields

$$\dot{s}_e = (D+K)E + v_s + c_n e_{n+1} + \ldots + c_2 e_3 + c_1 e_2 \quad (57)$$

and substitution from Eq. (55)

$$\dot{s}_e = -\eta_e sign(s_e) \quad (58)$$

Now, consider the following Lyapunov function:

$$V(t) = \frac{1}{2}s_e^2 \quad (59)$$

Taking derivative of $V(t)$ with respect to time leads to

$$\dot{V} = s_e\dot{s}_e = -\eta_e|s_e| \quad (60)$$

This is negative-definite. Integrating of Eq. (60) leads to Eq. (54) (see proof of Theorem 3).

The second part of input control signal $v(t)$ consists of a discontinuity to force, the system states toward the sliding surface and overcome the matched uncertainty W and also preserve the invariance property in the sliding mode in the presence of the uncertainty W. This part of input control signal consists of a sign function. Therefore, the following adaptive input control signal is proposed, which consists of equivalent control part plus discontinuous control part

$$v(t) = KE + \dot{u}_d + v_s - (q_1+1)\beta\, sign(s) \quad (61)$$

where $(q_1+1)\beta$ is a switching gain, $q_1 > 0$ is a constant design parameter, and $\beta(t)$ is an adaptive term such that

$$\beta(t) > 0 : \forall t \quad (62)$$

Here, we propose an adaptive method for calculating $\beta(t)$. In the sequel, we make the following assumption:

Assumption 6. *We assume that* $W(X, x_{n+1}, u, \dot{u})$ *is bounded by a known function* $\Omega(X, u, \dot{u})$, *i.e.*

$$|W(X, x_{n+1}, u, \dot{u})| \le \Omega(X, u, \dot{u}) < \infty \quad (63)$$

In general, inequality (63) is not restrictive [1–3,12,32,33].

Remark 4. Considering other existing approach of handling uncertainty [1–15], the proposed approach requires only that a bound exists, but the magnitude of this bound Ω need not to be known (see remark 6).

Remark 5. Note that the sign function affects $v(t)$ (see Eq. (61)), and by considering Eq. (42), input control signal, $u(t)$, will be continuous and free of switching.

5 Adaptive Approach

A method for selection of $\beta(t)$ is as follows:

$$\dot{\beta} = \dot{\Omega} + q_2|s|, \quad \beta(0) = \beta_0, \quad \Omega(0) = \Omega_0 \quad (64)$$

where $q_2 > 0$ is a constant design parameter and β_0 and $\Omega_0 = \Omega(X(0), u(0), \dot{u}(0))$ are the constant initial values of β and Ω, respectively. Note that we can select β_0 arbitrarily. Despite the fact that this equation shows that the switching gain is increased until the error trajectory is driven into sliding mode, this approach has three severe practical disadvantages.

1. In case of a large initial distance from the sliding surface, the switching gain will increase quickly due to this error and not because of perturbation. This may result in a switching gain, which is significantly larger than necessary.
2. Noise on the measurements will prevent $|s|$ to ever become exactly zero, so the adaptive gain will continue to increase. This cause instability of the closed loop system. The rate of increase depends on the value of q_2.
3. The adaptation law can only increase the gain but never decrease it. Thus, if the circumstances change such that a smaller switching gain is permitted, the adaptation law is not able to adapt to these new circumstances.

Therefore, we propose a method, which can decrease or increase the switching gain according to the circumstances such that does not have these problems. In the proposed approach, $\beta(t)$ is defined as follows:

$$\dot{\beta} = \dot{\Omega} + q_2(|s| - \psi(\beta)), \quad \beta(0) = \beta_0, \quad \Omega(0) = \Omega_0 \quad (65)$$

and

$$\psi(\beta) = \frac{\varepsilon_1}{2}(sign(\beta - \Omega - \varepsilon_0) + 1) \ge 0 \quad (66)$$

Constants $q_2 > 0$, $\varepsilon_1 > 0$, and $\varepsilon_0 > 0$ are design parameters and β_0 and $\Omega_0 = \Omega(X(0), u(0), \dot{u}(0))$ are the bounded initial values of β and Ω, respectively. Note that we can select β_0 arbitrary. Integrating Eq. (65) leads to

$$\beta(t) = \Omega(t) + \beta_0 - \Omega_0 + q_2\int_0^t (|s(\tau)| - \psi)d\tau \quad (67)$$

Lemma 1. *If the following condition is satisfied*:

$$\beta_0 - \Omega_0 > \varepsilon_0 \quad (68)$$

Then, Eqs. (65) and (66) guarantee that

$$\beta - \Omega \ge \varepsilon_0 : \forall t > 0 \quad (69)$$

Proof. Letting $\theta = \beta - \Omega$, results in $\theta(0) = \theta_0 = \beta_0 - \Omega_0$. From Eqs. (66) and (67) we can write

$$\theta(t) = \theta_0 + q_2\int_0^t |s(\tau)|d\tau - \frac{q_2\varepsilon_1}{2}\int_0^t (sign(\theta - \varepsilon_0) + 1)d\tau \quad (70)$$

The right hand side of the above equation is the sum of continuous functions. Therefore, $\theta(t)$ is a continuous function such that $\theta_0 > \varepsilon_0$ (assumption (68)). Before $\theta(t)$ becomes smaller than ε_0, it must passes ε_0 at a time t_1 such that

$$\theta(t) > \varepsilon_0 : \forall t \in (0, t_1) \quad (71)$$

where

$$t_1 \ge \frac{2(\theta_0 - \varepsilon_0)}{q_2\varepsilon_1} \quad (72)$$

and at $t = t_1$, we have $\theta = \varepsilon_0$, i.e.

$$\varepsilon_0 = \theta_0 + q_2\int_0^{t_1} |s(\tau)|d\tau - q_2\varepsilon_1 t_1 \quad (73)$$

Now suppose that there is a time t_2 such that

$$\theta(t) < \varepsilon_0 : \forall t \in (t_1, t_2) \quad (74)$$

Then

$$\theta(t) = \theta_0 + q_2\int_0^{t_1} |s(\tau)|d\tau + q_2\int_{t_1}^{t} |s(\tau)|d\tau - q_2\varepsilon_1 t_1 \quad (75)$$

Using Eq. (73), we can write

$$\theta(t) = \varepsilon_0 + q_2 \int_{t_1}^{t} |s(\tau)| d\tau \tag{76}$$

It means that

$$\theta(t) \geq \varepsilon_0 : \forall t \in (t_1, t_2) \tag{77}$$

and this contradict with assumption (74), i.e.

$$\theta(t) \geq \varepsilon_0 : \forall t \tag{78}$$

Theorem 2. *Consider the error dynamic (44) with the adaptive input control signal of Eqs. (61), (65), and (66). Then, the error trajectory converges to the sliding surface of Eq. (45) if condition (68) is satisfied.*

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\left(s^2 + q_2^{-1}(\Omega - \beta)^2\right) \tag{79}$$

Taking derivative of $V(t)$ with respect to time leads to

$$\begin{aligned}\dot{V} &= s\dot{s} + q_2^{-1}(\Omega - \beta)(\dot{\Omega} - \dot{\beta}) \\ &= s(v - \dot{u}_d + W - KE - v_s) + q_2^{-1}(\Omega - \beta)(\dot{\Omega} - \dot{\beta})\end{aligned} \tag{80}$$

By substitution from Eqs. (61) and (65) into Eq. (80)

$$\dot{V} = s(W - (q_1 + 1)\beta\, sign(s)) + q_2^{-1}(\Omega - \beta)(-q_2(|s| - \psi)) \tag{81}$$

or

$$\dot{V} \leq \Omega|s| - (q_1 + 1)\beta|s| - (\Omega - \beta)(|s| - \psi) \tag{82}$$

Then

$$\dot{V} \leq -q_1\beta|s| + \psi(\Omega - \beta) \tag{83}$$

and by Lemma 1

$$\dot{V} \leq -q_1\beta|s| \tag{84}$$

By taking $\omega(t) = q_1\beta|s(t)|$, one can write

$$\dot{V} \leq -\omega(t) \leq 0 \tag{85}$$

and integrating the above equation from zero to t yields

$$0 \leq \int_0^t \omega(\tau)d\tau \leq \int_0^t \omega(\tau)d\tau + V(t) \leq V(0) \tag{86}$$

when $t \to \infty$, the above integral exist and is less than or equal to $V(0)$. Since $V(0)$ is positive and finite, according to Barbalat's lemma [2,28], we obtain

$$\lim_{t\to\infty} \omega(t) = \lim_{t\to\infty} q_1\beta|s(t)| = 0 \tag{87}$$

Since q_1 and $\beta(t)$ are greater than zero, Eq. (87) implies that, $\lim_{t\to\infty} s = 0$ proving the theorem.

Theorem 2 guarantees convergence to sliding surface asymptotically; however, in SMC finite time convergence is necessary so that the invariance property is retained after finite time. The next theorem shows the finite time convergence to sliding surface.

Theorem 3. *Consider the error dynamic of Eq. (44) with the adaptive input control signal of Eq. (61), (65), and (66). The error trajectory converges to the sliding surface (45) in finite time t_f, if β_0 is selected such that inequality (68) is satisfied. Moreover*

$$t_f \leq \frac{|\hat{x}_{n+1}(0) - y_{n+1}(0)|}{\varepsilon_0} \tag{88}$$

Proof. Consider the Lyapunov function (79). Taking derivative of $V(t)$ with respect to time leads to

$$\begin{aligned}\dot{V} &= s\dot{s} + q_2^{-1}(\Omega - \beta)(\dot{\Omega} - \dot{\beta}) = s\dot{s} + q_2^{-1}(\Omega - \beta)(-q_2(|s| - \psi)) \\ &= s\dot{s} - (\Omega - \beta)|s| + \psi(\Omega - \beta)\end{aligned} \tag{89}$$

From Eq. (83) we have

$$s\dot{s} - (\Omega - \beta)|s| + \psi(\Omega - \beta) \leq -q_1\beta|s| + \psi(\Omega - \beta) \tag{90}$$

Thus

$$s\dot{s} - (\Omega - \beta)|s| \leq -q_1\beta|s| \tag{91}$$

or

$$s\dot{s} \leq -(\beta - \Omega + q_1\beta)|s| \leq -(\beta - \Omega)|s| \tag{92}$$

From Lemma 1, we can write

$$-(\beta - \Omega) \leq -\varepsilon_0 \tag{93}$$

Substituting Eq. (93) into Eq. (92) yields

$$s\dot{s} \leq -\varepsilon_0|s| \tag{94}$$

where

$$\varepsilon_0 > 0 \tag{95}$$

Suppose t_f is the reaching time, i.e., $s(t_f) = 0$ and consider two cases:

First case: $s > 0$, from Eq. (94)

$$\dot{s} \leq -\varepsilon_0 \tag{96}$$

Integrating Eq. (96) between $t = 0$ and $t = t_f$ leads to

$$-s(0) \leq -\varepsilon_0 t_f \tag{97}$$

Therefore

$$t_f \leq \frac{s(0)}{\varepsilon_0} \tag{98}$$

Second case: $s < 0$, from Eq. (94)

$$\dot{s} \geq \varepsilon_0 \tag{99}$$

Integrating Eq. (99) between $t = 0$ and $t = t_f$ leads to

$$-s(0) \geq \varepsilon_0 t_f \tag{100}$$

Therefore

$$t_f \leq \frac{-s(0)}{\varepsilon_0} \tag{101}$$

In general, from Eqs. (98) and (101), we can write

$$t_f \leq \frac{|s(0)|}{\varepsilon_0} \tag{102}$$

From Eqs. (45) and (48), we have

$$s(0) = \sigma e_{n+1}(0) = \hat{x}_{n+1}(0) - y_{n+1}(0) \tag{103}$$

Hence, proof is complete.

Remark 6. Using a conservative value of the bound Ω may lead to an unsuitable control effort. Therefore, from an engineering point of view, a controller that can be auto-adjusted seems interesting. If Ω is set to a large positive constant then, $\dot{\Omega} = 0$ and we will have

$$\dot{\beta} = q_2(|s| - \psi) \tag{104}$$

and

$$\psi(\beta) = \frac{\varepsilon_1}{2}(sign(\beta - \varepsilon_0) + 1) \geq 0 \tag{105}$$

In this case, the variable $\theta = \beta - \Omega$ can be considered as an estimator for the uncertainty bound Ω, which leads to $\dot{\theta} = \dot{\beta} - \dot{\Omega} = \dot{\beta} = q_2(|s| - \psi)$. Integrating Eq. (104) leads to $\beta(t) = \beta_0 + q_2 \int_0^t (|s(\tau)| - \psi)d\tau$. Moreover, to satisfy (62) we should choose $\beta_0 > \varepsilon_0 > 0$. In this case, Ω is only used to prove Theorems 2 and 3 and does not appear in the controller and Eqs. (104) and (105) are as an estimator of the bound Ω. Then, proposed approach will be applicable to practical systems where, this bound may not be known

This method has the following advantages:

1. In the case of a large initial distance from the sliding surface, the switching gain will increase quickly resulting in the distance to shrink. Once this distance is smaller than ε_1, the switching gain will decrease again.
2. Noise on the measurements does not disturb the adaptation procedure if the constant ε_1 is not chosen very small.
3. The adaptation law can increase $\beta(t)$ again according to Lemma 1 $\beta(t)$ will not converge to zero.

The following two remarks are true in the global sense of this paper.

Remark 7. In the proposed method the singular case, as discussed in Ref. [34], will not occur.

Remark 8. To preserve the invariance property during the reaching phase and to prevent peaking phenomena [35,36], one can use the following sliding surface (peaking phenomena can lead to instability of closed loop system [35], as the input signal of actuators are limited)

$$s(t) = \sigma e_{n+1}(t) - \int_0^t ([\sigma D(\tau) + K(\tau)]E(\tau) + v_s(\tau))d\tau - h(t) \tag{106}$$

where $h(t) = \sigma e_{n+1}(0)\, e^{-\alpha t}$ and $\alpha >> 0$. Notice that $s(0) = 0$. It is easy to prove that if this surface is zero then the error vector converges to zero.

6 Simulation Results

In this section, we apply the proposed controller to a model of induction servomotors in the form of Eq. (2). Consider a three-phase Y-connected, two-pole, 800 W, 60 Hz, and 120 V/5.4 A induction servomotor with the following model [20–22]:

$$J_t\ddot{\theta} + B_t\dot{\theta} + T_l = T_e \tag{107}$$

where J_t is the moment of inertia, B_t is the damping coefficient, θ is the rotor position, T_l represents the external load disturbance, and T_e denotes the electric torque. With the implementation of field-oriented control [21] and defining $x_1 = \theta$ and $x_2 = \dot{\theta}$, we can write system (107) in the form of servomotors

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_2 + bu(t) + cd(t) \end{cases} \tag{108}$$

where $u(t) = T_e/K_t$ is the torque current command (control input), $a = -B_t/J_t$, $b = K_t/J_t$, $c = -1/J_t$, $d(t) = T_l$, and K_t is the torque constant, $K_t = (3n_p/2)(L^2{}_m/L_r)i^*_{ds}$, with n_p being the number of pole pairs, L_m is the magnetizing inductance per phase, L_r is the rotor inductance per phase, and i^*_{ds} is the flux current command. From Eq. (31), one can write

$$\begin{cases} \hat{x}_3 = \hat{f}(x_1, x_2, u) \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = \hat{x}_3 \\ \dot{\hat{x}}_3 = \varphi + \gamma\dot{u} \end{cases} \tag{109}$$

where

$$x_3 = f(x_1, x_2, u) = ax_2 + bu(t) + cd(t) \tag{110}$$

and $\hat{x}_3$ is the neural identified of unknown variable x_3. We choose a RBFNN with three inputs (x_1, x_2, u), nine RBF neurons in the hidden layer as follows, which is depicted in Fig. 1

$$\xi_i(x_1, x_2, u) = \exp\left(-\left(\sqrt{x_1^2 + x_2^2 + u^2} - (5 - i)\right)/5\right) : i = 1, 2, \ldots, 9 \tag{111}$$

The RBFNN output is $\hat{f}(x_1, x_2, u)$. The neural network tuning parameters are chosen as $K_w = 5$, $K_x = 70$, and $K_e = 30$. Other parameters are chosen as

$$P = \begin{bmatrix} 400 & 300 \\ 400 & 500 \end{bmatrix}, \quad A_s = \begin{bmatrix} 0 & 1 \\ -9 & -8 \end{bmatrix} \tag{112}$$

The initial conditions of the weight vector are chosen as $w(0) = [0, 0, \ldots, 0]^T$. Notice that these initial conditions can be chosen arbitrary. The objective is to make the states of system (108) track the states of the following linear system

$$\frac{y}{u_d} = \frac{1}{S^2 + 3S + 5} \tag{113}$$

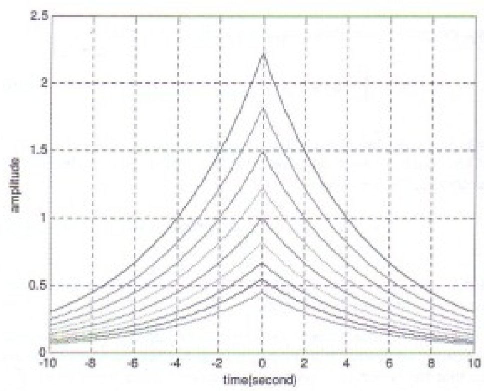

Fig. 1 Functions $\xi_i(t) = \exp\left(-\left(\sqrt{t^2} - (5 - i)\right)/5\right) : i = 1, 2, \ldots, 9$

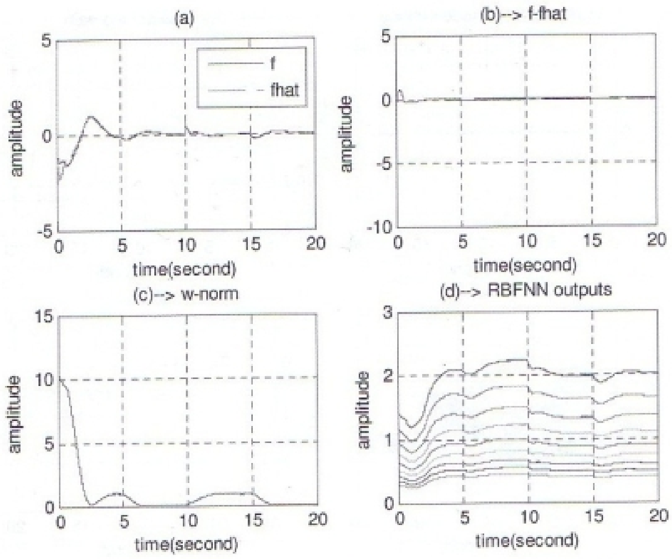

Fig. 2 *(a)* *f* **and its estimation** $\hat{f}$ **(output of RBFNN);** *(b)* **error between the** $\hat{f}$ **and its actual value** *f*; *(c)* **the norm of the weight vector;** *(d)* **the outputs of the each neuron in RBFNN**

Then, we can write

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -5y_1 - 3y_2 + u_d \end{cases} \tag{114}$$

or in the form of an augmented system

$$\begin{cases} y_3 = -5y_1 - 3y_2 + u_d \\ \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = -5y_2 - 3y_3 + \dot{u}_d \end{cases} \tag{115}$$

We assume Ω to be a large positive constant, i.e., $\dot{\Omega} = 0$. Moreover, u_d is a periodic step signal and

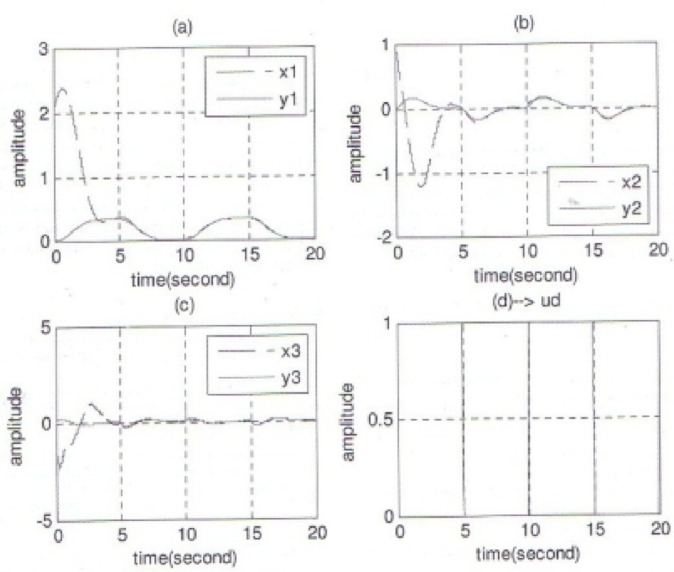

Fig. 3 **Reference signal tracking of augmented system:** *(a)* **first state;** *(b)* **second state;** *(c)* **third state;** *(d)* **input control signal of reference system**

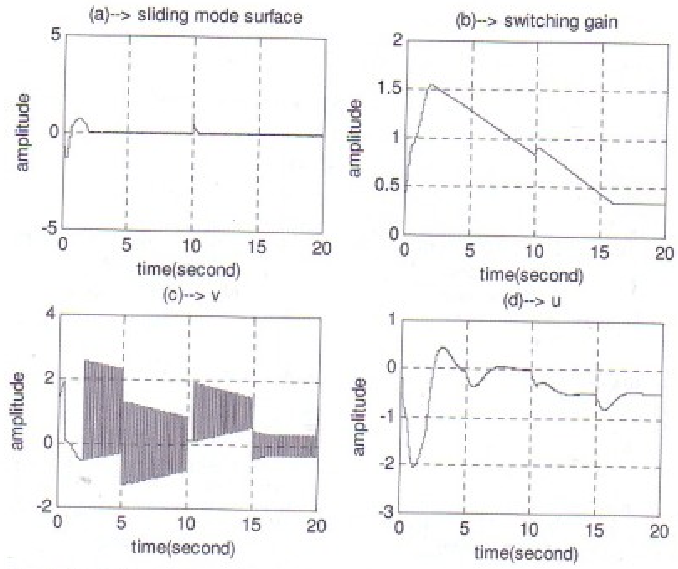

Fig. 4 **(*a*) Sliding surface; (*b*) switching gain; (*c*) input control signal of state feedback; (*d*) input control signal of system**

$$v_s = 0 \tag{116}$$

Since we have $D = [0, -5, -3]$, matrix M is chosen as $M = [-3, -5, -3]$ and, therefore, $K = [-3, 0, 0]$. According to Eq. (53)

$$\dot{e}_3 = ME \tag{117}$$

We use the following parameter values:

$$\begin{aligned} K_t &= 4.851 \times 10^{-3}\ \text{N.m/A} \\ J_t &= 4.78 \times 10^{-3}\ \text{N.m/s}^2 \\ B_t &= 5.34 \times 10^{-3}\ \text{N.m.s/rad} \end{aligned} \tag{118}$$

Note that these parameters are used in the system simulation not in the controller. Because we have assumed Ω to be a large positive constant, estimates of K_t, J_t, and B_t are not needed, i.e., the controller is model free. The initial values of the system states are assumed to be 2 and 1, respectively, i.e., $X(0) = [x_1(0), x_2(0)]^T = [2, 1]^T$. We simulate the proposed adaptive approach with the following parameters:

$$\beta(0) = 0.05, \quad q_1 = 1.30, \quad q_2 = 0.5, \quad \varepsilon_0 = 0.15, \quad \varepsilon_1 = 0.09 \tag{119}$$

The simulations are done by MATLAB with sample time of 0.001. The procedure for calculating u is as follows:

1. define and calculate $\xi(X, u)$ as Eq. (111)
2. calculate weight vector w from Eq. (16)
3. calculate $\hat{f}(X, u)$ from Eq. (29)
4. calculate of K and v_s via Eqs. (52) and (55), respectively (we can set $v_s = 0$)
5. calculate sliding surface using Eq. (45)
6. calculate β using Eqs. (65) and (66) or Eqs. (104) and (105)
7. calculate v via Eq. (61)
8. calculate $\dot{u}$ using Eq. (42)
9. calculate u by numerical integrating of $\dot{u}$

Figures (2)–(4) show the simulation results. From Fig. 4(*b*), we can see that the switching gain increases at first to force the error trajectories toward the sliding surface, but it decreases when the trajectories reach to the surface, while the input control of system is without switching (Fig. 4(*d*)). In simulation, we applied an external load disturbance $d(t) = 0.5$ N.m at $t \geq 10^s$. From Fig. 4(*b*), we can see that at $t = 10^s$ the switching gain increases to overcome on this disturbance and then starts to decreases again.

7 Conclusion

In this paper, a new method for designing dynamic sliding mode controller based on variable structure control technique is proposed for nonlinear systems. In the proposed method, use of an upper bound for uncertainty is not necessary for designing controller (see remark 6) but, if it is used the performance of the controller will be better. The proposed method also preserves all the main properties of SMC such as invariance and simplicity in implementation. The proposed method is applied to position control of induction servomotors. Simulation results show the effectiveness of this method.

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