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Multi-Projects Scheduling with Resource Constraints and Priority Rules and Fuzzy Activities

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Abstract:

In order to perform a project, all project managers are facing with high quality standards and benefiting from shortest period of time, possible resources and continuous fights. There are a lot of social and economic factors for a project manager to have an optimized benefit from limited accessible resources either from man power point of view or reserve of materials. Various tools and several techniques are introduced within recent decades for scheduling of projects, but most of them are involved with scheduling of single projects and least for multi-projects. We can model different Courses of Action (COAs) evaluated during the operational military planning as many different activity networks. The corresponding project scheduling model is sufficiently general to be able to represent the most part of military missions. Recently, a project scheduling mathematical model has been proposed where each activity called an action in the military context has different execution modes depending on the resource combination selected. This paper is a combination of Simulated Annealing Algorithm and the best Priority rules for solving the problem of scheduling in multi projects with limited resources that has been tested with numerical examples. The numerical tests make it clear that proposed method is better than multi-applicable rules of scheduling.

Keywords: RCMPSP, scheduling, simulated annealing algorithm, priority rules

1. Introduction

Today there is an ever-increasing in benefit from project management for obtaining most organizational goals. Project management is used for finding some exceptional results by applying limited resources and within sensitive time limitations. Project scheduling is a base for supervision and controlling of project activities and includes in major tools for project management. Most of current scheduling methods of project are applicable in scheduling of all activities of a single project with little submission of any methods or new method for scheduling of multi-projects activities.

The planning process is critical to the success of any mission. The planning process has six steps. Each step of this process begins with an input from the previous one and builds upon. Nevertheless, this process is iterative and recursive. The initiation step commences with the reception of the mission statements or in an anticipation of a new mission. During this step, the task is assigned or assumed, major combat and logistic resources and strategic transportation assets are identified for planning purposes, the intelligence process initiated, and the groundwork is laid for planning to begin. As soon as the new mission is received, the staff prepare for the mission analysis by gathering a set of tools (e.g. maps of area of operations, both own and higher headquarters' standing operating procedures (SOPs), appropriate documents, estimates). Moreover, during this step, the staff issues a warning order to other supporting and subordinate units. The orientation step is crucial to the CF OPP. It allows the Commander to

begin the analysis and definition of the mission, the preparation of the planning guidance and the description of the end states of the operation. The orientation step includes the analysis of the government orders, initial intelligence, assessment of specified, implied and essential tasks, review of the available assets, estimation of the constraints, identification of the critical facts and assumptions, risk assessment, Commander's critical information requirements, initial reconnaissance, mission analysis briefing, development of initial Commander's intent, and issue of Commander's guidance. The Course of Action (COA) development step involves the entire staff. The Commander's guidance and intent helps the staff to focus on the development of comprehensive and flexible plans within the time available. These COAs "should answer fundamental questions of when, who, what, where, why and how". Each COA should be suitable, feasible, acceptable, exclusive and complete. During the COA development step, staff should analyze the relative combat power (friendly possible actions, enemy's perspective, enemy's vulnerabilities and powers, additional resources, resources allocation, etc.), generate comprehensive COAs (defeat all feasible enemy's COAs, brainstorming, decisive point, cross-fertilization of COAs), and determine initial forces necessary to accomplish the mission. The decision step is based on the analysis and comparison of the proposed COAs. The analysis of the COAs provides the Commander with precious information to evaluate the quality of these COAs. The main approaches used to analyze the COAs are war-gaming, advantages / disadvantages and comparison criteria. The COA comparison highlights each COA advantages and disadvantages with respects to each other. The COS



decides which one he will recommend to the Commander during the Commander's Decision Brief. The COS will decide what detail is necessary to ensure that the Commander is provided with adequate information to make a decision. COA approval consists of a choice of the best COA according to the Commander's beliefs and estimates. If the Commander rejects all the proposed COAs, then the staff should start the process over again. While the Commander chooses a COA, he may refine his intent, guidance and priorities for execution planning. By deciding on a COA, the Commander assesses what residual risk is acceptable. Based on the Commander's decision and final guidance, the staff refines the COA, complete the planning process and issue orders. The aim of the plan development step is to provide a detailed plan or orders to subordinate headquarters based on the Commander's decision. The plan should go through review and analysis processes. Orders and plans provide all necessary information to subordinates, allies and supporting units to initiate planning or execution of different operations. Finally, the Commander reviews and approves orders before any dissemination. Scheduling and allocation of resources of multi projects is somehow more difficult than single projects. There is a considerable increase in time limits for calculations in scheduling of multi projects or when there is a great scheduling problem. Therefore, any benefiting from optimization traditional methods is not effective. Even if it is applicable it would waste a lot of time. As a result the researchers go towards heuristic and meta-heuristic methods. Right now, there are a lot of software packages for solving any scheduling problems of multi-projects. But this software has different violations in providing of optimized answer against the major optimized reply. Also the researchers intend to find more efficient algorithms with more optimized answers. There are different studies which may involve the researchers such as increasing the efficiency of heuristic methods, extending the scope of solvable problem with heuristic methods, presenting new combined methods with higher calculation efficiency. The presented combined method of simulated annealing algorithm and famous resource-constrained multi-project scheduling problem (RCPSP) rules in this paper is a new method for solving any problems of time scheduling. The proposed method has been tested with numerical examples along with comparing the results with famous rules of time scheduling. The obtained results make the priority of proposed method clear than famous scheduling rules in all

types of problems. Different types of scheduling techniques and project scheduling have been evaluated within recent two decades and under limited resources condition (Diranloo, 2006) We have used two methods for solving the problem of scheduling as follows: heuristic methods and Meta heuristic methods. All performed researches up to now reveal that there is not an applicable method for solving of great and average problems. Therefore it is possible to consider the present research a new one accordingly.

2. Project modeling of a COA

The execution of a mission requires the realization of certain activities executed by actors and using specific resources, according to some order and in a given place. The mission can be broken down to its elementary tasks. Assigning resources to each activity and deciding the order in which those activities are to be executed (when) and where constitutes a COA. Following the project concepts, at point where one obtains a set of primary objectives. To devise one COA as any project, one proceeds with the project technique known as work breakdown structure. This technique is illustrated in COA activities. The followed process results into the object of each activity, the other activities and the resources required for its complete execution resources allocated to the execution of an activity defines its execution also implies the identification and the obtainment of the pool of Preparation and evaluation of different COAs during mission planning is one of the main Commander's roles. As the Commander may design different COAs to fulfill his mission, one gets as many project instances by modifying the nature of activities, each COA creating new activities, by removing others or by changing their technological orders. This set of COAs represents as many as project alternatives A_i any COA can be seen as a set of [al., 2000]. Project modeling consists in breaking down generic activities into sub i . It decomposes and organizes a military mission into primitive actions, Figure poses Figure 2 - Set of COA alternatives to fulfill a mission interrelated activities known as a project, cf. primitive activities interrelated to accomplish 1 - Work Breakdown Structure A_i , as illustrated in Figure available resources. activities up to the Figure precedence execution. Each combination of resources allocated to the execution of an activity defines its execution mode. The planning process also implies the identification and the obtainment of the pool of available resources.

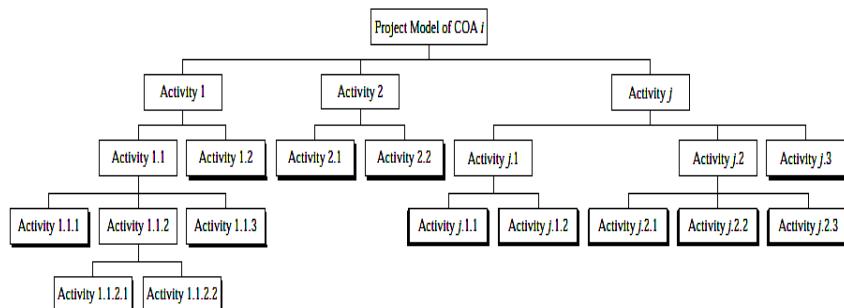




Figure 1 - Work Break-down Structure

Preparation and evaluation of different COAs during mission planning is one of the main Commander's roles. As the Commander may design different COAs to fulfill his mission, one gets as many project instances by modifying the nature of activities, each COA being

obtained by creating new activities, by removing others or by changing their technological orders. This set of COAs represents as many as project alternatives A_i , as illustrated in Figure 2.

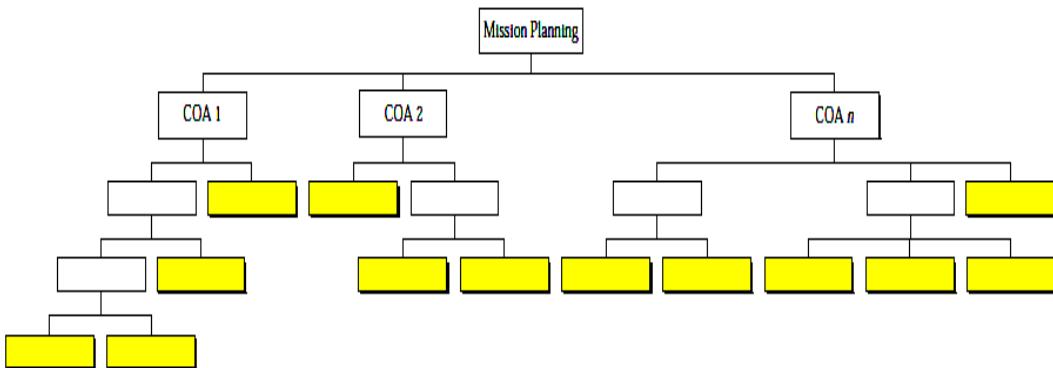


Figure 2 - Set of COA alternatives to fulfill a mission

The Commander has to select the most promising alternative with respect to available resources and to mission objectives. It is a complex and demanding task, which implies the analysis and the processing of multiple information regarding the operational zone, the capabilities of the task force and the available resources. Without appropriate tools, the analysis of different alternatives and the selection of the best COA considering mission objectives become a painful and hazardous process. This decision problem can be symbolically represented by a decision matrix shown in Figure 3. In this type of problem, the Commander is considering a collection of predetermined alternatives $A = \{A_1, A_2, \dots, A_n\}$, designated as COA, from which he must select the "best" one. Associated to these alternatives is a set of criteria $C = \{C_1, C_2, \dots, C_v\}$. Then the values c_{ij} represent the payoff obtained by applying COA A_i evaluated according to criterion C_j . As in many problems meet in our daily lives, all decision problems have multiple and generally conflicting criteria.

	C_1	C_j	C_v
A_1			
A_i		c_{ij}	
A_n			

Figure 3 - Decision matrix

Familiar success criteria of the project are defined to appreciate objective accomplishment, such as the project duration and the project cost. Other criteria intervene when

selecting a specific project, the most notable in a combat situation being the risk and the impact. From a practical point of view, one can consider the problem as a multi-attribute decision making when all the alternatives are known and predetermined. One can consider the problem as a multi objective problem when alternatives have not been predetermined a priori.

Multiple Mode Resource-Constrained Project Scheduling Problem of a COA The Multiple Mode Resource-Constrained Project Scheduling Problem (MRCPSP) modeling a COA was introduced by [Guitouni et al., 2000], see also [Guitouni et al., 2002]. During the activity engineering, the Commander may specify for each activity different execution modes where a mode corresponds to a specific combination of resources and a given activity duration. Again, these activities grouped according to a temporal and logical sequence define an activity network with multiple modes. Project instances are obtained by changing the resource combinations of activities and the corresponding activity durations. These project instances constitute variants of the same COA i that must be evaluated. The evaluation process of the set of alternatives of one COA with multiple modes is the object of the MRCPSP. Project scheduling with fuzzy activity durations

Project scheduling refers to the process of defining the best sequence and the starting times of activities according to the objectives pursued. It results in a project plan defining the time t at which each activity should be accomplished with the selected resource combination. Resource constrained project scheduling problems are NP-hard problems and they can't be solved to optimality in



polynomial time. Considering a fuzzy activity duration with each execution mode constitutes a new problem that is more complex to solve than the one where activities have deterministic durations. This paper introduces a fuzzy Multiple Mode Resource-Constrained Project Scheduling model to evaluate a COA having activities with multiple execution modes and/or fuzzy activity durations. It describes the mathematical foundation developed to perform the project network analysis and it proposes a scheduling procedure to determine the fuzzy project duration. Section two introduces the uncertainty modeling considered and the required mathematical operations to apply the CPM with fuzzy time parameters. Section three describes the mathematical model of the scheduling problem. Section four presents the fuzzy priority rule heuristics proposed to evaluate the project duration corresponding to the completion duration of the considered mission. These fuzzy priority rule heuristics are deduced from the scheduling theory in project scheduling. The final section gives conclusions about the advantages of modeling a military mission with uncertainty and future research works. Though fuzzy numbers can take various shapes, triangular and trapezoidal fuzzy numbers are the most common in fuzzy scheduling literature. Triangular fuzzy numbers are represented by a triplet (a, m, b) and trapezoidal fuzzy numbers are represented by a quadruple $(a, m, \underline{m}, \bar{m})$ where a and b are the lower and the upper bounds of the left-hand and right-hand spreads, while the parameters $m, \underline{m}, \bar{m}$ are the lower and upper modal values, respectively. And, as a generalization, a fuzzy triangular number (FTN) can be viewed as a special case of the fuzzy trapezoidal number (FTrN) for which the lower and the upper modal values are equal (e.g. $m = \bar{m}$). Thereafter, we use the L-R type representation of fuzzy numbers of [Dubois and Prade, 1988], denoted by $(a, m, \underline{m}, \bar{m})_{LR}$. A fuzzy number M is a normalized convex fuzzy subset of the real line $\mathbb{A} : M = \{x, m_M(x) | x \in \mathbb{A}\}$ where $m_M(x)$ is the membership function taking values within $[0,1]$ indicating the degree of appurtenance of x to M can be expressed by means of two functions L and R , with four parameters $(m, \underline{m}, \bar{m}) \in \mathbb{A}^2$ and a, b in the form:

Note that, as a fuzzy interval, an ordinary real number t is written $(0, t, t, 0)$. We can also characterize a FTrN by the interval of confidence at a level a . This is a useful concept to describe different groups of possible values by applying a level cut to fuzzy subset. Let a $\hat{I}[0,1]$. The a -level of set M is the set defined by :

As a practical way of getting suitable membership functions of fuzzy activity durations, it is proposed that the Commander, acting as an expert, specifies the prominent membership levels, see [Rommelfanger, 1990], e.g.. $m_M(x) = 1$ means that the Commander believes that the value x certainly belongs to the subset of admitted values $[\underline{m}, \bar{m}]$. $m_M(x) > 1$ means that the Commander believes that the value x has a good possibility to belong to the subset of possible values. $m_M(x) < 1$ means that the Commander believes that the value x has a little possibility to belong to the subset of possible values. A general graphical representation is shown in Figure 4.

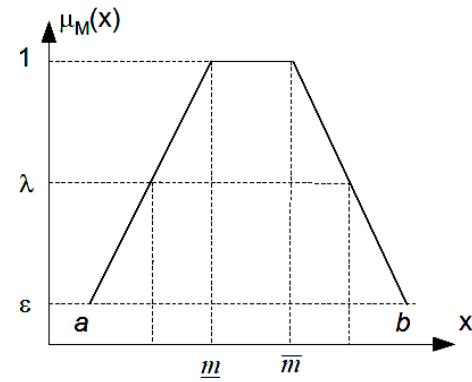


Figure 4 – Linear pieces fuzzy number

Computation with fuzzy quantities

In order to apply scheduling procedures processing fuzzy numbers representing time, one must first determine how to establish and how to compare the sequences of activities to be considered according to their fuzzy durations. Required operations are the addition, the subtraction, the division, the multiplication, the extended minimum, the extended maximum and the comparison. A major advantage of trapezoidal fuzzy numbers is that many operations based on the max-min convolution can be replaced by direct arithmetic operations, cf. [Dubois and Prade, 1988]. Addition of fuzzy numbers

Addition operation on two FTrN gives a FTrN.

$$\begin{aligned} M_1 (+) M_2 \\ = (a_1, \underline{m}_1, \bar{m}_1, b_1) (+) (a_2, \underline{m}_2, \bar{m}_2, b_2) \\ = (a_1 + a_2, \underline{m}_1 + \underline{m}_2, \bar{m}_1 + \bar{m}_2, b_1 + b_2) \end{aligned}$$

Subtraction of fuzzy numbers

$$\begin{aligned} M_1 (-) M_2 \\ = (a_1, \underline{m}_1, \bar{m}_1, b_1) (-) (a_2, \underline{m}_2, \bar{m}_2, b_2) \\ = (a_1 + b_2, \underline{m}_1 - \underline{m}_2, \bar{m}_1 - \bar{m}_2, a_2 + b_1) \end{aligned}$$

Symmetric

Symmetric of a FTrN is defined as:

$$(-)M_1 = (-a_1, \underline{m}_1, \bar{m}_1, -b_1)$$

Maximum and minimum operators

These operators are defined as follows:

$$\max(M_1, M_2) = (\max(a_1, a_2), \max(\underline{m}_1, \underline{m}_2), \max$$

$$(\bar{m}_1, \bar{m}_2), \max(b_1, b_2))$$

$$\min(M_1, M_2) = (\min(a_1, a_2), \min(\underline{m}_1, \underline{m}_2), \min$$

$$(\bar{m}_1, \bar{m}_2), \min(b_1, b_2))$$

Comparison of fuzzy numbers

We consider the comparison of fuzzy numbers proposed by [Roubens, 1990] in the particular case of L-R fuzzy numbers. It is based on the compensation of areas and it is reduced to the comparison of upper and lower bounds of a



cuts defined by the following proposition.

Proposition. Let M_1 and M_2 be L-R fuzzy numbers with parameters $(-a_1, \underline{m}_1, \bar{m}_1, -b_1)$, $(a_2, \underline{m}_2, \bar{m}_2, b_2)$, and reference functions (L_1, R_1) , (L_2, R_2) , then M_2 iff

$$\text{Sup } I(M_1, A_{M_1-R}) + \text{Inf}_{x \in R} I(M_1, a_{M_1,L}) \geq \text{sup}_{x \in R} I(M_2, a_{M_2,R}) + \text{Inf}_{x \in R} I(M_2, a_{M_2,L})$$

Where, if $n=1,2$.

$$a M_{n,R} =$$

$$R_n(\int_0^1 R_n^{-1}(a)da) \text{ and } a M_{a,L}(\int_0^1 L_n^{-1}(a)da).$$

Then, in the case of trapezoidal fuzzy numbers, one obtains:

$$\begin{aligned} M_1 \geq M_2 \text{ iff } & \underline{m}_1 + \bar{m}_1 + \frac{1}{2}(b_1 - a_1) \\ & \geq \underline{m}_2 + \bar{m}_2 + \frac{1}{2}(b_2 - a_2) \end{aligned}$$

Critical path analysis with fuzzy activity duration

Previous work on network scheduling based on fuzzy set theory, provides methods for determining the expected fuzzy early times of each event [Chanas and Kamburowski, 1981], [McCaughan, 1987], [Dubois and Prade, 1988]. Most of the priority rule heuristics rely on PERT/CPM (Critical Path Method) computation. One of the first attempts to apply the calculation of the PERT analysis with fuzzy duration estimates is by [Dubois and Prade, 1988]. These methods, however, do not support backward pass calculations in a direct manner similar to the one used in the forward pass. This is mainly due to the fact that fuzzy subtraction is not the inverse of fuzzy addition, cf. [Gazdik, 1983], [McCaughan, 1993], [Nasution, 1994]. According to the scheduling literature, a project is represented by a directed acyclic activity network $G(N, P)$ where N is the set of activities j , $j = 1, \dots, J$, and P is the set of precedence relations network $G(N, P)$ where N is the set of activities j , $j = 1, \dots, J$, and P is the set of precedence relations between activities. A primitive activity j is designated by a name. For each activity j , one denotes by d_{jm} the activity duration corresponding to the resource combination of activity j executed under mode $m, m = 1, \dots, m_j$. Without loss of generality, execution mode numbers are ordered according to the increasing activity duration.

PERT calculation proposed by [Dubois and Prade, 1988] is adapted below to an activity-on-node representation and the network analysis method has to handle activity having multiple execution modes with uncertain duration. Denote by t_0 the time origin of the planning process of the activity network and denote by T^V the latest finish time of successors of activity j . The numbering of activity nodes according to the topological order is required to guarantee that $A_i \in P_j, i < j$. To calculate the earliest and latest starting

times of the activities, we proceed as follows:

The earliest starting time of an activity j , noted T_j^{es} , is given by the forward algorithm:

$$T_j^{es} = \begin{cases} \max\{T_j^{ss} + \tilde{d}_{j1} \mid i \in P_{j0}\} \text{ if } P_j \neq \emptyset \\ 0 \text{ if } P_j = \emptyset \end{cases}$$

$$\forall j \in N \quad (1)$$

Then, one obtains the earliest finish time of the project

$$T_j^{ss} = \max\{T_j^{es} \mid j \in N\}. \text{ Where } T_j^{ss} = T_j^{es} + \tilde{d}_{j1}.$$

It corresponds to the critical path length of the project when any resource constraint applies. Denoting by T^V the latest finish time of the project. The latest finish time of activity is obtained by the following backward algorithm.

$$T_j^{ls} = \begin{cases} \min\{T_j^{ls} \mid T_j^{ls} + \tilde{d}_{j1} = T_j^{ss}\} \text{ if } S_1 \neq \emptyset \\ T_j^{ss} \text{ if } S_1 = \emptyset \end{cases}$$

$$\forall j \in N \quad (1)$$

The interval $[T_j^{es}, T_j^{ls}]$ represents the slack time of activity j noted \tilde{s}_j . And it is obtained by setting

$$\tilde{s}_j = T_j^{ls} - T_j^{es}.$$

But the criticality of activity j becomes more or less uncertain according to how the fuzzy intervals $[T_j^{es} - T_j^{ls}]$ overlap. Within the CPM. An activity is considered critical when the interval between the earliest finish time T_j^{es} and the latest finish time T_j^{ls} of an activity j is null. This is meaningless with imprecise durations. For this reason, [Dubois and prade, 1988] propose to define the latest finish time of the project T^f independently in an imprecise than their successors, cf. [Lorterapong, 1995]. When the activities have imprecise durations, represented by fuzzy intervals, the traditional CPM algorithm is still good, if the operations of addition, subtraction, maximization and minimization are replaced by their extensions to fuzzy arguments.

So, representing the trapezoidal number T_j^{ls} by $(a_j^{ls}, \underline{m}_j^{ls}, \bar{m}_j^{ls}, b_j^{ls})$, the trapezoidal number of the fuzzy duration \tilde{d}_{j1} of activity executed under mode 1 (the shortest mode must be selected for CPM calculation) by $(a_{j1}^d, \underline{m}_{j1}^d, \bar{m}_{j1}^d, b_{j1}^d)$ and applying the fuzzy subtraction operation, one obtains:

$$t_j^{ls} = (a_j^{lf} - a_{j1}^d, \underline{m}_{j1}^{lf}, \bar{m}_j^{lf} - \underline{m}_{j1}^d, b_{j1}^d)$$

$$(3)$$

This time is valid only if the differences in (3) are non-negative, and

$$\bar{m}_j^{lf} - \underline{m}_{j1}^d \geq \underline{m}_{j1}^{lf} - \underline{m}_{j1}^d \quad (4)$$

Usually, these conditions are more often satisfied for those activities at the end of the project network but not for those



activities at the beginning of the project network. In general, the equation $\tilde{t}_j^{ls} + \tilde{d}_{j1} = \tilde{t}_j^{lf}$ in (2) must be approximate by the fuzzy equation $\tilde{t}_j^{ls} + \tilde{d}_{j1} \equiv \tilde{t}_j^{ls}$ with the additional restriction that $\tilde{t}_j^{ls} + \tilde{d}_{j1}$ does not exceed \tilde{d}_{j1} , cf. [Lorterapong, 1995]. [Ramik and Rommelfanger, 1995]. Then equation of the form (3) must be used when (4) is fulfilled where the spreads are given by:

$$\begin{aligned}\tilde{t}_j^{ls} &= (\max(0, a_j^{lf} - b_{j1}^d), \underline{m}_j^{lf}, \bar{m}_{j1}^d - \\ &\quad \bar{m}_t^{lf} - \max(0, b_{j1}^d - a_j^{lf}))\end{aligned}\quad (5)$$

in the case

$$\underline{m}_t^{lf}, \bar{m}_{j1}^d < \underline{m}_t^{lf} - \bar{m}_{j1}^d \quad (6)$$

We adopt the approximation proposed by [Ramik and Rommelfanger, 1995]. Equation (3) is then evaluated by the following formula:

$$\begin{aligned}\tilde{t}_j^{ls} &= \left(\max(0, a_j^{lf} - a_{j1}^d + \left(\bar{m}_t^{lf} - \underline{m}_j^{lf} - \bar{m}_{j1}^d \right) \right. \\ &\quad \left. - \left(\underline{m}_j^{lf} - \bar{m}_{j1}^d \right) \right). \\ \bar{m}_j^{lf} - \bar{m}_{j1}^d, \bar{m}_{j1}^d - \max(0, b_{j1}^d - a_j^{lf})\end{aligned}\quad (7)$$

When condition (6) prevails and additionally $b_j^{lf} - b_{j1}^d < 0$, then the extended sum $\tilde{t}_j^{ls} + \tilde{d}_{j1}$ on all membership levels a smaller than 1, if \tilde{t}_j^{ls} is calculated according to (7). If we want to avoid this optimism. [Ramik and Rommelfanger. 1995] propose the following formula to calculate the latest starting times:

$$\tilde{t}_j^{ls} = (a_j^{lf}, \underline{m}_j^{ls}, \bar{m}_j^{ls}, b_j^{lf}) \quad (8)$$

Where

$$\begin{aligned}a_j^{ls} &= \max(0, a_j^{lf} - a_{j1}^d) - \max(0, (\underline{m}_j^{lf} - \\ &\quad \bar{m}_{j1}^d) - (\bar{m}_j^{lf} - \bar{m}_{j1}^d)) - \max(0, b_{j1}^d - b_j^{lf})\end{aligned}$$

$$\begin{aligned}\underline{m}_j^{lf} &= \min(\underline{m}_j^{lf} - \bar{m}_{j1}^d, \bar{m}_j^{lf} - \bar{m}_{j1}^d) - \\ &\quad \max(0, b_{j1}^d - b_j^{ls})\end{aligned}\quad (10)$$

$$\underline{m}_j^{lf} = \underline{m}_j^{lf} - \bar{m}_{j1}^d, \max(0, b_{j1}^d - b_j^{lf}) \quad (11)$$

$$b_j^{lf} = \max(0, b_j^{lf} - b_{j1}^d) \quad (12)$$

Note that the processing of availability time and due-date of each activity j can be introduced within the previous algorithms (1) and (2).

3. Fuzzy Multimode Resource-Constrained Project Scheduling Problem

Suppose the project (e.g.a COA) may require a set R to K renewable resources where each resource type k $\in R$ has a

variable resource availability over the time horizon, denoted by Q_{kt} . Defining the zero-one decision variables x_{jmr} equal to one of the activity j executed according to the mode m is completed at the end of period t and zero otherwise, then figure 5 formulates by zero-one programming the fuzzy Multimode Resource-Constrained Project scheduling problem (FMRCPSP) with fuzzy activity durations. The corresponding notation is given in table 1.

The objective function (13) minimizes the project duration. This is obtains by scheduling the last activity J as soon as possible. Constraint set (15) specifies the precedence relation between activities. Availability of renewable resources is verified by constraint set (16). The sum of resources of each type k required by activities executed at period t must not exceed the amount available Q_{kt} . Constraint set (17) specifies the binary values of decision variables x_{jmr} .

The solution of the FMRCPSP is a schedule determining the finish time noted F_j , and the selected mode m of each project activity j. The problem can be solved by one of the fuzzy zero-one programming methods. However, because of the computational complexity of the problem in real-world mission, these exact methods are restricted to small scheduling problems. So one adopts approximate method for solving these problems. Approximate methods subdivide mainly between truncated exact methods, construction methods and improvement methods like Tabu search and simulated annealing. Up to now, most of the researches have been dedicated to construction methods based on priority rule heuristics because of their low computational complexity, cf [Boctor, 1990], [Alvarez-Valdes, 1989], [Kolisch, 1995].

4. fuzzy project scheduling procedures

4.1 scheduling schemes

Construction method are characterized according to a scheduling scheme often referred as the serial and the parallel approaches. The serial approach ranks all the project activities according to their priority rule and it schedules each eligible activity j at a starting time, noted \tilde{S}_j , where required resources are available. The finish time

\tilde{F}_j of each activity j is then determined by the relation:

$$\tilde{F}_j = \tilde{S}_j + \tilde{d}_{jm_j^s}, \forall j$$

Here, an activity is considered eligible as soon as all its



predecessors have been scheduled. The starting time $\tilde{s}_j \geq \tilde{F}_j$, $\forall j \in P_j$. Within the parallel method, eligible activities considered at time \tilde{t} according to their respective priorities, but if an activity having a higher priority can't be scheduled due to resources unavailability. The next prioritized activity is considered. If no more activity can be scheduled, the time t is advanced to the next period where an activity in progress will terminate and release resources or where there is enough available resources to schedule an eligible activity. Within the serial approach, the activity priorities remain the same during the whole scheduling process while, within the parallel approach, eligible activity priorities are reevaluated at the beginning of each stage.

4.2 formulation of fuzzy priority rules

A lot of studies have been conducted to determine the relative efficiency of the priority rules used within construction methods mainly for the single mode RCPS, see [Alvarez-Valdes, 1989]. [Boctor, 1990], [Davis and Patterson, 1975]. [Kolisch, 1995]. For the multiple mode case, fewer.

results are available. These researches try to identify the best priority rule according to the privileged objective function. Priority rule definitions depend on the scheduling scheme retained. And, depending on the priority rule calculation method, activity priorities may remain the same even if the priority rule is applied within the parallel approach. But then, the resulting quality with the application of parallel approach may be less less. [Boctor, 1993] studied different priority rule combinations for the MRCPS. Selection of activities according to priority rules where experimented with three mode selection rules: the shortest feasible mode (SFM), the least criticality ratio or least critical resource (LCR) and the least resource proportion (LRP). Now, we define activity selection priority rules on the basis of traditional ones and we applied them within the construction methods considering fuzzy times. They are presented in Table 2. In these definitions, an immediate candidate is an activity j which is schedulable if activity j' is scheduled to start at the considered period while the remaining work is defined as the sum of the shortest possible durations of the activity j considered and all its successors (figure 6).

Priority name	Priority symbol	Priority Value
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Minimum SLack(SLK) time	MIN SLK	MIN SLK -
LATEST Finish Time (LFT)	MIN LFT	min LFT -
Maximum number of immediate successors	MAX NIS	MAX
Maximum remaining work	MAX RWK	MAX RWK -
Maximum processing time	MAX PTM	MAX
Minimum processing time	MIN PTM	MIN
Maximum number of immediate candidate	MAX CAN	MA CAN

5. Hybrid Genetic Algorithm (HGA)

The Hybrid Genetic Algorithm (HGA) is used as a globally search technique which is same as simple genetic algorithm with a little deviation from the generation of initial solution. In HGA, some heuristics help to generate initial feasible solution and then the procedure of simple genetic algorithm is used by the population according to population size. HGA can be describing as follows:

Step1: initialization and evaluation

- Generation of initial sequence with special heuristics (SH) called as one of the chromosome of population is the first step in this algorithm.
- Sequences are generated randomly as per population size (Ps)
- Initial sequences combination is obtained by special heuristics with the sequence that was generated randomly in order to form equal population size sequence.

Step2: Reproduction

Then, a set of new populations are created through the algorithm. At each generation, the individuals in this current generation are used by the algorithm to let the next generation to be generated. In this process, the following steps are performed by the algorithm:

- Fitness computation let us to score each member of the current population.
- Parents are selected based on the fitness function.
- The best fitness individuals in the current population are used as elite which are utilized in the next population.
- Offspring is produced by crossing over from the pair of parents or a single parent is changing randomly (mutation).
- The current population with children are replaced to form the next generation.

Step 3: stopping limit. Stopping condition is used to terminate the algorithm for certain numbers of generations. Diagram of HGA comes in figure 5 .

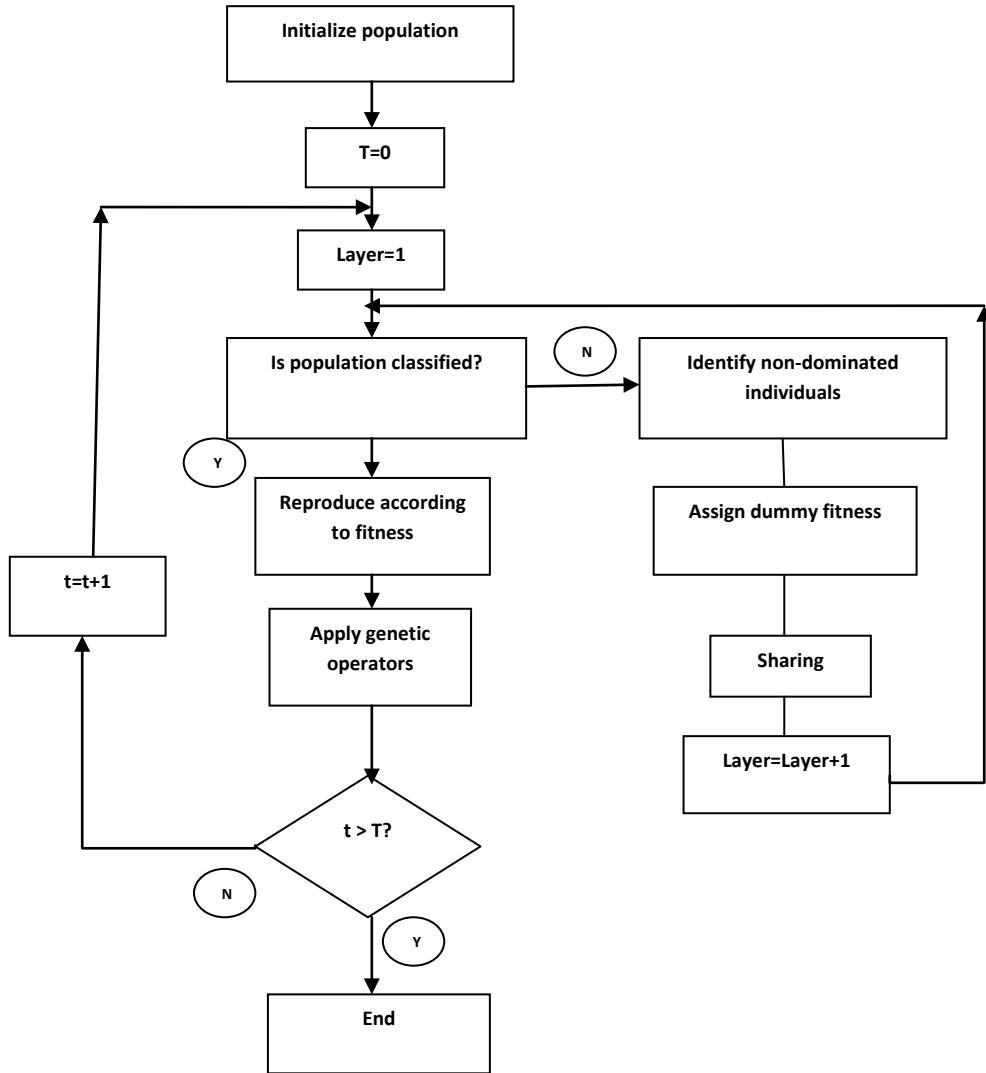


Figure 5 the used HGA diagram for mentioned problem

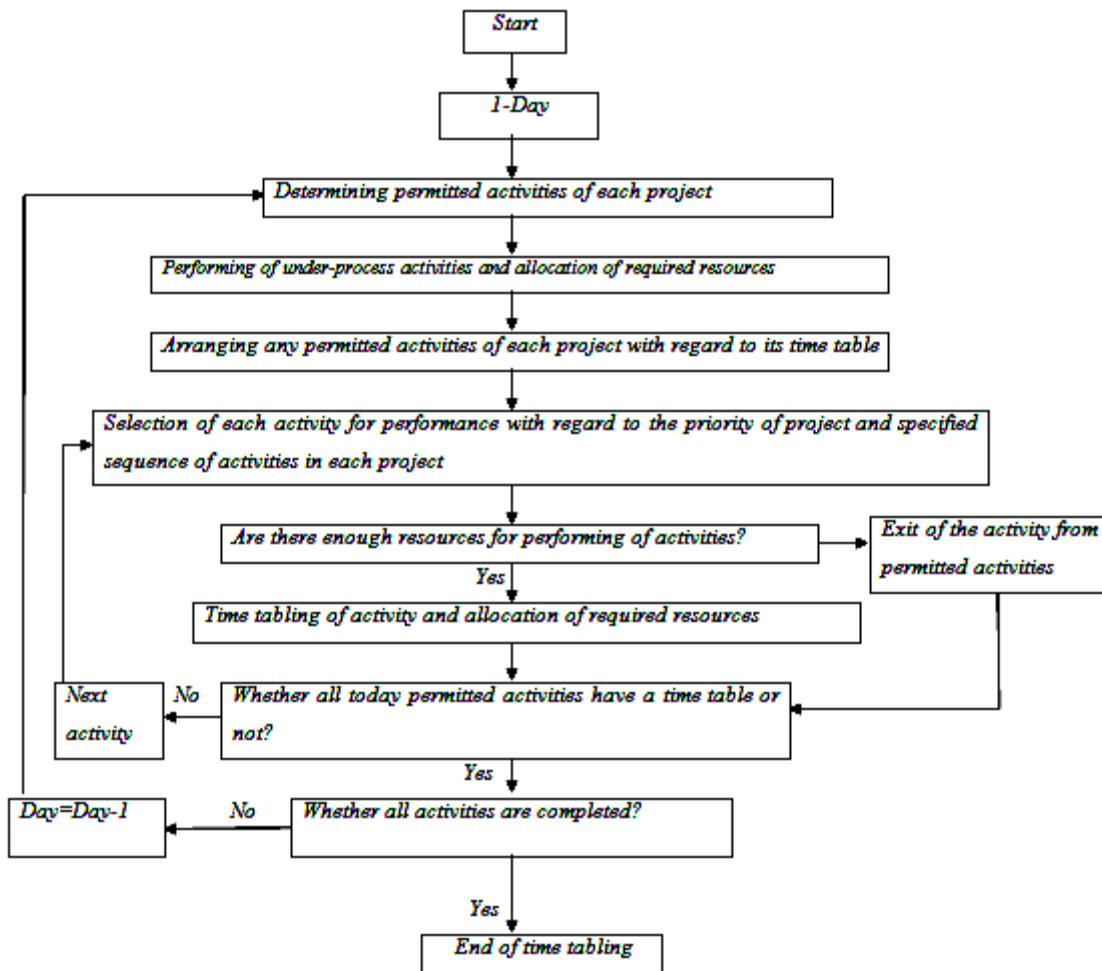


Figure (6): Project Scheduling procedure

In this section, 12 experimental issues of optimization Completion time of Multi-projects scheduling with resource constraints and Priority rules in different dimensions that produced randomly, were done by HGA. For programming the HGA is used MATLAB 7.5. And for running the algorithm, is used in a PC with 3.2 PIV, 2 GB RAM. The results of experiments come in Table



Table3: the result of experiments by use of simulated annealing in different dimensions.

Problem name	Number of projects	Number of activities	Computational time	Best answer
Vmd1	5	8	0:00:27	12days
Vmd2	5	20	0:00:41	16days
Vmd3	10	10	0:00:52	20days
Vmd4	10	20	0:01:28	22days
Vmd5	30	30	0:02:49	59days
Vmd6	30	50	0:03:36	63days
Vmd7	40	50	0:04:02	70days
Vmd8	40	60	0:04:16	138days
Vmd9	100	125	0:04:11	151days
Vmd10	100	150	0:04:01	242days
Vmd11	150	200	0:04:06	341days
Vmd12	200	250	0:04:22	515days

6. Parameter setting

In this section, the results of the computational experiments are used to evaluate the performance of the proposed algorithm for optimization of completion time of multi-projects scheduling with resource constraints and priority rules. There are 12 instances for each problem size. At this point, some information about parameter analysis would be useful. Initially, several experiments were conducted on test problems in order to determine the tendency for the values of parameters. Six test problems were used for this purpose.

In each step, only one of the parameters was tested. Each test was repeated four times. We considered the following values for the several parameters required by the proposed HGA:

Crossover probability (pc): four levels (0.90, 0.85, 0.80 and 0.75).

Mutation probability (pm): four levels (0.02, 0.04, 0.06 and 0.08).

Number of initial population (np): three levels (300, 200 and 100).

Number of generation (ng): one level (200).

Test results showed that these values were suitable for the problem. Later, additional tests were conducted in order to determine the best values. After completing the tests, Taguchi analysis is applied for the different values of parameters. The best values of the computational experiments for Two Metaheuristic Algorithms for Two-Echelon Location-Routing Problem were obtained for pc = 0.85, pm = 0.06,

np = 100 and ng = 200. These values were set as the default value of the Parameters.

7. Conclusions

In this paper, we recall the approach used to model a COA according to the project decomposition method. A COA having multiple execution modes and fuzzy activity durations is then modeled as a Fuzzy Multiple Mode Resource-Constrained Project Scheduling Problem. To take into account the uncertainty of activity durations encountered during mission execution, these activity durations are represented by trapezoidal numbers of L-R-type. The Commander can define different multiple mode COAs and the preferred COA can then be selected according to a multicriteria

method. To evaluate the duration of a given multiple mode COA, a fuzzy parallel construction method based on the parallel scheme and the SFM heuristic rule as activity selection priority rule is proposed. The aim of the fuzzy parallel scheduling procedure proposed is to evaluate the efficiency of different resource combinations to execute a specific COA. A statistical

experiment must be designed to assess the performance of the suggested fuzzy parallel method and the priority rules described. The overall valuation methodology can be integrated into an interactive decision support system where the Commander can select one COA on the basis of selected performance criteria and his preferences. The proposed fuzzy multiple mode resourceconstrained project scheduling model may be extended to consider fuzzy multiple objective functions simultaneously.

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