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VIBRATION ANALYSIS OF NANO-FLUID CONVEYING CARBON NANOTUBES EMBEDDED IN PASTERNAK-WINKLER TYPE ELASTIC FOUNDATION WITH CONSIDERATION OF SURFACE EFFECTS

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In this paper, the vibration analysis of carbon nanotubes (CNTs) conveying nanofluid and resting on an elastic foundation is studied based on the nonlocal Timoshenko beam model. Both Winkler type and Pasternak type models are considered to simulate the interaction between the nanotube and the elastic foundation. Moreover, the effects of the inner and outer surface layers on the CNT walls and the small-size effect of the nanoflow are incorporated. The natural frequencies and the critical flow velocity at which instability occurs are obtained using Differential Quadrature Method (DQM). The validation of the present solution method is confirmed by comparing the results with those obtained from literature. Furthermore, the rapid convergence of DQM results proves the suitability, reliability and rapidity of DQM. The effects of the main parameters on the vibrational characteristics of the CNT are also investigated. The present model is absent in the literature which could include the above parameters for the vibration and instability of the CNT conveying fluid.

1. Introduction

Carbon nanotubes (CNTs) have attracted great attention of many researchers because of their remarkable mechanical, thermal, chemical and electrical properties. These outstanding properties of CNTs have lead to its usage in nanobiological devices and nanomechanical systems such as fluid conveyance and drug delivery.[1-4]

In particular, the vibration behaviour of CNT filled with fluids has become a significant and challenging research topic because it is a critical issue in the design of the CNT-based fluidic devices. The influence of internal moving fluid on free vibration and flow-induced instability of both the cantilevered and supported CNTs based on the Euler-Bernoulli beam model is studied.[5-9] For the first time in the literature, the application of the classical Timoshenko beam model has been investigated to study the vibration of stubby multi-walled carbon nanotubes (MWCNTs) conveying a non-viscous fluid.[10] The influences of flow velocity on the fundamental frequency and mode shape of a fluid-conveying single-walled carbon nanotubes (SWCNTs) have been investigated using the classical Timoshenko beam theory.[11] For the first time, the nonlocal Timoshenko beam model has been implemented to investigate the vibration response of SWCNTs embedded in an elastic medium.[12] The effects of nonlocal small-scale coefficient, Winkler modulus parameter, Pasternak shear modulus parameter and the aspect ratio of the SWCNT on frequency of the SWCNT have been studied. The surface effects on the vibration and stability of fluid-conveying CNTs has been exam-

ined based on the classical Euler-Bernoulli beam model.[13] The small-size effects of the flow field on the vibrational characteristics of CNTs conveying fluid have been explored using classical Euler-Bernoulli beam model.[14] The nonlocal Timoshenko beam model has been proposed to analyze the vibration and instability of the viscoelastic CNTs conveying fluid and embedded in the viscous fluid including the thermal effects and rotary inertia.[15]

In this paper, we will analyze the vibration and instability of the CNTs conveying nanofluid and embedded in the elastic foundation using the nonlocal Timoshenko beam model. To obtain more accurate results, the small-size effect of the nanoflow will be incorporated by the assumption of the slip boundary conditions between the flow and the nanotube walls. Moreover, the surface effects of both inner and outer layers will be also taken into account. Both Pasternak-type and Winkler-type models will be used for simulating the interaction between the CNT and the elastic foundation. It should be remembered that no model exists in the literature able to take into account the combined effects of the different design parameters, including the small-scale effect of CNT, the small-size effect of the nanoflow, the influence of the surface layers, the effects of the Winkler elastic modulus and the Pasternak shear modulus. The numerical solutions of the equations of motion will be obtained based on Differential Quadrature Method (DQM), for a clamped-clamped CNT. Furthermore, the validity and rapid convergence of DQM will be confirmed by comparing the current results with those presented in the literature. Finally, the influences of the main parameters on the fundamental frequencies and the critical flow velocity at which instability occurs will be investigated.

2. Analytical model

The CNT is described as a uniform tubular beam with cross-sectional area A, length L, flexural rigidity EI, shear modulus G, mass per unit length m_c, mass moment of area per unit length I_c, conveying fluid of mass per unit length m_f, mass moment of area per unit length I_f, with a steady axial flow velocity U.

The coupled equations of motion for the nonlocal Timoshenko beam model with considering the surface effects and the small-size effect of nanoflow are given by[12-15]

$$\begin{aligned}&\left(m_c+m_f\right)\frac{\partial^2 w}{\partial t^2}+2m_f\left(VCF\right)U\frac{\partial^2 w}{\partial x\partial t}+\left(m_f\left(VCF\right)^2U^2-\Pi_0\right)\frac{\partial^2 w}{\partial x^2}+K_w w-K_G\frac{\partial^2 w}{\partial x^2}\\&-\left(e_0a\right)^2\left[\left(m_c+m_f\right)\frac{\partial^4 w}{\partial x^2\partial t^2}+2m_f\left(VCF\right)U\frac{\partial^4 w}{\partial x^3\partial t}+\left(m_f\left(VCF\right)^2U^2-\Pi_0\right)\frac{\partial^4 w}{\partial x^4}\right.\\&\left.+K_w\frac{\partial^2 w}{\partial x^2}-K_G\frac{\partial^4 w}{\partial x^4}\right]=KGA\left(\frac{\partial^2 w}{\partial x^2}-\frac{\partial\phi}{\partial x}\right).\end{aligned}\tag{1}$$

$$\left(EI+h\right)\frac{\partial^2\phi}{\partial x^2}+KGA\left(\frac{\partial w}{\partial x}-\phi\right)=\left(I_c+I_f\right)\left[\frac{\partial^2\phi}{\partial t^2}-\left(e_0a\right)^2\frac{\partial^4\phi}{\partial x^2\partial t^2}\right].\tag{2}$$

where w is the transverse displacement, ϕ is the rotation angle of cross section of the beam, K is the shear correction coefficient equal to $\pi^2/12$, e_0a is the nonlocal parameter, K_w is the Winkler elastic modulus and K_G is the Pasternak shear modulus.

The average velocity correction factor VCF including small-size effect of the nanoflow is a function of Knudsen number Kn given by[14]

$$VCF=\frac{v_{avg,slip}}{v_{avg,(no-slip)}}=\left(1+cKn\right)\left(4\left(\frac{2-\sigma_v}{\sigma_v}\right)\left(\frac{Kn}{1+Kn}\right)\right).\tag{3}$$

where $v_{avg,slip}$ and $v_{avg,(no-slip)}$ are average fluid flow velocities with slip boundary conditions and without slip boundary conditions, respectively. c is a coefficient and is given by[14]

$$c = c_0 \frac{2}{\pi}\left[\tan^{-1}\left(c_1 Kn^B\right)\right]. \tag{4}$$

The values, c_1 = 4 and B = 0.4, are some practical parameters. Coefficient c_0 equals to 64/(15π) for the second-order term of slip conditions. σ_v is tangential moment accommodation coefficient and for most practical purposes, is considered to be 0.7.

The additional flexural rigidity h due to the inner and outer surface layers may be written as[13]

$$h = \pi E_s t_0 \left(R_{in}^2 + R_{out}^2\right). \tag{5}$$

where E_s and t_0 are the Young's modulus and thickness of both surface layers, respectively. R_{in} and R_{out} are the inner and outer radii of the beam, respectively. The constant Π_0 determined by the residual surface tension and the shape of cross section is given by[13]

$$\Pi_0 = 4\tau_0 \left(R_{in} + R_{out}\right). \tag{6}$$

where τ_0 is the residual surface tension.

In what follows, a CNT with both ends clamped will be considered. The corresponding boundary conditions can be written as

$$w(0,t) = \phi(0,t) = 0, \qquad w(L,t) = \phi(L,t) = 0\,. \tag{7}$$

3. Results and discussion

The Differential Quadrature Method (DQM) is a relatively new numerical technique used to formulate solutions to Eqs. (1), (2) and (7).[16-17] This method was introduced to problems of applied mechanics in 1996 and will be used here directly.[18]

The essence of DQM is that a derivative of a function at a grid point is approximated as a weighted linear sum of the functional values at all grid points within the computational domain. A computer program is written based on DQM. The validity of the suggested solution method is checked by comparing the obtained results with those given in the literature. Also, its convergence is examined. Moreover, the effects of the main parameters including of the surface elasticity and residual surface tension**,** the Winkler elastic modulus, the Pasternak shear modulus and the Knudsen number on the frequencies and the structural instability of the CNTs are also studied.

3.1 Validation and convergence of the DQM

In order to show the accuracy and stability of present solution approach, validation of DQM results has been carried out. Critical flow velocity results are compared with results available in the literature. The comparison of present nonlocal DQM results without considering the surface elasticity and residual surface tension**,** the Winkler elastic modulus, the Pasternak shear modulus and the Knudsen number is presented in Table 1 for different nonlocal parameters against previous nonlocal Galerkin results.[15] It should be noted that, in this analysis, the critical velocity (u_{cr}) is the flow velocity at which the frequency reaches to zero at the first mode. In the following calculations, the same parameters as reported by the above reference for a CNT are used. The parameters of the CNT are the outer radius R_{out} = 0.5 nm, the thickness t_c = 0.35 nm, the aspect ratio $L/2R_{out}$ = 20, the mass density ρ_c = 2.3 g/cm^3, Young's modulus E = 1 TPa and Poisson ratio ν = 0.2. The mass density of water is ρ_f = 1 g/cm^3. Excellent agreement of the results is observed in Table 1, with relative errors less than 2%. This obviously shows the suitability and reliability of present solution method for the analysis of clamped-clamped CNTs.

Table 1. Comparison of the critical velocity obtained from the present and previous study for a clamped-clamped CNT conveying fluid.

$e_0a(nm)$	Present study	Previous study[15]
0	8169	8245
1	7793	7874
2	6915	6987

In order to determine the minimum number of grid points required to obtain a stable and accurate results, the convergence test is performed. Figure 1 shows the variation of critical flow velocity error percent with number of grid points. Critical flow velocity error percent is defined as

$$\text{Critical flow velocity error percent} = \left| \frac{u_{cr} - u_{cr}^{converged}}{u_{cr}^{converged}} \times 100 \right|. \tag{8}$$

As shown in Fig. 1, the error percent reduces very rapidly as number of grid points increases and after $N = 7$, converged results are obtained. But in Galerkin method[15], the converged results are obtained after $N_0 = 12$, where N_0 is the number of vibrational modes. Therefore, it is found that numerical results can be obtained very accurately and rapidly in DQM. As a consequence, very little computational effort and time are needed when the DQM is used.

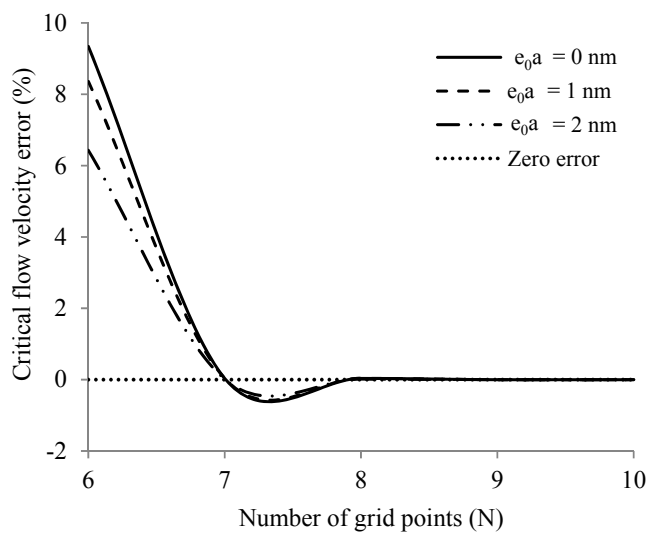

Figure 1. Convergence of critical flow velocity results.

3.2 Effects of the surface layers

Figure 2a shows the variation of the first natural frequency (fundamental frequency) with respect to the flow velocity, for two cases of including and excluding surface effects. For numerical calculation in this case, the same physical properties used in Section 3.1 with $e_0a = 2$ nm, $E_st_0 =$ 5.1882 N/m and $\tau_0 = 0.9108$ N/m are considered.[13] It can be seen that the fundamental frequency decreases with increasing flow velocity. For a given velocity, the fundamental frequency predicted by the current model is generally higher than that predicted by the Timoshenko beam model without surface effects. Another aspect that may be immediately found in Fig. 2a is that the fundamental frequency will become zero at a high flow velocity (u_{cr}), indicating the nanotube is buckled and a divergence instability occurs. Moreover, the structural stability of CNT enhances due to surface layers.

Figure 2b displays the critical flow velocity deviation percent versus CNT thickness for three different aspect ratios 10, 20 and 30. Critical flow velocity deviation percent is defined as

$$\text{Critical flow velocity deviation percent} = \left| \frac{u_{cr}^{\ with\ \ surface\ \ effect} - u_{cr}^{\ without\ \ surface\ \ effect}}{u_{cr}^{\ without\ \ surface\ \ effect}} \times 100 \right|. \qquad (9)$$

It can be seen that the influence of surface layers on the critical flow velocity increases as the CNT thickness decreases or the aspect ratio increases. For example, in the case of $L/2R_{out} = 30$ and t = 0.05 nm, the critical flow velocity predicted with the surface effects is about 1.61 times higher than that predicted without the surface effects, showing that the stability of the nanotube is enhanced due to surface layers.

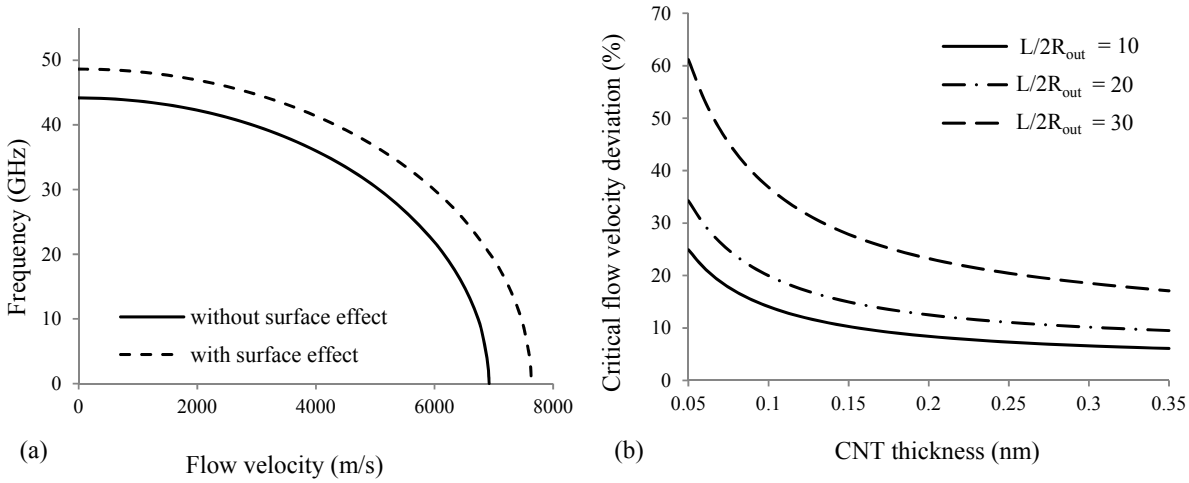

Figure 2. Effect of the surface layers on (a) the fundamental frequency and (b) the critical flow velocity for various values of CNT thickness and different aspect ratios.

3.3 Effects of the Winkler elastic modulus

The effect of the Winkler elastic modulus on the fundamental frequency for three different values of modulus K_w = 0, 50 and 100 MPa is shown in Fig. 3a. For numerical calculation in this case, the same physical properties used in Section 3.1 with e_0a = 2 nm are considered. It is clear that for a flow velocity, the natural frequencies of CNT increase as the Winkler elastic modulus increases. Furthermore, increasing the Winkler elastic modulus results in increasing the critical flow velocity.

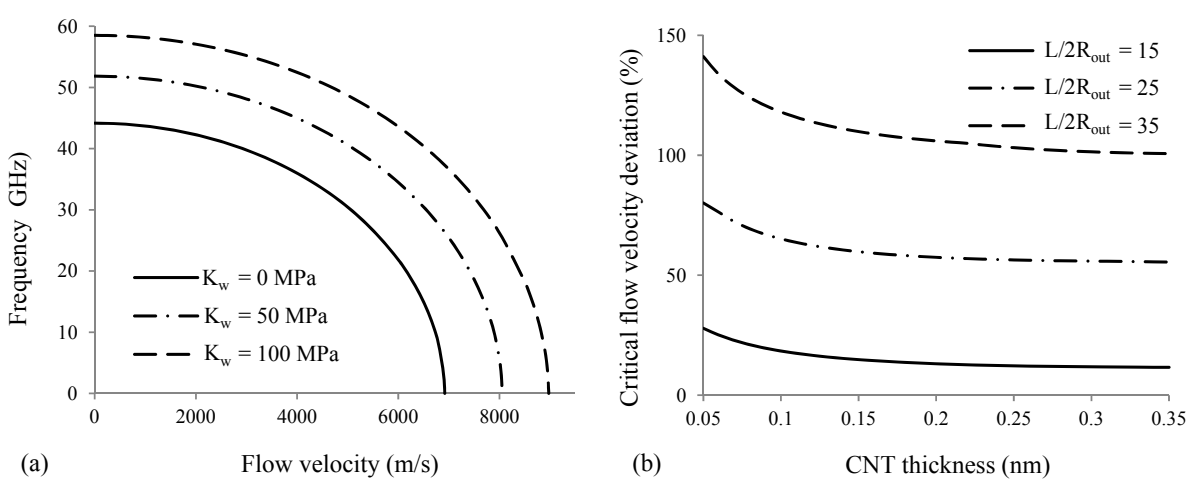

Figure 3. Effect of the Winkler elastic modulus on (a) the fundamental frequency and (b) the critical flow velocity for various values of CNT thickness and different aspect ratios.

Figure 3b shows the critical flow velocity difference percent as function of CNT thickness for three different aspect ratios 15, 25 and 35. Critical flow velocity difference percent is defined as

$$\text{Critical flow velocity difference percent} = \left| \frac{u_{cr}^{K_w=K_w} - u_{cr}^{K_w=0}}{u_{cr}^{K_w=0}} \times 100 \right| . \tag{10}$$

As it can be seen from Fig. 3b, the effect of Winkler elastic modulus on the critical flow velocity enhances as the CNT thickness decreases and the aspect ratio increases.

3.4 Effects of the Pasternak shear modulus

Figure 4a depicts the variation of the fundamental frequency with flow velocity, for three different values of Pasternak shear modulus 0, 5×10^{-10} and 10×10^{-10} N. For numerical calculation in this case, the same physical properties used in Section 3.3 are considered. It is well-known that the natural frequencies increase as the shear modulus increases. In addition, increasing the shear modulus increases the critical flow velocity.

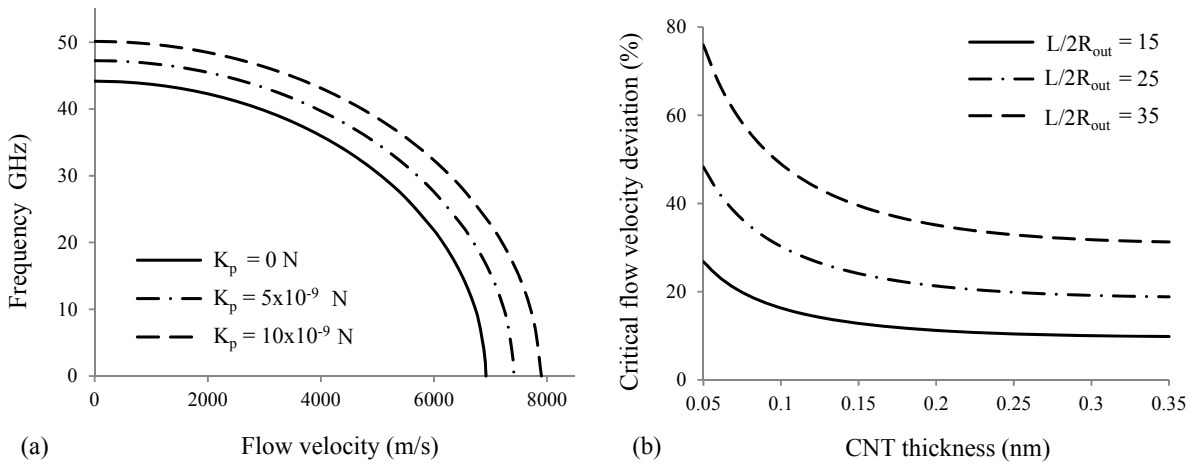

Figure 4. Effect of the Pasternak shear modulus on (a) the fundamental frequency and (b) the critical flow velocity for various values of CNT thickness and different aspect ratios.

The effect of the Pasternak shear modulus on the critical flow velocity, for various values of CNT thickness and three different aspect ratios 15, 25 and 35, is shown in Fig. 4b. Critical flow velocity variation percent is defined as

$$\text{Critical flow velocity variation percent} = \left| \frac{u_{cr}^{K_G=K_G} - u_{cr}^{K_G=0}}{u_{cr}^{K_G=0}} \times 100 \right| . \tag{1}$$

As shown in Fig. 4b, the effect of Pasternak shear modulus on the critical flow velocity enhances as the CNT thickness decreases and the aspect ratio increases.

3.5 The small-size effect of nanoflow field

Figure 5 illustrates the variation of the fundamental frequency with the flow velocity for various values of Knudsen number. As shown in Fig. 5, the small-size effect of nanoflow on the fundamental frequency increases as the flow velocity increases. It can be seen that the fundamental frequency decreases as the Knudsen number increases. In addition, increasing the Knudsen number decreases the critical flow velocity.

It should be noted that a nanogas flow is much more sensitive to small-size effects than a nanoliquid flow, because for liquids Kn could not reach large values but gas flows could reach any Kn continuously.[14]

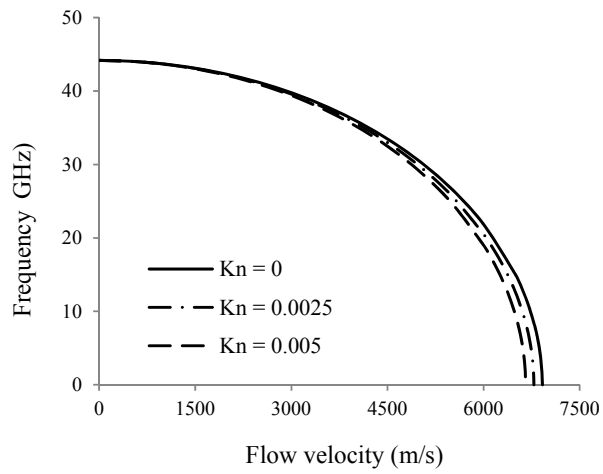

Figure 5. Fundamental frequency as a function of flow velocity for different values of Knudsen number.

4. Conclusion

In this paper, the nonlocal Timoshenko beam model was presented to analyze the vibration of CNTs conveying nanofluid and embedded in Winkler-Pasternak type foundation. In this model, the small-size effect of the nanofluid and the effects of the inner and outer surface layers on the CNT vibrational and stability characteristics were considered. The governing equations for the nonlocal Timoshenko CNTs were solved using DQM. The validation and rapid convergence of the present solution method were justified by comparing the results with some available known data in literature. Results showed that considering the effects of surface layers enhanced the fundamental frequencies and the structural stability of the CNT. Furthermore, increasing the Winkler elastic modulus and the Pasternak shear modulus increased the fundamental frequencies and the critical flow velocity. It was observed that the effects of the surface layers, the Winkler elastic modulus and the Pasternak shear modulus on the critical flow velocity are significant especially for small CNT thickness or large aspect ratio. In addition, it was found that incorporating the small-size effect of the nanoflow resulted in decreasing the frequencies and the structural stability of the CNT.

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