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A New Method for Linear-Phase FIR Filter Designing with Desired Shape

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Abstract

This paper introduces a new method for designing linear-phase FIR filters with a desired shape. The proposed algorithm adjusts desired filter frequency response based on least squared (LS) method. It can design full-shaped filters with specified deviation in the entire the frequency band. Also, the transition bands can be considered as short as possible without any difficulty although it may not be necessary. The simulation results show that the proposed algorithm can successfully adjust and regulate the deviations of LS design methods for filters.

Key Words

Linear-Phase, FIR, Desired-Shape Filter, Recursive method

1. Introduction

This paper aims to introduce a simple recursive method for designing desired shape FIR filters. The proposed algorithm which is based on LS method, updates the coefficients of predefined filter FIR filter to close its specification as much as possible to desired one. The main criteria in this approach are the frequency response which has to be the same for design and desired filters. Therefore, it is clear that the transition bands between the frequencies are not considered. This is because that the transition bands are not really a demand of designing. Furthermore, the consideration of the transition bands [1,2] are usually introduced to reduce the Gibbs effect for the least square approximation or to use Chebyshev approximation.

For example, a low pass filter normally may design in such a way to covey the pass, transition and stop bands respectively. In most practical cases there is no separation between the passband and stopband for the transition band. In other words, the spectrum of the desired and undesired signals often overlap and it is hard to specify a point that separates the pass and stop bands and certainly it is impossible to state a band for separating them. In this way, the transition band is introduced to reduce or remove the oscillations in the frequency response near the band edges caused by Gibbs effect. It should be noted that when there are large peaks in the transition band of filters such as Chebyshev filters, we must care and alter the specifications so that the peaks are eliminated [3].

In addition to transition band problem, to design a FIR filter with a special shape, some methods are introduced [4]. Some of these filter design algorithms are a kind of optimization, in which the deviation between the desired and designed filters are minimized [5,6,7]. Some other design algorithms are based on genetic algorithms [8,9]. Both these methods are computationally expensive. In this paper a simple effective recursive algorithm is proposed for desired shape FIR filters and predefined deviation in frequency band. The simulation results show that the algorithm is very flexible for obtaining the main characteristics of the desired filter such as transition bands, and less complexity at the same time.

The paper is organized as follows: first the designing of the linear-phase FIR filters using LS method is introduced. The proposed algorithm is described in section 3. The section 4 explains some considerations to reduce the computational complexity of proposed algorithm, and finally, the simulation results are presented and concluded in section 5.

2. Linear-phase Filter design with LS method

A linear-phase filter with impulse response $h[n]$, may have even symmetric as

$$h[n] = h[-n] \tag{1}$$

Where, its corresponding frequency response can be interpreted by

$$A(e^{jw}) = h[0] + \sum_{n=1}^{L} 2h[n]\cos(wn) \tag{2}$$

It is assumed that the values of the $A(e^{j\omega})$ at $\underset{i=1,2,\dots,N}{0 \le \omega_i \le 2\pi}$,which are called *designing frequencies*, are known, and denoted by $\underset{i=1,2,\dots,N}{H(\omega_i)}$. The problem is to compute $\underset{n=0,1,\dots,L}{h[n]}$ coefficients optimally. To minimize the error of designing according to LS method, this can be done by setting to zero the derivatives of the following objective function with respect to $\underset{n=0,1,\dots,L}{h[n]}$ [11].

$$E=\frac{1}{2}\sum_{i=1}^{N}\left[H(\omega_i)-A(e^{j\omega_i})\right]^2=\frac{1}{2}\sum_{i=1}^{N}\left[h[0]+\sum_{n=1}^{L}2h[n]\cos(\omega_i n)-H_i\right]^2 \qquad (3)$$
$$H_i=H(\omega_i)$$

Hereafter, we call $H(\omega)$ "predefined filter". In continuous frequency domain the above function is reduced to

$$E=\frac{1}{2}\int_0^{2\pi}\left[H(\omega)-A(e^{j\omega})\right]^2 d\omega \qquad (4)$$

Although the LS method is very simple and powerful, and the least square is the main criteria, other designing requirements can not be taken into account. One of the most important requirements is deviation limitation. It can be seen in the literature that many methods are proposed to overcome this drawback [5-7], which in turn the flexibility of the methods may be lost. The fundamental factors which determine the complexity and affect the results of the LS method are objective function, designing frequencies, and predefined filter. The method is usually built on changes of the objective function although the adjustment of the predefined filter may also be considered.

3. New proposed method

The new proposed algorithm for designing any FIR filters with desired shape is an iterative algorithm based on adjustment of predefined filter. At each step of the algorithm, with characteristics of the current predefined filter, the design is accomplished. Then, the error between the desired and design filters is computed as an error signal. Based on this error signal, the predefined filter specifications for the next step are updated. This process is continued; until the error signal tends to zero. Therefore, the filter coefficients will be optimized.

Let us define the frequency response of the desired filter as $d_0(f)$, and $b_n(f)$ and $d_n(f)$ denote the designed and predefined filters at n-th step, which are shown in Fig.1. At 0-th step the desired filter $d_0(f)$ is the same as predefined filter. With LS method $b_0(f)$ is obtained which the error can be defined as

$$e_n(f)=\mathrm{sgn}(d_n(f)-b_n(f))\times\max(|d_n(f)-b_n(f)|-\delta(f),0) \qquad (5)$$

The acceptable deviation, $\delta(f)$, is defined the difference between the obtained designed and desired filters,

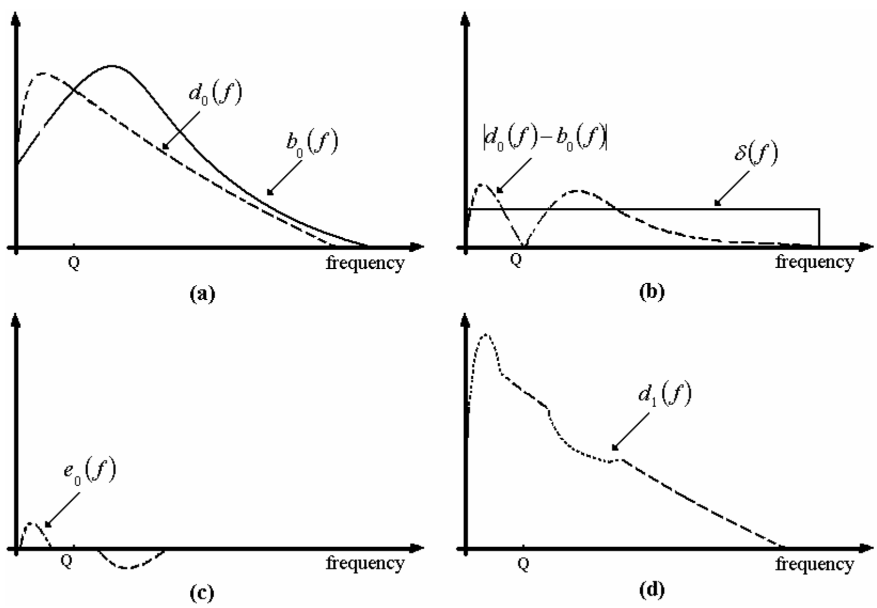

Fig.1

and *sgn* indicates the sign function. Fig.1-b and c show the computation process of the error signal. The error signal is nonzero at each frequency in which the designed filter deviation is larger than the acceptable deviation. In addition, we have,

1. if $d_0(f) \ge b_n(f)$ then, $e_n(f) \ge 0$.
2. if $d_0(f) \le b_n(f)$ then $e_n(f) \le 0$.

Now, the main problem is to minimize the error signal. It is obvious that to decrease the error, and make the designed filter shape as much as desired one, $b_n(f)$ has to be increased where $e_n(f) \ge 0$, and decreased where $e_n(f) \le 0$. This can be done by increasing $d_n(f)$ any where $e_n(f) \ge 0$ and decreasing of $d_n(f)$ any where $e_n(f) \le 0$. Therefore, to realize this mechanism, $d_{n+1}(f)$ is computed as follows

$$d_{n+1}(f) = d_n(f) \times (1 + h \times e_n(f)) \tag{6}$$

which *?>0* shows the convergence speed, and is selected appropriately. The small value of the convergence speed makes the computation time very long, and selecting it large, the algorithm may become unstable. During of the computing the next step predefined filter, it may be negative in some frequencies. To avoid this, we add a sufficient large number to desired filter source before starting the algorithm, and then subtract it from $h[0]$ after the algorithm convergence.

To consider the transition bands in the designing, It is simply realized by choosing a large acceptable deviation, $d(f)$, in the transition bands. Designing the filter according to LS and with no deviation consideration, it is sufficient to choose a large $d(f)$ in entire the frequency band. Therefore, from this point of view, the proposed algorithm is an extension of conventional LS method.

4. Computational complexity

Computational complexity of the LS filter design is related to the following equation set

$$P = \begin{bmatrix} N & 2\sum_{i=1}^{N}\cos(w_i) & \cdots & 2\sum_{i=1}^{N}\cos(Lw_i) \\ \sum_{i=1}^{N}\cos(w_i) & 2\sum_{i=1}^{N}\cos(w_i)\cos(w_i) & \cdots & 2\sum_{i=1}^{N}\cos(w_i)\cos(Lw_i) \\ \vdots & \vdots & & \vdots \\ \sum_{i=1}^{N}\cos(Lw_i) & 2\sum_{i=1}^{N}\cos(Lw_i)\cos(w_i) & \cdots & 2\sum_{i=1}^{N}\cos(Lw_i)\cos(Lw_i) \end{bmatrix}, \quad P \times \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_L \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} H_i \\ \sum_{i=1}^{N} H_i \cos(w_i) \\ \vdots \\ \sum_{i=1}^{N} H_i \cos(Lw_i) \end{bmatrix} \tag{7}$$

Computation of P is the most expensive part of the LS computation. But, as eq.(7) shows, it is obvious that predefined filter ($\underset{i=1,2,\cdots,N}{H_i}$) contributes only at right side of the equation. Thus, changing the predefined filter at each step, it is necessary to recompute this part only. To reduce the complexity, P can be simplified by choosing w_i as

$$w_i = (i-1) \times p / (N-1) \tag{8}$$

$$P = \begin{bmatrix} N & 0 & 2 & \cdots & 0 \\ 0 & N & 0 & \cdots & 2 \\ 1 & 0 & N & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 2 & 0 & \cdots & N \end{bmatrix} \tag{9}$$

Further computation decrease can be achieved when we update only summations parts of the eq.(9) in which $\underset{i=1,2,\cdots,N}{H_i}$ have been changed. For example, suppose only $\underset{1<k<N}{H_k}$ is changed, it is sufficient to update $\underset{j=0,1,\cdots,L}{H_k \cos(jw_k)}$ in summation parts of eq.(9). The proposed algorithm can be called Extended LS(ELS), or Recursive LS(RLS).

5. Simulation results

The proposed algorithm has been simulated for designing different filters and compared the results with LS method. As an example, the simulation results are shown in Fig.2-3 for three different samples. *0.01*, *1000* and *1000* which are selected for *?*, *N* and *R*, respectively.. Each figure has 7 parts included,

a. *Desired filter* characteristics such as transition bands, maximum acceptable deviation.
b. *Designed filter by LS*.
c. *Designed filter by ELS*.
d. *Error signal of LS designed filter* computed by eq.(5).
e. *Error signal of RLS designed filter* computed by eq.(5).
f. *Changes of error* during convergence process.
g. *The final filter* obtained by ELS.

Examples are designing of low-pass and band-stop filters. The comparison of Fig.2-d, and Fig.3-d with Fig.2-e and Fig.3-e, show that ELS designed filters have very smaller error than LS designed filters. By comparison of these figures, we can realize the performance of ELS.
Furthermore, the comparison of Fig.2-b and Fig.3-b with Fig.2-c and Fig.3-c is shown that ELS increases the oscillations in the entire of the band to reduce unacceptable large deviations.
The Fig.2-g and Fig.3-g show the final predefined filters. It can be seen that ELS increases predefined filter value at near band edges to enforce the deviations below the acceptable range. In Fig.(2-f), (3-f), & (4-f) the error changes of filter during convergence is shown.
In spite of the RLS flexibility, its main drawback is computational complexity of its computations compared to LS, which is because of its iterative structure. Thus, ELS may be not good for real-time processing, but very suitable approach to design any desired-shape FIR filters.

5. Conclusions

In this paper a new iterative algorithm for designing FIR filters with any desired shape is introduced, which is called ELS method. The simulation results show a good performance of ELS with computationally expensive with respect to other methods such as LS method. This new method may be not good for real-time processing, but very suitable approach to design any linear-phase desired-shape FIR filter.

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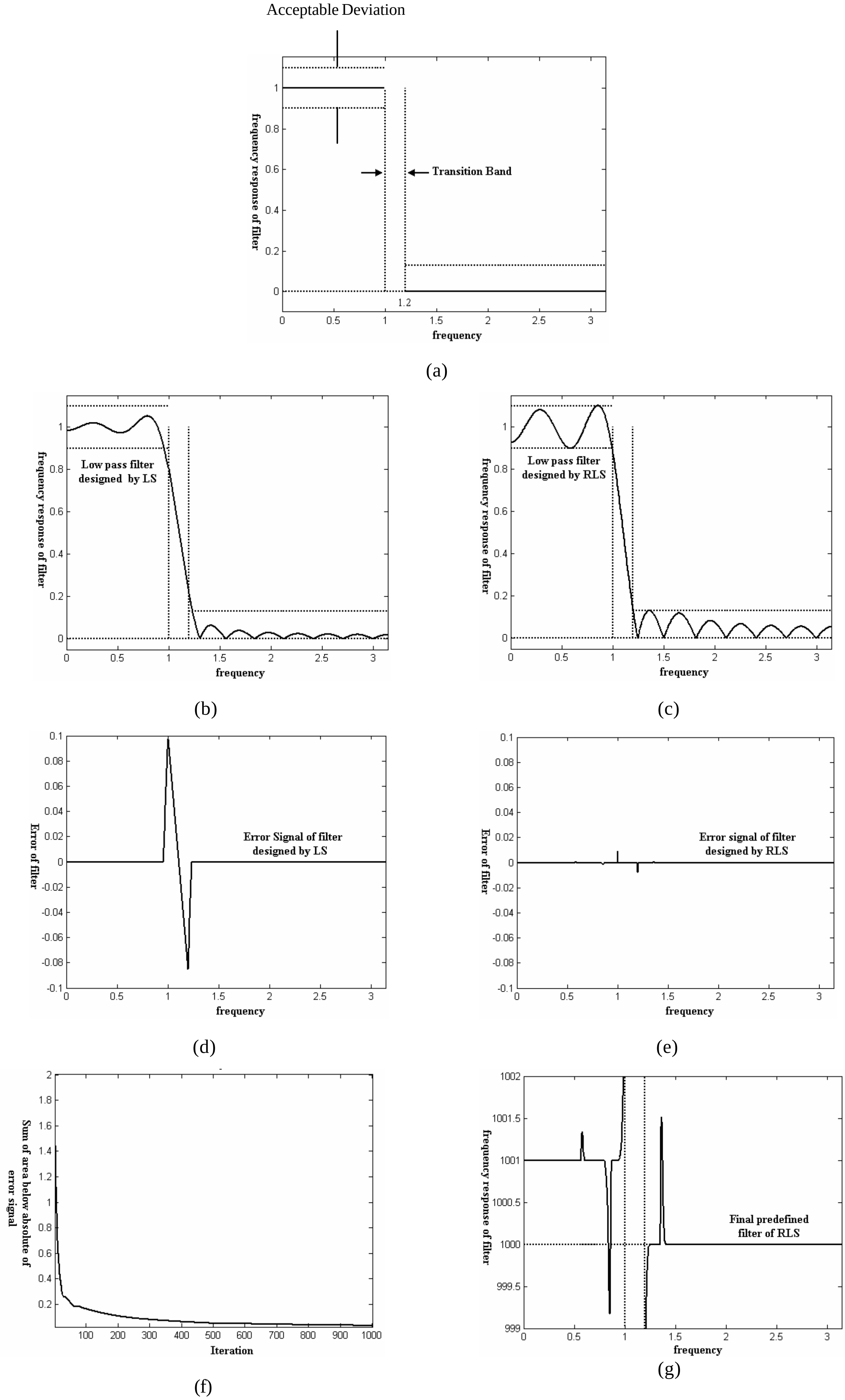

Fig. 2. A low-pass filter designing, with degree of 21

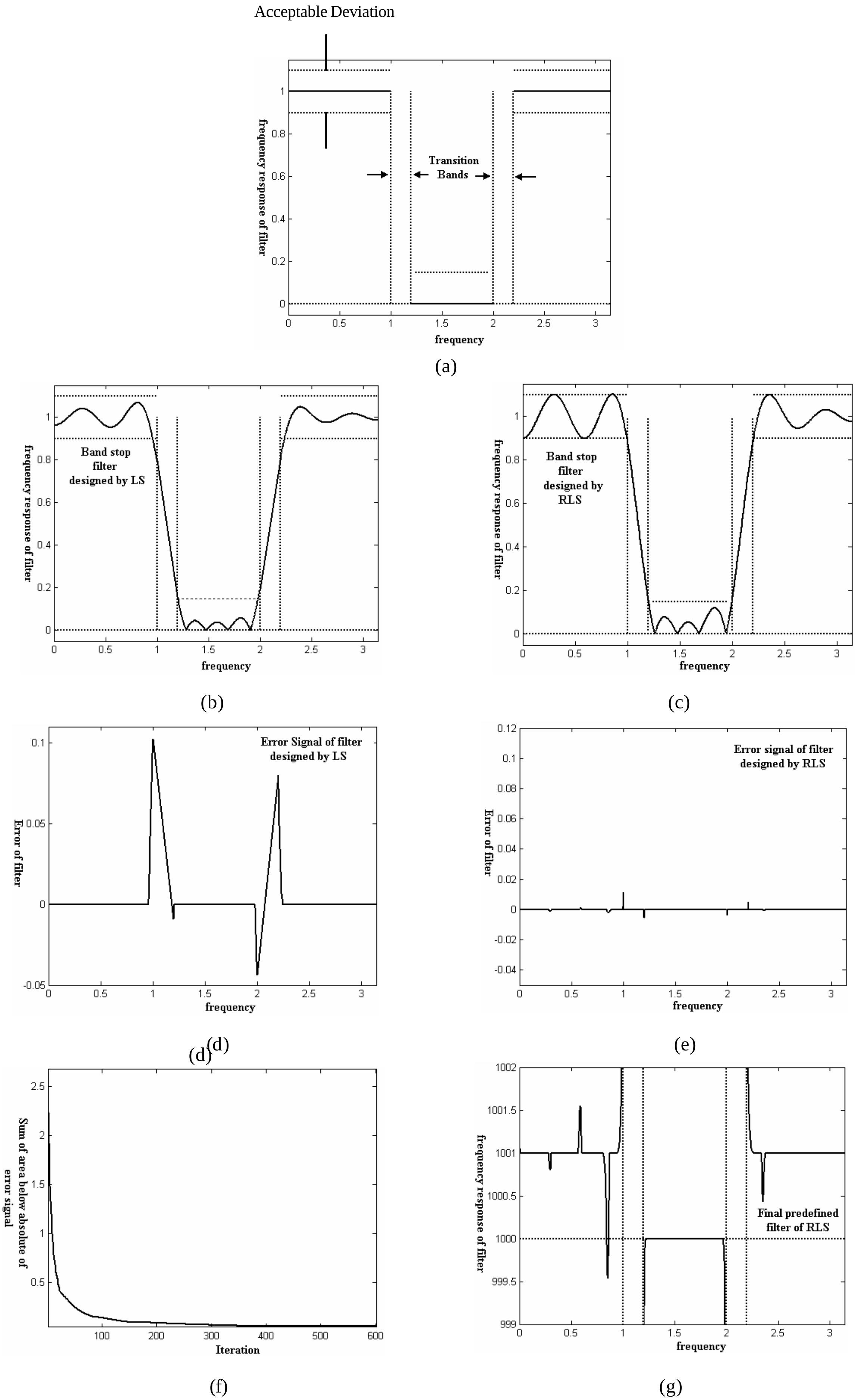

Fig.3. A band-stop filter designing, with degree of 21