IJST, Transactions of Mechanical Engineering, Vol. 36, No. M2, pp 165-180
Printed in The Islamic Republic of Iran, 2012

NUMERICAL STUDY OF MIXED CONVECTION IN AN ANNULUS BETWEEN CONCENTRIC ROTATING CYLINDERS WITH TIME-DEPENDENT ANGULAR VELOCITY*

A. ABEDINI AND A. B. RAHIMI**
Faculty of Engineering, Ferdowsi University of Mashhad, P.O. Box No. 91775-1111, Mashhad, I. R. of Iran
Email: rahimiab@yahoo.com

Abstract– Numerical study of mixed convection in an annulus between concentric rotating horizontal cylinders is presented in this work by using finite volume method. The mixed convection is due to natural convection because of the temperature difference of the cylinders, and forced convection is a result of cylinder rotation with different time functions and in different directions. Here, the effect of different cylinders motions are investigated on flow parameters such as streamlines, and heat transfer parameters such as temperature contours, Nusselt numbers, along with required torque for rotation. The non-dimensionalization of the governing equations is different in this study such that the non-dimensional Reynolds number does not appear in the equations and the velocity magnitudes are Reynolds numbers themselves that can be used locally for analyzing flow in solution field. The obtained results are compared with the existing analytical and numerical of simpler cases which prove good agreements.

Keywords– Mixed convection, annulus, rotating concentric cylinders, time-dependent angular velocity

1. INTRODUCTION

Mixed convection problem in an annulus between concentric cylinders has attracted special attention because of its wide engineering applications such as mixtures, thermal energy storage systems, cooling of electrical components, double pipe heat exchangers designed for chemical processes and food industries, and nuclear reactors. Because of small dimensions and low velocities, flow in these devices is assumed laminar. Such laminar flow is often influenced by body forces which tend to produce secondary flow in the annular cross section. A good overview on concentric and eccentric annuli has been carried out by Kuehn and Goldstein [1, 2]. They measured the heat transfer coefficients in air and water in concentric and eccentric horizontal annuli. Their experimental data is commonly used to validate most of the recent numerical studies. Some years later Shahraki [3] demonstrated the effect of temperature-dependent properties on the streamlines and temperature distributions in a concentric annulus. The varying viscosity had the strongest effect on the flow fields, while the thermal conductivity had the strongest effect on temperature profiles. An unsteady natural convection in a horizontal annulus was investigated by Mizushima et al. [4]. Instability analyses for natural convection in horizontal annuli are extensively addressed in literature. Numerical results of the turbulent flow in a two dimensional domain were presented by Farouk and Guceri [5]. In their work, the natural convection was considered in the annulus between two horizontal concentric cylinders. The stream-vorticity equation was discretized by finite difference technique and turbulence was modeled by the κ-ε approach for Rayleigh numbers above 105. Padilla et al. [5] investigated laminar and unsteady natural convection at low Rayleigh number in a

*Received by the editors October 27, 2011; Accepted March 13, 2012.
**Corresponding author

concentric annulus. In most of the numerical studies on buoyancy driven flow in an annulus, the flow is assumed to be invariant in the axial direction, leading to a two dimensional approach of the problem. However, in practical applications, the viscous shearing effects at the end walls could lead to erroneous results. Vafai and Ettefagh [6] performed a three-dimensional buoyancy-driven flow simulation in a closed horizontal annulus. They furnished a detailed analysis of the three-dimensional flow and temperature field. Desai and Vafai [7] and Lee [8, 9] with sufficiently long annulus and a range of limited parameters, showed that there exists a core region with a certain Reynolds number, which can be approximated by a two dimensional model. As far as heat transfer phenomenon, studies show that its rate is bounded by the inner area cylinder. In some applications, to increase the heat transfer rate fin is used in the axial direction. Chai and Patankar [10], Desrayaud et al. [11], Patankar et al. [12], and Rustum and Soliman [13] investigated the effect of fins attached on the inner and outer cylinders on the fluid flow and heat transfer in annuli by analyzing the result obtained from the numerical solution of two-dimensional and steady state governing equations. Dyko et al. [14] presented numerical and experimental buoyancy driven flow in an annulus between two horizontal coaxial cylinders at Rayleigh numbers exceeding the critical value. Numerical investigations of three-dimensional natural convection flow were accomplished by Yeh [15]. Most works for mixed convection problems in rotating systems have been performed for the flows in vertically cylindrical annuli. Relatively few studies, however, have been made for the flows in horizontal annuli. Lee [16] treated the problem over a good range of Rayleigh numbers, allowing for both horizontal and vertical eccentricities of the inner cylinder. It was concluded that the mean Nusselt number increased with the Rayleigh number at a given angular velocity and decreased with the rotation speed, all other parameters being kept constant. Mixed convection of air between two horizontal concentric cylinders with a cooled rotating outer cylinder has been done by Yoo [17]. Fusegi et al. [18] did not consider eccentric cases but treated the problem for both high and low values of the Froude number, σ , which expresses the relative magnitude of buoyancy versus rotation effects. Choudhury and Karki [19] studied this problem with the eccentricity along the vertical line. They concluded that the eccentricity introduces additional non-uniformity in the flow and temperature fields. Habib and Negm [20] investigated numerically the fully developed laminar mixed convection of horizontal concentric annuli for the case of non-uniform circumferential heating. Two heating conditions were explored, one in which the top half of the inner surface of the inner cylinder is uniformly heated while the bottom half is kept insulated, and the other in which the heated and the insulated surfaces were reversed. It was found that the bottom heating arrangement gives rise to a vigorous secondary flow, resulting in average Nusselt numbers much higher than those of pure forced convection. Alawadhi [21] studied the natural convection flow in a horizontal annulus enclosure with a transversely oscillating inner cylinder numerically. Most heat transfer studies on the laminar mixed convection problem have been carried out using a staggered grid arrangement and stream function-vorticity, vorticity- velocity or primitive variables formulations based on the finite difference or finite element methods with the inner cylinder rotating. Also, some researchers such as Glakpe et al. [22], Watkins and Gakpe [23], Hessami et al.[24], Kumar [25], Soliman and Mirza [26], Tsui and Tremblay [27], Yang et al. [28], and Hamer and Vane Sande [29] did experimental and numerical studies for investigating geometry, boundary condition, fluid properties, steady and unsteady effects in a concentric and eccentric annulus.

To date there has been no publication about the time-dependent boundary conditions in mixed convection in a horizontal annulus. This work is concerns the different kinds of time-dependent boundary conditions which are brought about because of the time-dependent angular velocities of the inner and / or outer cylinders. These situations mostly have industrial applications such as in different kinds of mixtures.

2. PROBLEM FORMULATION

The fluid is contained between two infinite horizontal concentric circular cylinders, which are held at different uniform temperatures (T_c and T_i). Inner and outer cylinders can rotate with different kinds of time-dependent functions and their effects are studied here. Density changes in the fluid are neglected everywhere except in considering the buoyancy effects (Boussinesq approximation), and all the other physical properties of the fluid are assumed constant. Because of low velocity in mixed convection phenomenon, dissipation in the energy equation is also neglected. We consider a two-dimensional problem, and use the cylindrical coordinates (r, ϕ), the angular coordinate ϕ being measured counter-clockwise from the right horizontal axis through the center of the cylinders. The non-dimensional parameters are defined in the following forms:

$$L = r_o - r_i \qquad \mathbf{R} = \frac{\mathbf{r}}{L} \qquad \mathbf{U_R} = \frac{u_r L}{\upsilon} \qquad \mathbf{U}_\phi = \frac{u_\phi L}{\upsilon} \qquad P = \frac{pL^2}{\rho \upsilon^2}$$

$$\theta = \frac{T - T_c}{T_h - T_c} \qquad \tau = \frac{t\upsilon}{L^2} \qquad \Pr = \frac{\upsilon}{\alpha} \qquad Gr = \frac{\beta g (T_h - T_c) L^3}{\upsilon^2}$$

Note that kinematics viscosity has been used here to non-dimensionalize the velocity components. In this way velocity is changed to Reynolds number, which can be used for better interpretation of the results. The equations governing conservation of mass, momentum and energy are put into non-dimensional forms as follows:

$$\frac{1}{R}\frac{\partial}{\partial R}(RU_R) + \frac{1}{R}\frac{\partial}{\partial \phi}(U_\phi) = 0 \tag{1}$$

$$\begin{aligned} &\frac{\partial U_R}{\partial \tau} + \frac{1}{R}\frac{\partial}{\partial R}(RU_R U_\phi) + \frac{1}{R}\frac{\partial}{\partial \phi}(RU_R U_\phi) - \frac{U_\phi{}^2}{R} = \\ &-\frac{\partial P}{\partial R} + \left[\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial U_R}{\partial R}) + \frac{1}{R^2}\frac{\partial}{\partial \phi}(U_R) - \frac{U_R}{R^2} - \frac{2}{R^2}\frac{\partial U_\phi}{\partial \phi}\right] + Gr\theta\sin(\phi) \end{aligned} \tag{2}$$

$$\begin{aligned} &\frac{\partial U_\phi}{\partial \tau} + \frac{1}{R}\frac{\partial}{\partial R}(RU_R U_\phi) + \frac{1}{R}\frac{\partial}{\partial \phi}(U_\phi{}^2) + \frac{U_R U_\phi}{R} = \\ &-\frac{1}{R}\frac{\partial p}{\partial \phi} + \left[\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial U_\phi}{\partial R}) + \frac{1}{R^2}\frac{\partial^2 U_\phi}{\partial \phi^2} - \frac{U_\phi}{R^2} + \frac{2}{R^2}\frac{\partial U_R}{\partial \phi}\right] + Gr\theta\cos(\phi) \end{aligned} \tag{3}$$

$$\frac{\partial \theta}{\partial \tau} + \frac{1}{R}\frac{\partial}{\partial R}(RU_R\theta) + \frac{1}{R}\frac{\partial}{\partial \phi}(U_\phi\theta) = \left[\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial \theta}{\partial R}) + \frac{1}{R^2}\frac{\partial^2 \theta}{\partial \phi^2}\right] \tag{4}$$

Boundary conditions are:

$$\begin{aligned} R = R_i &\quad \rightarrow \quad \theta = 1 \\ R = R_0 &\quad \rightarrow \quad \theta = 0 \end{aligned} \tag{5}$$

Boundary conditions for momentum equations are applied by different kinds of time-dependent inner and outer cylinders rotation functions. By defining the Nusselt number as the ratio between the actual heat transfer rate and the heat transferred by pure conduction, the local Nusselt number along the inner and outer cylinder is calculated as:

$$Nu_i(\phi) = \frac{-R\frac{\partial\theta}{\partial R}}{Nu_{cond}} \qquad \text{at} \quad R = R_i \tag{6}$$

$$Nu_0(\phi) = \frac{-R\frac{\partial\theta}{\partial R}}{Nu_{cond}} \qquad \text{at} \quad R = R_o \tag{7}$$

in which Nu_{cond} is Nusselt number in the case of heat transfer through the annulus by pure conduction and is given by:

$$Nu_{cond} = \frac{1}{Ln(\frac{R_0}{R_i})} \tag{8}$$

The average Nusselt number is calculated from integrating the local value over the circumference of the cylinder as follows:

$$\overline{Nu_i} = \frac{1}{2\pi}\int_0^{2\pi} Nu_i(\phi)d\phi \tag{9}$$

$$\overline{Nu_o} = \frac{1}{2\pi}\int_0^{2\pi} Nu_o(\phi)d\phi \tag{10}$$

Similarly, we can calculate torque by:

$$TQ = \int_0^{2\pi}(\tau.r^2)d\phi, \qquad \tau = R\frac{\partial}{\partial R}(\frac{U_\phi}{R}) \tag{11}$$

Dimensionless torque is defined as the actual torque on the cylinder divided by the torque obtained with the Couette velocity distribution in which velocity profile can be extracted from analytical solution of momentum equation as:

$$V(r) = AB + \frac{B}{R}, \quad A = \frac{R_o}{R_o^2 - R_i^2}, \quad B = -\frac{R_o^2 R_i^2}{R_o^2 - R_i^2} \tag{12}$$

3. SOLUTION PROCEDURE

The geometry of the problem and its computational mesh are as shown in Fig. 1. The governing equations are solved by using the finite volume technique and simple algorithm developed by Patankar [30]. Face values are estimated by power law scheme. The number of grids used is (10 × 80). Both Nusselt number and torque are calculated from numerical differentiation of dimensionless temperature and velocity distribution. The discretized equations are solved by the Gauss-Seidel method. The iteration method used in this program is a line-by-line procedure, which is a combination of the direct method and the resulting Tri-Diagonal Matrix Algorithm (TDMA). The computer program was constructed in a general form to solve governing equations and investigate the time-dependent boundary conditions. First, the result is compared with pure forced convection (Eq. (12)) in Fig. 2. A similar problem has been investigated by Yoo [17]. He solved the mixed convection between concentric annulus in which the outer cylinder is rotating with constant angular velocity. The program was run by the same parameters that Yoo used. The dimensionless Nusselt number is compared in Fig. 3. Results have good agreement with the available analytical and numerical results.

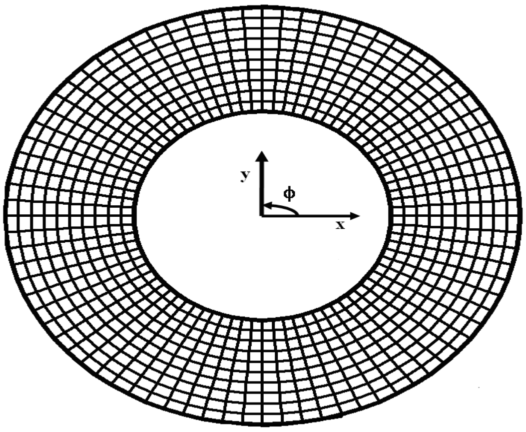

Fig. 1. Sample of the computational mesh

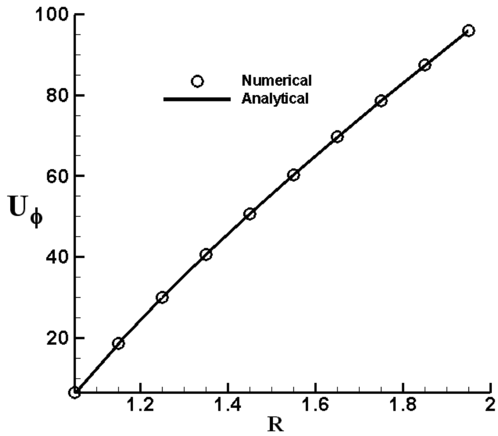

Fig. 2. Comparison of tangential velocity ($\mathbf{Re_O = 100}$)

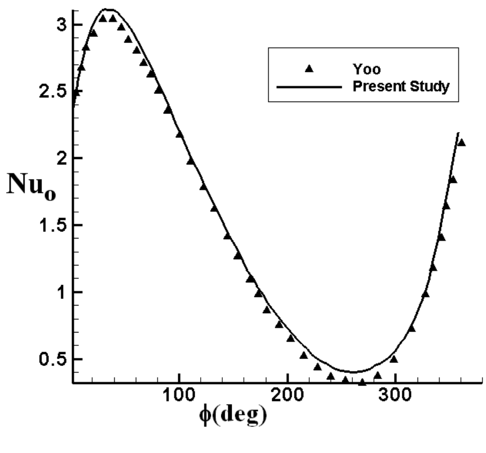

Fig. 3. Comparison of local Nusselt number at the outer cylinder
(Ra=5000; Re=100)

4. RESULTS AND DISCUSSIONS

All the results are presented here for $2R_i / L$, where L is the gap width. Prandtl number throughout the paper is constant and equal to 0.7. The results including stream lines, isotherms, average Nusselt number and torque are shown here for different Rayleigh and Reynolds numbers.

The following results are for the case of outer cylinder rotation velocity $\Omega_{out} = 10Exp(t)$ and stationary inner cylinder and at $Ra = 1000$:

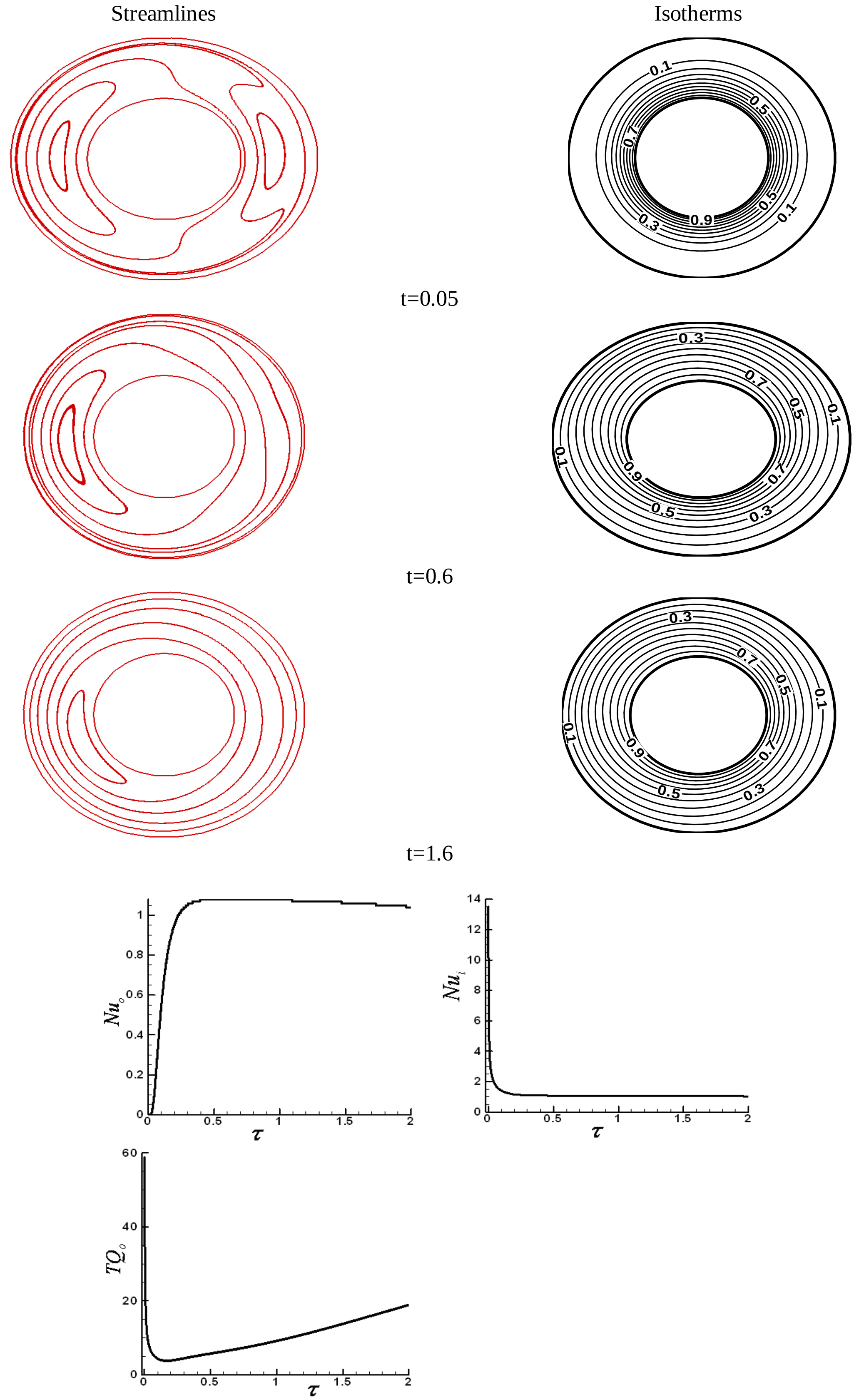

Fig. 4. Streamlines, isotherms, Nusselt number and dimensionless torque

At the beginning, as it can be seen from these figures, the dominant heat transfer mechanism is free convection and as expected, two large vortices are observed. As time passes and the outer cylinder velocity increases, the left vortex becomes smaller and the right one narrower. With further time increase, the right vortex is affected from the above and becomes smaller as well. As far as the isotherms, since at low Rayleigh numbers the dominant heat transfer mechanism is conduction, which is evident from the final Nusselt number (1.09) and lower velocity because of free convection or forced convection, these isotherms have concentric forms and continue to be concentric throughout the time passage.

To start the fluid rotating from rest, a relatively large amount of torque is needed. At the beginning the flow at the vicinity of the cylinder wall is only under the influence of viscosity effects and tangential stress until natural convection effect comes into existence at the right side. Then this natural convection effect and the viscosity effects reinforce each other and, in some parts of the cylinder periphery a negative torque is created locally and, as time goes by the torque needed to rotate the cylinder is decreased. This phenomenon can be seen clearly in Fig. 5. Here, after $t = 0.3$ the right vortex becomes smaller with an increase of exponential velocity of outer cylinder and the dominant heat transfer phenomenon is forced convection and the torque needed to create rotation is increased.

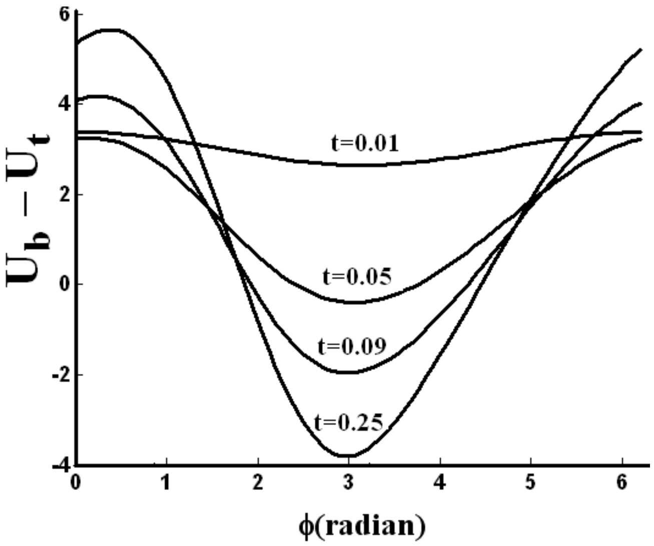

Fig. 5. Local relative tangential velocity

The following results are for the case of outer cylinder rotation velocity $\Omega_{out} = 10Exp(-t)$ and stationary inner cylinder and at $Ra = 1000$:

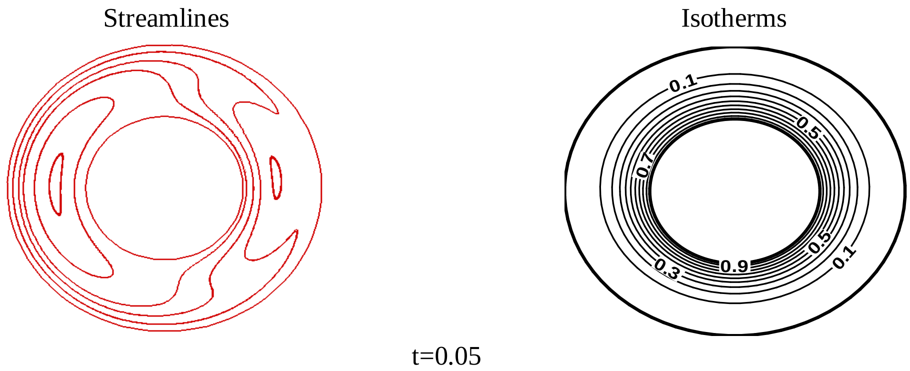

Fig. 6. Streamlines, isotherms, Nusselt number and dimensionless torque

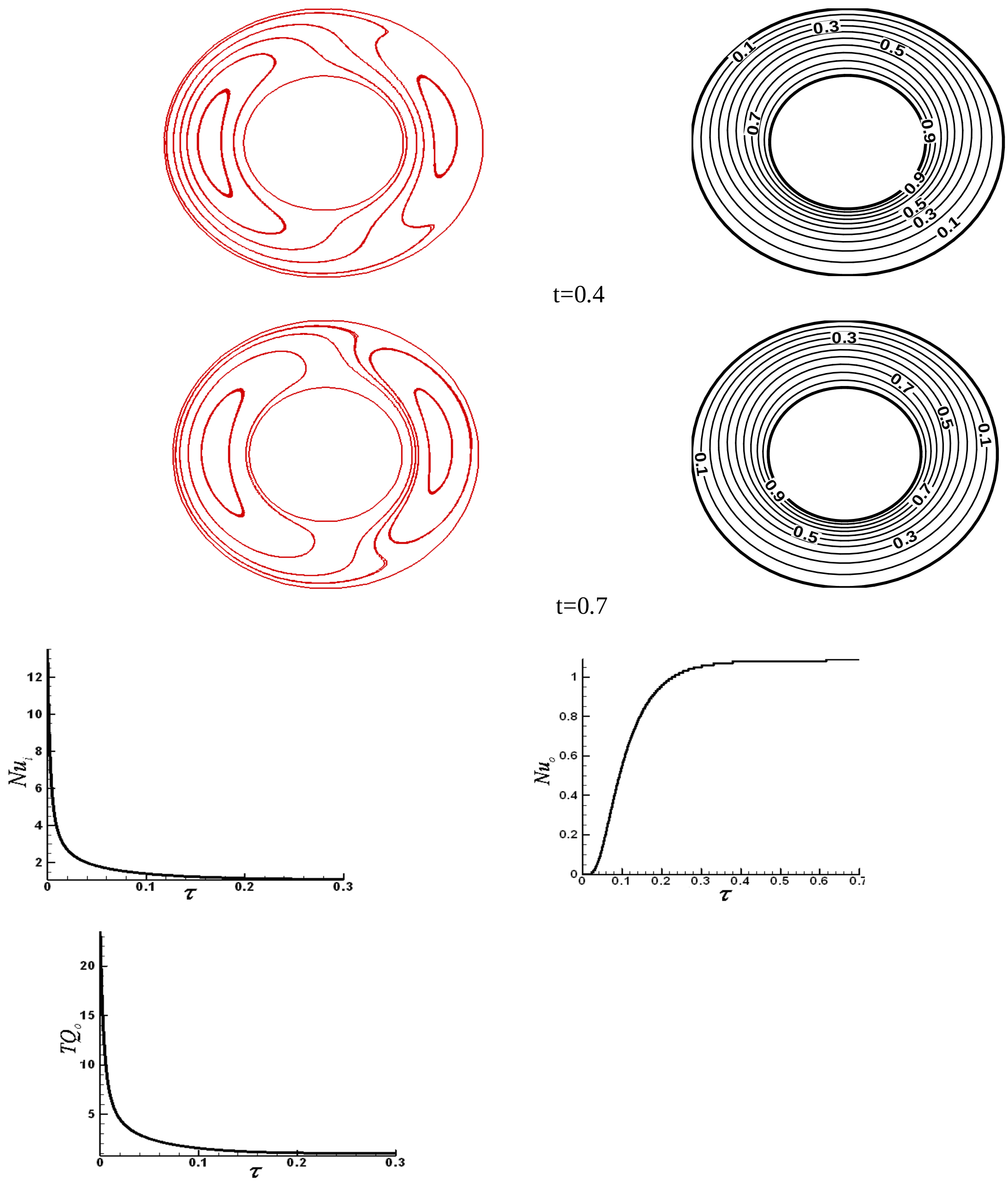

Figure 6 Continued

The isotherms are concentric circles like before but the streamlines are different. At the beginning both heat transfer phenomenon of forced and natural convection play an important role, but as time passes the outer cylinder velocity decreases gradually, as expected, and finally the flow tends to natural convection flow. The Nusselt number and the required torque for cylinder rotation also tend to an expected trend and finally tends to values related to pure conduction.

The following results are for the case of outer and inner cylinders rotation velocity $\mathbf{\Omega = 100\sin 10t}$ in counterclockwise direction and at $\mathbf{Ra = 5000}$:

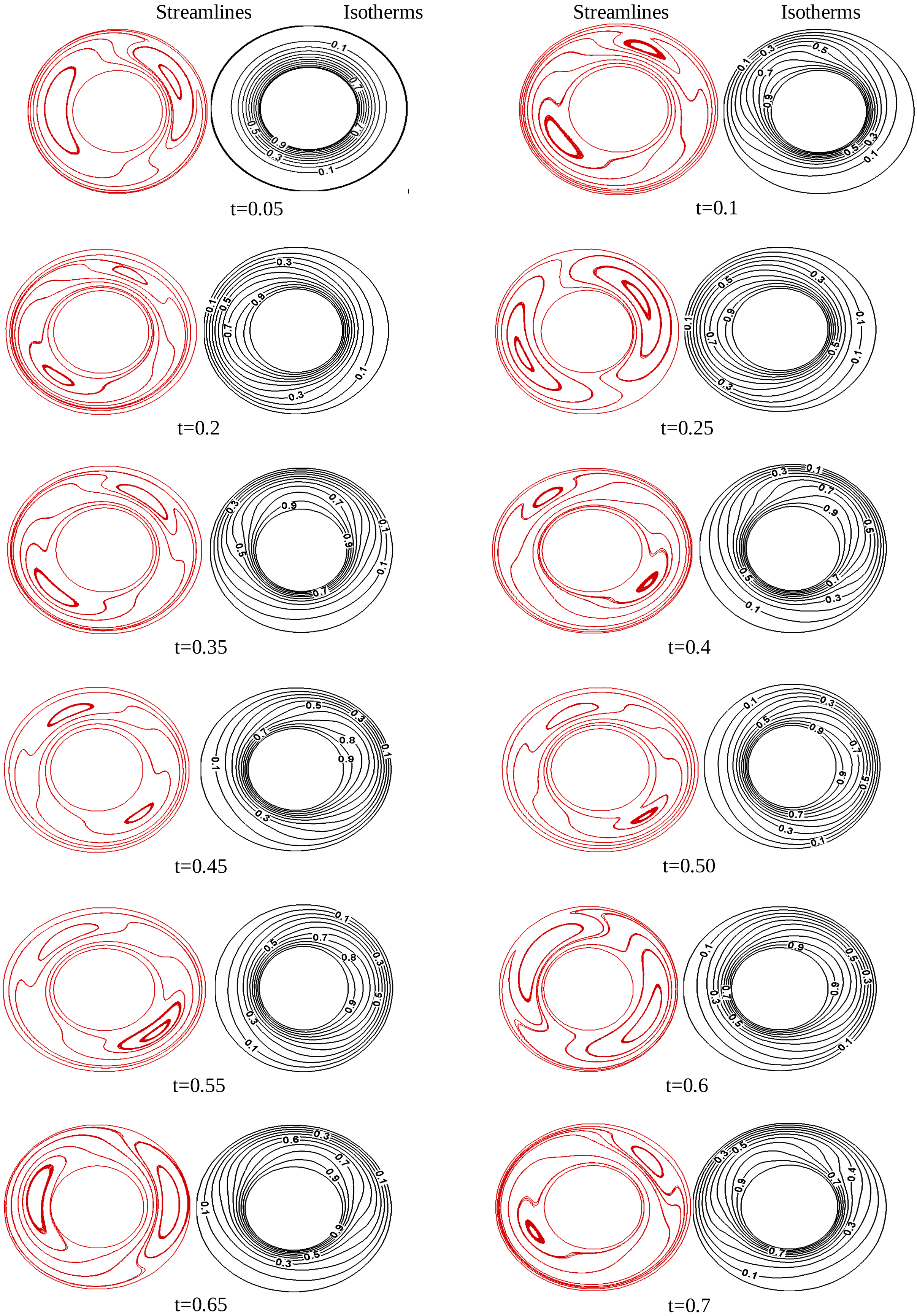

Fig. 7. Streamlines, isotherms, Nusselt number and dimensionless torque

Figure 7 Continued.

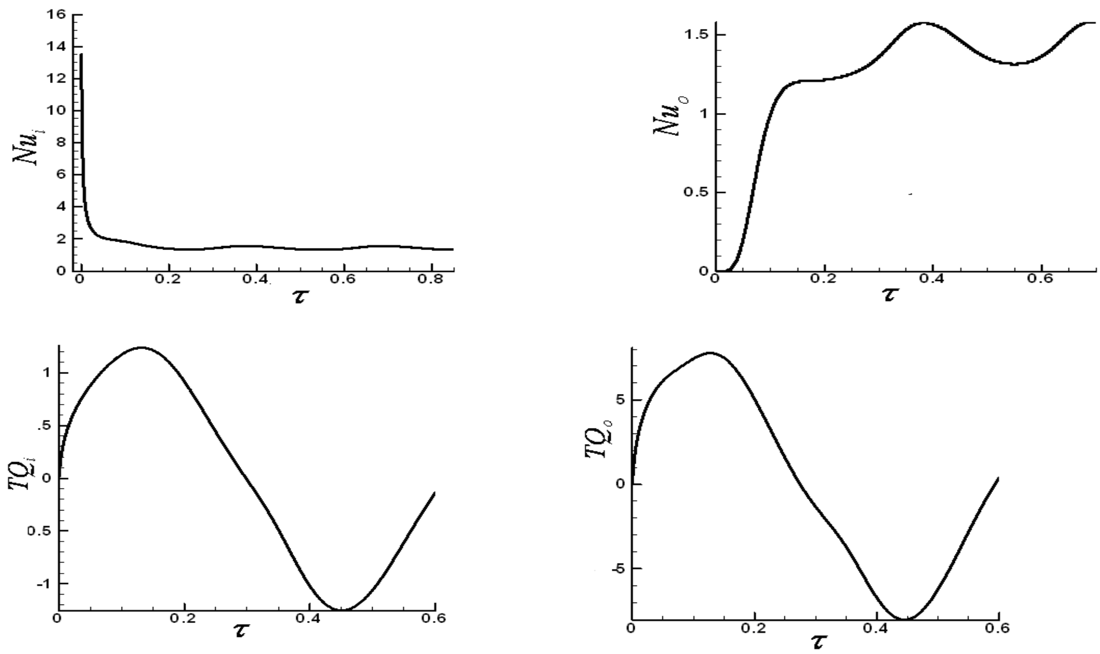

Fig. 7. Streamlines, isotherms, Nusselt number and dimensionless torque

In this case the role of natural convection is more dominant and the isotherms are no longer concentric and the largest value of eccentricity happens where the fluid particles are traveling in an almost vertical direction from the inner cylinder (warmer surface) to the outer cylinder (colder surface), see Fig. 8. But considering the changing velocity of the cylinders, the position and size of the vortices also change and the maximum temperature of the cylinders change where at the initial time values the flow is almost pure natural convection, and with the increase of the cylinder velocity and its effects on this natural convection, the mixed convection becomes dominant and the shape and size of the vortices change periodically as well as the position of the maximum temperature. The increase of Rayleigh number, contrary to Ra=1000, affects the Nusselt number greatly. As can be seen in the figure, the Nusselt numbers at the initial values of time are steady, while later on they change periodically. Note that because of the intense temperature gradients and its eccentricity this periodical situation shows itself more clearly in the vicinity of the inner cylinder.

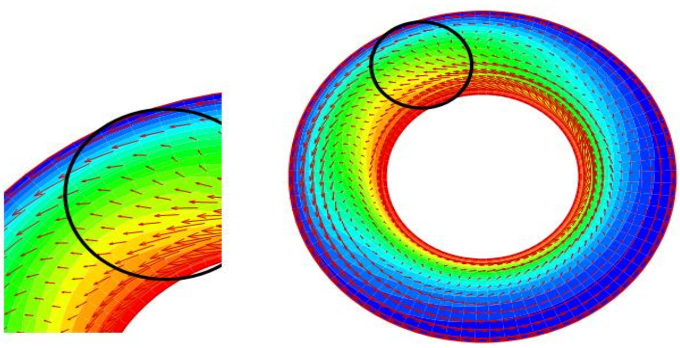

Fig. 8. Velocity vectors and their enlargement

The following results are for the case of outer and inner cylinders rotation velocity $\mathbf{\Omega = 50\exp(-2t)}$ in the same directions, counterclockwise, and at $Ra = 5000$:

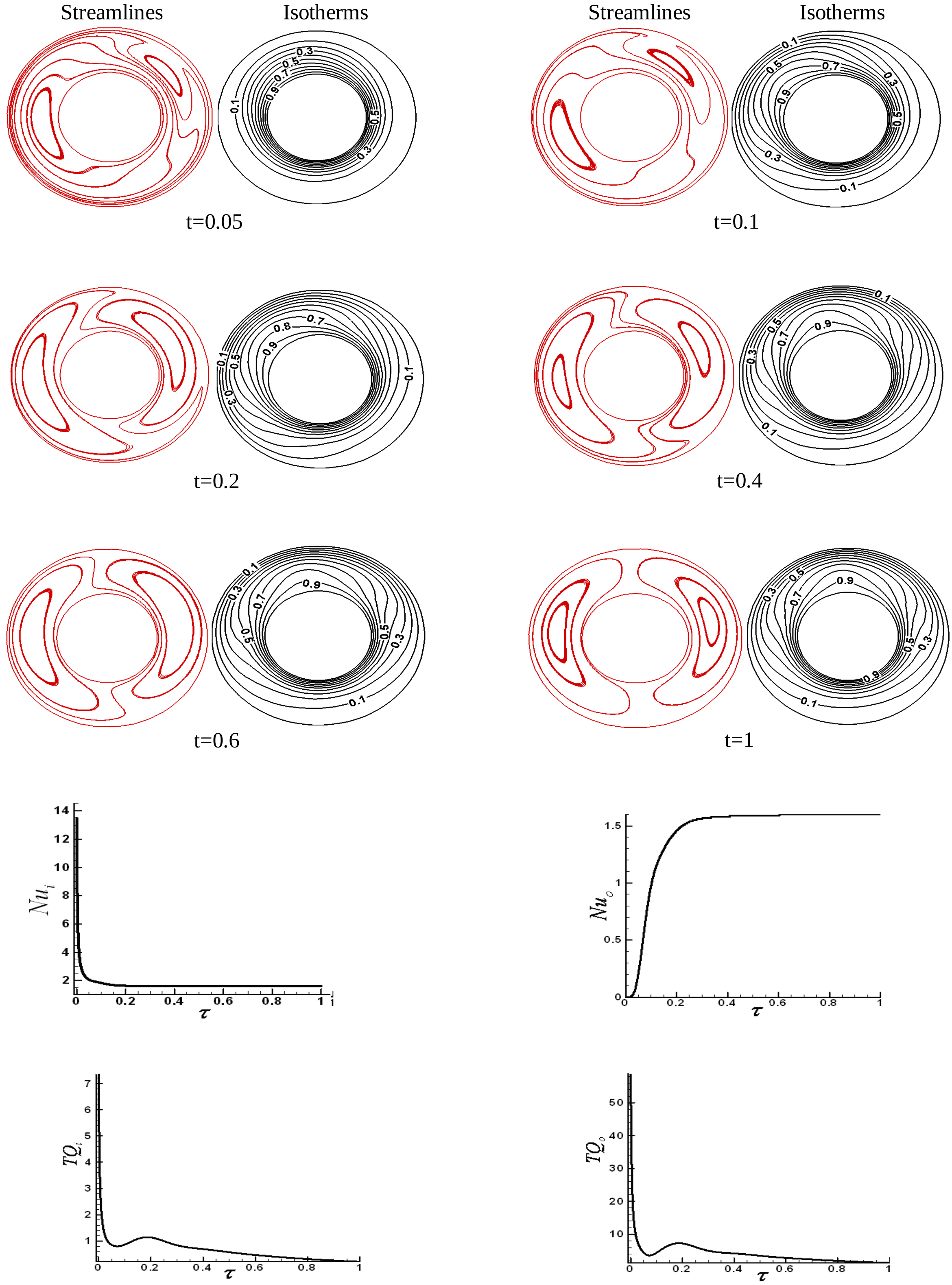

Fig. 9. Streamlines, isotherms, Nusselt number and dimensionless torque

The trend of flow is almost similar to conditions of Fig. 6 with the difference of effect of increase of Rayleigh number which was described earlier. The difference in the required torque curves for rotating the cylinders are also because of this Rayleigh number increase, which causes a strong vortex in the right side of the cylinder, while the left vortex covers the rest of the cylinder, $t = 0.2$. This vortex, having a strong inverse flow effect causes a sharp increase of velocity gradients in some parts of the cylinder periphery and in turn increases the required torque for rotation of the cylinder.

The following results are for the case of outer and inner cylinders rotation velocity $\mathbf{\Omega = 50\exp(t)}$ in different directions, inner cylinder rotation is clockwise, and at $Ra = 5000$:

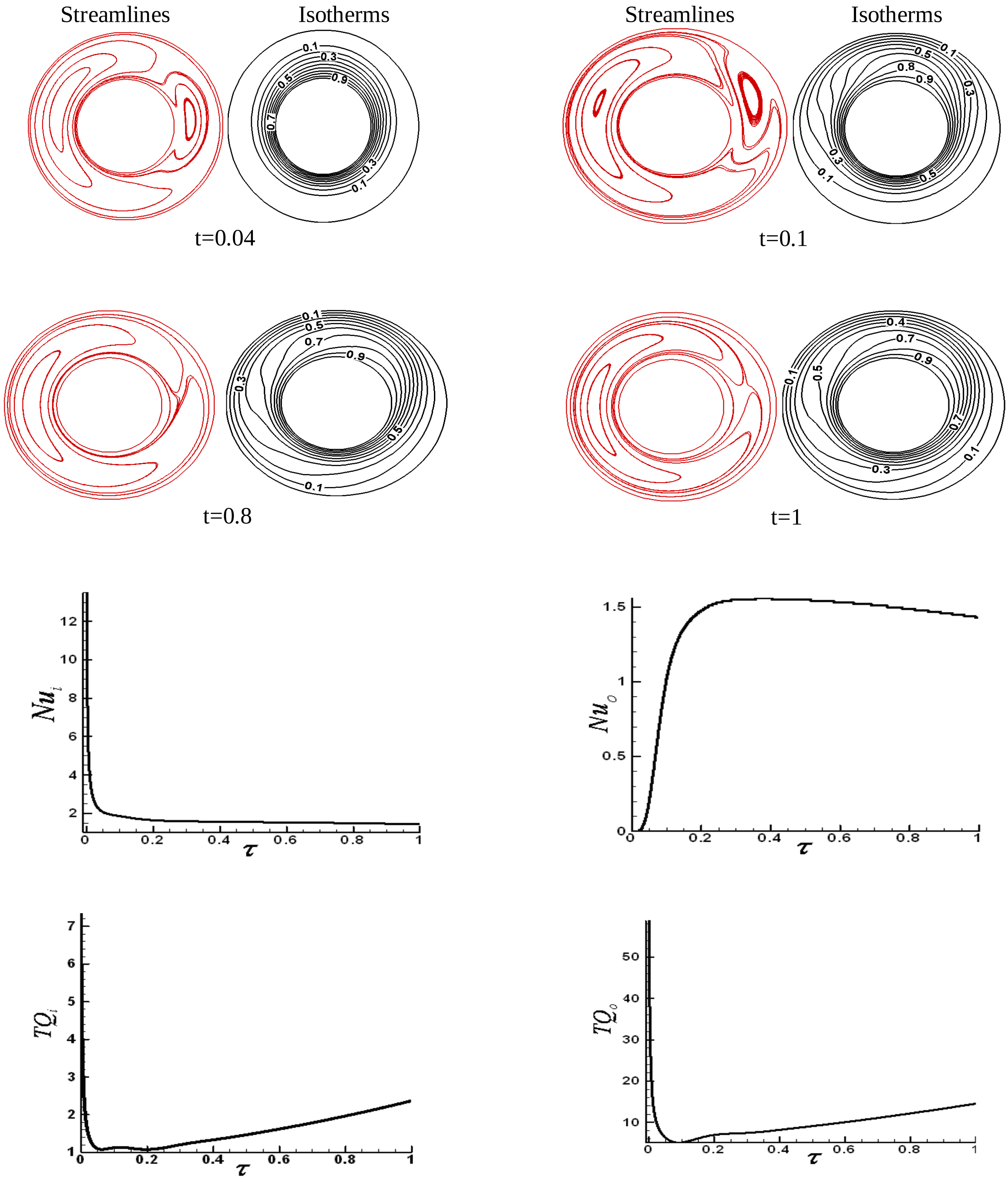

Fig. 10. Streamlines, isotherms, Nusselt number and dimensionless torque

The clockwise rotation of the inner cylinder helps the creation of natural convection flow in the left side of the cylinder and causes the vortex to be pulled toward the right hand side. The inverse of this happens at the left where the vortex in the left side becomes smaller and as time passes it fills the entire space. The isotherms do not change dramatically and the lines change as they are expected. The required torque for the inner cylinder in the period of 0.05-0.2 oscillates considerably and has two minimums. The first one is because of the initial fluid inertia and after the shape up of the complete effects of natural convection till $t = 0.05$ and coming togetherness of the vortices till $t = 0.1$, the left hand side vortex develops so much that it covers the surface of the inner cylinder completely and causes the decrease of torque in the period of $t = 0.1 - 0.2$. After $t = 0.2$, because of the increase of velocity and reduction of vortex effects in the vicinity of the wall, the required torque for inner cylinder rotation increases. The trend of change of the outer cylinder is as described in Fig. 4. The Nusselt number at the inner cylinder has a decreasing trend and at the outer cylinder this trend starts after $t = 0.4$, and this is because of the cancellation of the right hand side small vortex and therefore reduction in heat transfer.

The following results are for the case of outer and inner cylinders rotation velocity $\Omega_{out} = 50 \exp t$ and $\Omega_{in} = 50 \sin 5t$ respectively, in the same clockwise direction, and at $Ra = 5000$:

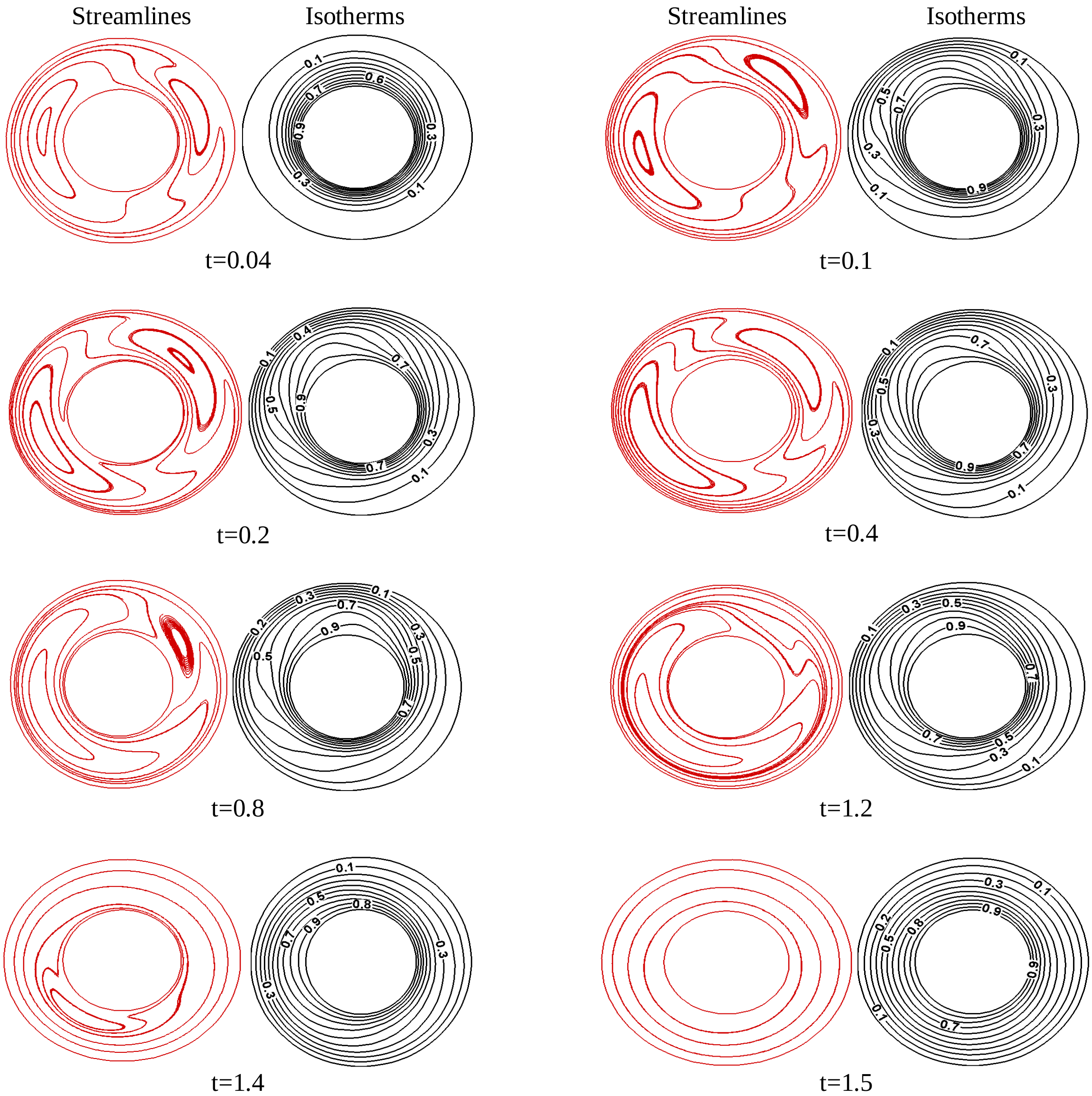

Fig. 11. Streamline, isotherms, Nusselt number and dimensionless torque

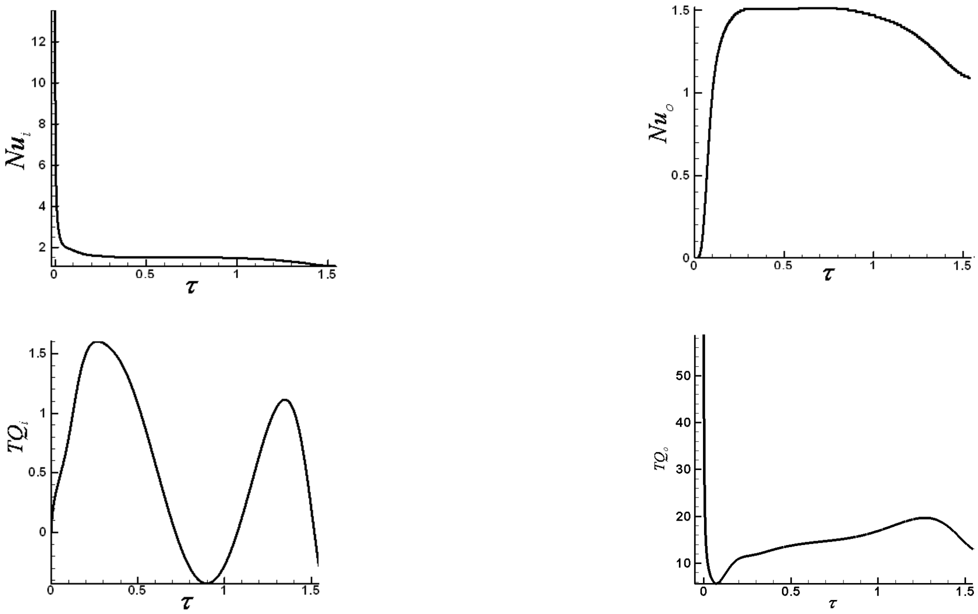

Figure 11 Continued

At the beginning, when the rotation effects and convection effects of both cylinders are at the same orders both of the left and right vortices are created and are in front of each other. This situation exists until around $t=1$ where because of the periodical change of inner cylinder the vortices, like in Fig. 7, change position and size. After $t=1$ on and inner cylinder velocity getting bigger, the flow tends to forced convection until $t=1.5$ when the last vortex disappears and the dominant flow is convection. The required torque for rotation passes through different stages. As it can be seen, the required torque for inner cylinder rotation has the same trend as Fig. 7 where the required torque for the outer cylinder is different. First it has a descending trend but after $t=0.05$ starts to increase dramatically. During the time $t=0.05-1.3$ the effect of periodical rotation of inner cylinder causes the required torque to have an ascending trend. Change of Nusselt number is similar to that in Fig. 10.

5. CONCLUSION

Numerical analysis of mixed convection flow between two concentric cylinders has been done using a finite volume numerical method. This study concentrates on the investigation of effects of time-dependent boundary conditions in which the effects of different rotation functions of inner and outer cylinders like exponential, sinusoidal is considered and also how they affect the fluid flow parameters and heat transfer has been studied. The nonlinear behavior of required torque for cylinder rotation is justified using the number and position of vortices. The trend of change of average Nusselt number can be studied by using different isotherms obtained in different time intervals.

NOMENCLATURES

g	acceleration of gravity	T	temperature
L	gap width of the annulus, $r_o - r_i$	T_h, T_c	temperatures at the inner and outer cylinders, respectively
Nu_{cond}	Nusselt number of pure conduction state	TQ_i, TQ_o	dimensionless torque at the inner and outer cylinders, respectively
Nu_i, Nu_o	local Nusselt numbers at the inner and outer cylinders, respectively	$\mathbf{u_r}, \mathbf{u_\phi}$	velocity components in the radial and angular directions, respectively.

$\overline{Nu_i}, \overline{Nu_o}$	mean Nusselt numbers at the inner and outer cylinders, respectively	U_R, U_ϕ	dimensionless velocity components in the radial and angular directions, respectively.
p	pressure		
P	dimensionless pressure	α	thermal diffusivity
Pr	Prandtl number, $\frac{\upsilon}{\alpha}$	β	coefficient of thermal expansion
		θ	dimensionless tempreture
R_i, R_o	dimensionless radii of the inner and outer cylinders, respectively	υ	kinematic viscosity
Ra	Rayleigh number based on the gap width, $\frac{\beta g(T_h - T_c)L^3}{\upsilon\alpha}$	ρ	density
		τ	dimensionless time
r	radial coordinate	Ω	cylinder angular velocity
R	dimensionless radial coordinate	ϕ	angular coordinate
t	time		

REFERENCES

1. Kuehn, T. H. & Goldstein, R. J. (1976). An experimental and theoretical study of natural convection in the annulus between horizontal concentric cylinders. *Journal of Fluid Mechanics*, pp. 695-719
2. Kuehn, T. & Goldstein, R. (1978). An experimental study of natural convection heat transfer in concentric and eccentric horizontal cylindrical annuli. *J. Heat Transfer*, Vol. 100, pp. 635–40.
3. Shahraki, F. (2002). Modeling of buoyancy-driven flow and heat transfer for air in a horizontal annulus: effects of vertical eccentricity and temperature-dependent properties. *Numer. Heat Transfer*, Vol. 42, pp. 603–621
4. Mizushima, J., Hayashi, S., Adachi, T. (2001). Transitions of natural convection in a horizontal annulus. *Int J Heat Mass Transfer*, Vol. 44, pp. 1249–57.
5. Padilla, E. L., Campregher, M. R. & Silveira-Neto, A. (2006). Numerical analysis of the natural convection in horizontal annuli at low and moderate Ra, *Thermal Engineering*, Vol. 5, No. 02, pp. 58-65
6. Vafai, K. & Ettefagh, J. (1991). An investigation of transient three-dimensional buoyancy-driven flow and heat transfer in a closed horizontal annulus. *International Journal of Heat and Mass Transfer*, Vol. 34, pp 2555-2570.
7. Desai, C. P. & Vafai, K. (1994). An investigation and comparative analysis of two- and three-dimensional turbulent natural convection in a horizontal annulus. *International Journal of Heat and Mass Transfer*, Vol. 37, pp. 2475–2504.
8. Lee, T. S. (1992). Numerical computation of fluid convection with air enclosed between the annuli of eccentric heated horizontal rotating cylinders. *Comput. Fluids (Int. J.)*, Vol. 1, No. 3, pp. 355–368.
9. Lee, T. S. (1994). Numerical studies of mixed recirculatory flow in annuli of stationary and rotating horizontal cylinders with different radius ratios. *Int. J. Num. Methods Heat Fluid Flow*, Vol. 4, pp. 561–573.
10. Chai, J. C. & Patankar, S. V. (1993). Laminar natural convection in internally finned horizontal annuli. *Numerical Heat Transfer*, Part A, Vol. 24, pp. 67-87.
11. Desrayaud, G., Lauriat, G. & Cadiou, P. (2000). Thermo Convective Instabilities in a Narrow Horizontal Air-filled Annulus. *Int. J. Heat Fluid Flow*, Vol. 21, pp. 65-73.
12. Patankar, S. V., Ivanovic, M. & Sparrow, E. M. (1979). Analysis of turbulent flow and heat transfer in internally finned tubes and annuli. *J. Heat Transfer*, Vol. 101, pp. 29-37.
13. Rustum, I. M. & Soliman, H. M. (1990). Numerical analysis of laminar mixed convection in horizintal internally finned tubes. *Int. J. Heat and Mass Transfer*, Vol. 33, pp. 1485-1496.
14. Dyko, M., Vafai, K. & Mojtabi, A. (1999). A nnumerical and experimental investigation of stability of natural convective flows within a horizontal annulus. *J. Fluid Mech.*, Vol. 381, pp. 27–61.

15. Yeh, C. (2002). Numerical investigation of the three-dimensional natural convection inside horizontal concentric annulus with specific wall temperature or heat flux. *Int. J. Heat Mass Transfer*, Vol. 42, pp. 775–84.
16. Le, T. S. (1992). Numerical computation of fluid convection with air enclosed between the annuli of eccentric heated horizontal rotating cylinders. *Comput. Fluids (Int. J.)*, Vol. 1, No. 3, pp. 355–368.
17. Yoo, J. S. (1998). Mixed convection of air between two horizontal concentric cylinders with a cooled rotating outer cylinder. *Int. J. Heat Mass Transfer*, Vol. 41, No. 2, pp. 293-302.
18. Fusegi, T., Farouk, B. & Enneth, S. B. (1986). Mixed convection flows within a horizontal concentric annulus with a heated rotating inner cylinder. *Numer. Heat Transfer*, Vol. 9, pp. 591–604.
19. Choudhury, D. & Karki, K. C. (1992). Laminar mixed convection in a horizontal eccentric annulus. *Numer. Heat Transfer*, Vol. 22, pp. 87–108.
20. Habib, M. A. & Negm, A. A. (2001). Laminar mixed convection in horizontal concentric annuli with non-uniform circumferential heating. *Heat and Mass Transfer*, Vol. 37, pp. 427-435.
21. Alawadhi, E. M. (2008). Natural convection flow in a horizontal annulus with an oscillating inner cylinder using lagrangian–eulerian kinematics. *Computers and Fluids*, Vol. 37, pp. 1253–1261.
22. Glakpe, E. K., Watkins, J. & Cannon, J. N. (1986). Constant heat flux solution for natural convection between concentric and eccentric horizontal cylinders. *Numerical Heat Transfer*, Vol. 10, pp. 279-295.
23. Glakpe, E. K. & Watkins, J. (1987). Effect of mixed boundary condition on natural convection in concentric and eccentric annular enclosure. *AIAA Paper*, pp. 1587-1591.
24. Hessami, M. A., Pollar, A. & Rowe, R. D. (1984). Numerical calculation of natural convection heat transfer between horizontal concentric isothermal cylinders- effect of variation of fluid properties. *Heat Transfer J.*, Vol. 106, pp. 668-671.
25. Kumar, P. (1988). Study of natural convection in horizontal annuli. *International Journal of Heat and Mass Transfer*, Vol. 31, pp. 1137-l 148.
26. Mirza, S. & Soliman, H. M. (1985). The influence of internal fins on mixed convection inside horizontal tubes. *Int. Commun. Heat Mass Transfer*, Vol. 12, pp. 191-200.
27. Tsui, Y. T. & Tremblay, B. (1984). On transient natural convection heat transfer in the annulus between concentric horizontal cylinders with isothermal surfaces. *Int. J. of Heat and Mass Transfer*, Vol. 21, pp. 103-111.
28. Yang, H. Q., Yang, K. T. & Loyd, J. R. (1988). Natural convection suppression in horizontal annuli by azimuthal baffles. *Int. J. Heat Mass transfer*, Vol. 31, pp 2123-2136
29. Van de Sande, E. & Hamer, B. J. G. (1979). Steady and transient natural convection in enclosures between circular cylinders (constant heat flux). *lnt. J. Heat Mass Transfer*, Vol. 22, pp. 361-370.
30. Patankar, S. V. (1980). *Numerical heat transfer and fluid flow.* McGraw-Hill, New York.