

## Angular dependence of pinning potential, upper critical field, and irreversibility field in underdoped BaFe<sub>1.9</sub>Co<sub>0.1</sub>As<sub>2</sub> single crystal

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# Angular dependence of pinning potential, upper critical field, and irreversibility field in underdoped BaFe<sub>1.9</sub>Co<sub>0.1</sub>As<sub>2</sub> single crystal

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Underdoped BaFe<sub>1.9</sub>Co<sub>0.1</sub>As<sub>2</sub> single crystal was studied by angular dependence of magneto-transport at fields up to 13 T over a wide range of temperature. Our results show that pinning potential,  $U_0$ , decreases slightly for  $\theta \leq 45^\circ$  and remains constant for  $\theta \geq 45^\circ$ , while the upper critical field,  $H_{c2}$ , and the irreversibility field,  $H_{irr}$ , increase with  $\theta$ . According to anisotropic Ginzburg-Landau theory, the anisotropy was determined by scaling the resistivity under different magnetic fields below the superconducting critical temperature,  $T_c$ . Anisotropy,  $\Gamma$ , in the underdoped crystal is found to be temperature dependent and decreases from 2.1 to 1.8 for as  $T$  is reduced from 17 to 12.5 K. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.3692582>]

The thermal activation behaviour of vortices in superconductors determines their magneto-transport properties, which are critical for practical applications. In highly anisotropic cuprates, a magnetic field perpendicular to the superconducting layers penetrates in the form of pancake vortices, while a parallel field creates Josephson vortices.<sup>1</sup> The interaction between pancake vortices and Josephson vortices creates vortex chains<sup>2</sup> or Josephson vortices decorated by pancake vortices<sup>3</sup> when the magnetic field is tilted. The pinning potential in cuprates is highly anisotropic and strongly field dependent due to the strong thermally activated behaviour of two-dimensional (2D) pancake vortices.

Iron based superconductors<sup>4</sup> exhibit relatively high transition temperature,  $T_c$ , very high upper critical field,<sup>5</sup>  $H_{c2}$ , and relatively low anisotropy.<sup>6-9</sup> Among pnictide superconductors, Ba-122 compounds have typical 2D layered crystal structure. However, they show nearly isotropic superconductivity<sup>6</sup> and very high intrinsic pinning potential, which is weakly field dependent.<sup>10</sup> These unique features make the 122 superconductors more favourable for practical application than other pnictide superconductors.

Thermally activated flux flow has been studied in NdFeAsO<sub>0.7</sub>F<sub>0.3</sub>,<sup>11</sup> Ba<sub>0.72</sub>K<sub>0.28</sub>Fe<sub>2</sub>As<sub>2</sub>,<sup>10</sup> BaFe<sub>1.9</sub>Ni<sub>0.1</sub>As<sub>2</sub>,<sup>12</sup> and Tl<sub>0.58</sub>Rb<sub>0.42</sub>Fe<sub>1.72</sub>Se<sub>2</sub> (Ref. 13) single crystals for  $H//ab$  and  $H//c$ . Very recently, studies of the angular dependence of the transport critical current density,  $J_c$ , have indicated that the  $J_c$  decreases monotonically with angle,  $\theta$ , for  $\theta < 90^\circ$ , where  $\theta$  is defined as the angle between the magnetic field and the  $c$ -axis. The ratio of  $J_c(H//ab)/J_c(H//c) = 7.5$  or  $1.8$  at  $H = 1$  T and  $T = 4.2$  K for La-1111 (Ref. 14) and Co-122 (Ref. 15) thin film, respectively. It should be noted that the determination of the angular dependence of the pinning potential,  $H_{c2}$ , and the irreversibility field,  $H_{irr}$ , are important for understanding how  $J_c$  changes with both

angle and field. So far, there has been no report on the angular dependence of these parameters in any pnictide superconductors. In this letter, we report the angular dependence of the upper critical field, the irreversibility field, and the pinning potential of underdoped BaFe<sub>1.9</sub>Co<sub>0.1</sub>As<sub>2</sub> single crystal by measuring magneto-transport at different magnetic fields and angles. Furthermore, by scaling the angular dependence of the resistance, based on the anisotropic Ginzburg-Landau (GL) theory, the anisotropy value ( $\Gamma$ ) was determined for different temperatures. Our results show that the pinning potential,  $U_0$ , is strongly angle dependent for  $\theta \leq 45^\circ$  and almost angle independent for  $\theta \geq 45^\circ$ , while both  $H_{c2}$  and  $H_{irr}$  increase monotonically with increasing angle.

The single crystal of BaFe<sub>1.9</sub>Co<sub>0.1</sub>As<sub>2</sub> was grown using the high temperature self-flux method. FeAs and CoAs were prepared by placing a mixture of As powder and Fe/Co powder in a quartz tube and reacting it at 600 °C for 10 h after it had been heated to 600 °C for 17 h. A mixture of FeAs/CoAs and Ba pieces was then placed in an alumina crucible. The whole assembly was sealed in a large quartz tube and heated to 1180 °C for 15 h, which was followed by a reaction at 1180 °C for 10 h.<sup>16</sup> The as-grown single crystal was cleaved and cut into a rectangular shape for measurements. The transport properties were measured over a wide range of temperature and magnetic fields up to 13 T with applied current of 5 mA, using a physical properties measurement system (PPMS, quantum design). The current was applied in the  $ab$  plane. The angular dependence of the resistivity was measured using the 13 T PPMS, with the angle,  $\theta$ , varied from 0° to 180°, where  $\theta = 0^\circ$  corresponded to the configuration of  $H//c$  and  $\theta = 90^\circ$  to  $H//ab$ , respectively.

The temperature dependence of the in-plane resistivity of the underdoped BaFe<sub>1.9</sub>Co<sub>0.1</sub>As<sub>2</sub> single crystal is shown in Fig. 1. The resistivity decreases with decreasing temperature from 300 K to 50 K, supporting metallic behaviour of this compound. The resistivity increases with further decreasing temperature and shows an anomaly at 49 K due to a

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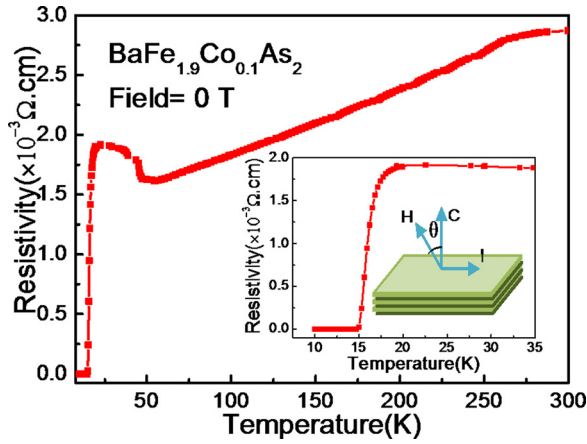


FIG. 1. (Color online) Temperature dependence of the in-plane resistivity of  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$ . The inset shows an enlargement of the region with  $T \leq 35$  K and a schematic diagram of the sample.

magnetic/structural phase transition.<sup>17</sup> The  $T_c^{\text{onset}}$  and  $T_c(0)$  were determined to be 17.4 and 15.2 K, respectively.

The angular dependence of the resistance at  $T = 17$  K is shown in Figure 2(a). All the curves show a symmetric dip-like structure, with a minimum at  $90^\circ$  and maximum resistance at  $0^\circ$  and  $180^\circ$ . The normal state resistance decreases with decreasing magnetic field and temperature, due to the enhancement of the superconducting state. The angular dependence of resistance is not very sharp with varying field and temperature, possibly due to the moderate anisotropy of the  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$  sample, which has also been reported in Nd-1111 single crystals.<sup>18</sup> Similar behaviour observed at  $T = 15$  and 12.5 K.

In layered superconductors, the variation in the superconducting order parameter,  $\psi$ , can be described by  $\delta\Psi/\delta z$  when the order parameter is quasi-continuous across the neighbouring layers. Therefore, the anisotropic Ginzburg-Landau approximation,  $H_{c2}^{ab}/\sqrt{\sin^2\theta + \Gamma^2\cos^2\theta}$ , can be used to estimate the anisotropy values for our sample. The angular dependence of the resistance can be scaled as  $R = R(0)f(H/H_{c2}^{GL})$ .<sup>19</sup> Then, the resistance measured under different magnetic fields should collapse into one curve at a certain temperature if the  $\Gamma$  parameter is properly scaled. The results of this scaling at  $T = 17$  K is shown in Figure 2(b). The estimated anisotropy value, is 1.8 for  $T = 12.5$  K, 1.9 for  $T = 15$  K, and 2.1 for  $T = 17$  K. The obtained anisotropy values are very close to the reported value of 2 at  $T = T_c$  for  $\text{BaFe}_{2-x}\text{Co}_x\text{As}_2$  single crystals.<sup>20</sup> The anisotropy values which are obtained from this rescaling analysis are more reli-

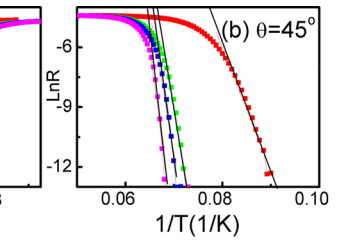
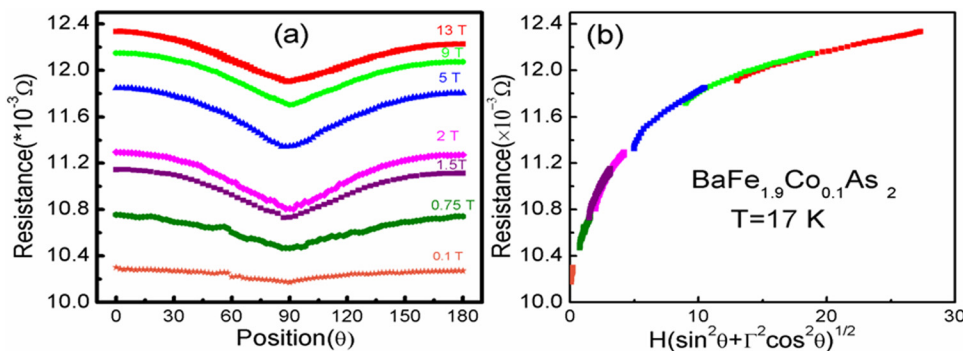


FIG. 3. (Color online) (a) Temperature dependence of resistance at different magnetic fields for  $\theta = 45^\circ$  where  $\theta$  is the angles between the applied magnetic field and the  $c$ -axis. (b) Arrhenius plots of the resistivity at the same angles in underdoped  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$  single crystal.

able compared to estimating the anisotropy using the ratio of  $H_{c2}$  in the  $ab$  plane to that along the  $c$ -axis.

The results of magneto-transport measurements at  $\theta = 45^\circ$  is shown in Fig. 3(a) where  $\theta$  is the angles between the applied magnetic field and the  $c$ -axis. According to the thermally activated flux flow model, the resistance can be described as

$$R = (2v_0LH/J)\sinh\left[\frac{JHVL}{T}\right]e^{i_c0HVL/T}, \quad (1)$$

where  $v_0$  is the attempt frequency for flux bundle hopping,  $L$  is the hopping distance,  $J$  is the applied current density,  $J_{c0}$  is the critical current density at 0 T, and  $V$  is the bundle volume.<sup>21</sup> If  $JHVL/T \ll 1$ , Eq. (1) can be written as

$$R = (2R_cU/T)e^{-U/T}, \quad (2)$$

where  $U = J_{c0}HVL$  is the thermally activated energy and the critical resistance,  $R_c = v_0LH/J_{c0}$ . Assuming that  $2R_cU/T$  is a temperature independent constant, defined as  $R_0$  and  $U = U_0(1-T/T_c)$  then Eq. (2) can be simplified as the Arrhenius equation

$$\ln R(T, H) = \ln R_0 - U_0/T,$$

Therefore, the activation energy,  $U_0(B)$ , is the slope of the lower part of the curve in the Arrhenius plot.

In order to study the flux motion in  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$  single crystal, we plot the electrical resistance as a function of  $1/T$  at different magnetic fields up to 7 T and different angles between the applied magnetic field and the  $c$ -axis. Fig. 3(b) shows the Arrhenius plot of the resistance for  $\theta = 45^\circ$ . The linear dependence of  $\ln R$  on  $1/T$  in the lower part of the curves indicates that this part can be described by the thermally

FIG. 2. (Color online) (a) The angular dependence of the magnetoresistance at  $T = 17$  K. (b) Scaling of the resistance as a function of  $H(\sin^2\theta + \Gamma^2\cos^2\theta)^{1/2}$  based on GL theory at  $T = 17$  K.



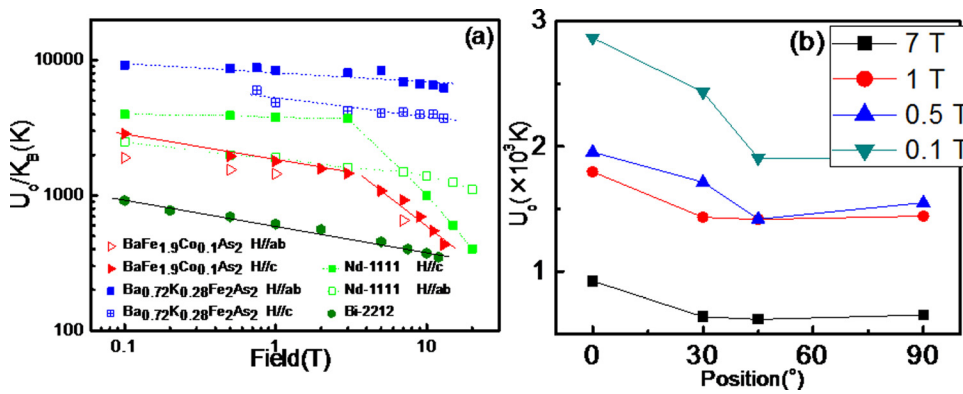


FIG. 4. (a) (Color online) Field dependence of  $U_0$  for  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$  single crystal. (b) Angular dependence of pinning potential for different magnetic fields.

activated flux flow model.<sup>11</sup> Similar measurements were performed for several other angles, from  $0^\circ$  up to  $90^\circ$ , and  $U_0(B)$  was calculated from the corresponding Arrhenius plots.

Fig. 4(a) shows the magnetic field dependence of  $U_0$  for  $H//c$  and  $H//ab$  in underdoped  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$  single crystal. The best fit of the experimental data yields pinning potential values ranging from  $U_0/k_B = 2900$  K to 1900 K, where  $k_B$  is the Boltzmann constant, for  $H//c$  and  $H//ab$ , respectively, at the low magnetic field of 0.1 T. These values are comparable to the reported values of  $U_0 = 3000$ –4000 K for  $\text{NdFeAsO}_{0.82}\text{F}_{0.18}$  single crystals.<sup>11</sup> For comparison, we also included the  $U_0$  for Bi-2212 single crystal in Fig. 4(a). The  $U_0$  value for underdoped  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$  is three times higher than for Bi-2212 single crystal.<sup>22</sup> However, this value is lower than the reported value of 9100 K for  $\text{Ba}_{0.72}\text{K}_{0.28}\text{Fe}_2\text{As}_2$  single crystal<sup>10</sup> for  $H//ab$ . The  $U_0$  value for our underdoped  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$  crystal with  $H//c$  exhibits a weak power-law dependence ( $U_0 \propto H^{-0.15}$ ) for  $H < 3$  T, where single vortex pinning dominates, followed by a power-law decrease with  $U_0 \propto H^{-0.81}$ , indicating collective pinning at higher fields,  $H > 3$  T. The pinning potential values decrease when the sample is rotated from  $H//c$  to  $H//ab$ . It should be noted that the  $U_0$  decreases slightly for  $0 < \theta \leq 45^\circ$ , and then remains constant for  $\theta \geq 45^\circ$  (Fig. 4(b)). Possibly, twin boundaries which can form below the temperature of the structural transition from tetragonal to

orthorhombic could act as barriers to vortex motion, resulting in the higher  $U_0$  value for  $H//c$  (Refs. 23 and 24) compared to that for  $H//ab$ .

The critical current density,  $J_c$ , vs. magnetic field is shown in Fig. 5(a).  $J_c$  was calculated from the magnetic hysteresis data at  $T = 2$  and 4.2 K by using the extended Bean model

$$J_c = 20 \times \Delta M / V / (a(1 - a/3b)) \quad \text{with } a < b,$$

where  $a$  and  $b$  are the sample dimensions,  $V$  is the sample volume, and  $\Delta M$  is the height of the magnetization loop. The  $J_c$  is as high as  $1.7 \times 10^5$  and  $1.5 \times 10^5$  A/cm<sup>2</sup> at  $T = 2$  and 4.2 K in zero magnetic fields, respectively. The  $J_c$  decreases with increasing magnetic field up to 1 T and after that become nearly field independent which is related to relatively high pinning potential and weakly anisotropic property in  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$  single crystal.

The upper critical field,  $H_{c2}$ , is characterized as the field at which the resistivity becomes 90% of the normal state resistivity; while the irreversibility field,  $H_{irr}$ , is defined by 10% of the normal state resistivity. Figure 5(b) shows  $H_{c2}$  and  $H_{irr}$  as functions of temperature for different angles between the field and the  $c$ -axis. From Fig. 5(c), the  $H_{c2}$  and  $H_{irr}$  follow the same trend with angle for  $\theta > 45^\circ$ . The estimated slopes  $dH_{c2}^{ab}/dT$  and  $dH_{irr}^{ab}/dT$  are 4.8 and 3.79 T/K for  $H//ab$ . The  $dH_{c2}^{ab}/dT$  is in good agreement with the reported value of  $dH_{c2}^{ab}/dT = 4.9$  T/K.<sup>20</sup> The slopes decrease as the sample is rotated from  $0^\circ$  to  $90^\circ$  (Fig. 5(d)), which is similar to the trend of  $U_0$  with angle.

In summary, we have investigated the angular dependence of  $U_0$ ,  $H_{c2}$ , and  $H_{irr}$  for underdoped  $\text{BaFe}_{1.9}\text{Co}_{0.1}\text{As}_2$  single crystal.  $U_0$  decreases while  $H_{c2}$  and  $H_{irr}$  increase with angle from  $H//c$  to  $H//ab$ . The anisotropy parameter decreased from 2.1 to 1.8 as  $T$  decreased from 17 to 12.5 K, using the anisotropic GL theory.

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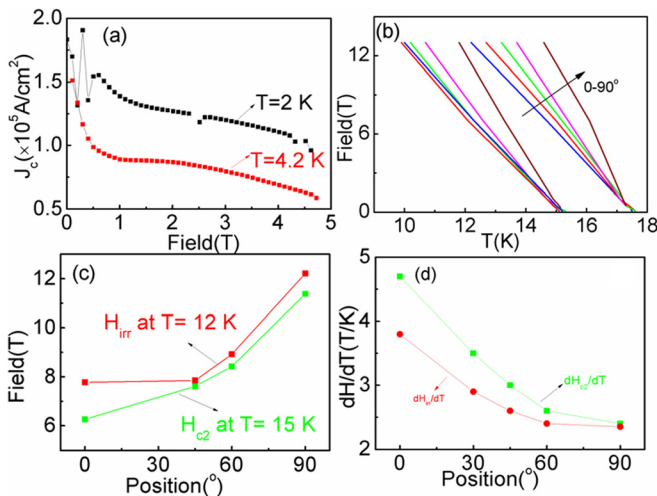


FIG. 5. (a) (Color online) Field dependence of  $J_c$  at  $T = 2$  and 4.2 K. (b) Angular dependence of  $H_{c2}$  and  $H_{irr}$  as a function of temperature. (c) Angular dependence of  $H_{c2}$  and  $H_{irr}$  at fixed temperature. (d) Angular dependence of  $dH_{c2}/dT$  and  $dH_{irr}/dT$ .

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