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Application of Game Theory to Field Crops in Khorasan-Razavi province

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ABSTRACT

In this study, two goals are pursued. The first, will be addressed the relationship between game theory and linear programming and then, the application of game theory is checked for crops Khorasan Razavi province. Actually, this is a theory that is related to decide when two or more than two competitors compete in a rational. In this study, game theory model is used for the province's major crops include irrigated wheat, rain fed wheat, irrigated barley, rain fed barley, irrigated sugar beet and irrigated cotton. The data included time series of gross product values of the investigated crops for the period 2000-2009. In this study, in game theory have been used the "Wald" decision-making criterion to determine the highest income under the worst conditions. The pattern results Show irrigated sugar beet cultivation is risky product for the period studied. Irrigated sugar beet is included in the optimization program since it will be the highest expected income in the worst conditions. On the other hand, it has the highest coefficient of variation compared to the other crops. As a result, the game theory model is a good indicator for selecting alternative management strategies for farmers.

Key words: game theory, linear programming, risk, Field crops.

INTRODUCTION

Achieving the objectives of agricultural development is only possible on the condition that the determinated policies and formulated appropriate programs in the agricultural sector, this situation largely depends on the awareness level of planner farmers' reaction. Since the agriculture programs results are signified in the future and there is no assurance what happens in the future, so agricultural systems programs are always accompanied with uncertainty and risk. In addition, if farmers want to have the chance to earn profits, they must accept the risk; because the profitable management strategy is not without risk. Farmers should balance between losses caused by weather conditions and the potential benefits of management strategies. In this construction, farm management finds more importance over the past years. There are risk-taking and uncertainty in marketing, production, investment, level of technology, political events and weather conditions, especially in agriculture which uncertainty about long-term plan is very high [2]. According to The ruling forces in agriculture, it demands that the technology and the economy should be compatible with these features. Therefore, consideration of different methods in planning such as game theory seems substantial in this study.

In game theory, players want their outcome, which the existing limitations influence on the amount of it, reach the optimal level. In a zero-sum game with two players when both players choose the best strategies, the highest acquired outcome of a player is equal to the lowest missing outcome of the opponent player [14]. Therefore, to maximize the exchange value of the outcome is exactly equivalent to minimize consequences of opponent.

Although the game methods application on agricultural issues may be useful to help farmers, game theory has been used less in agricultural economics research. Thus, cases have been reported using game theory to issues in agriculture, is negligible. Thus, reported cases using game theory in agricultural issues, are negligible. Initial

researches on game theory have been carried out by Langham (1963), McInerney (1967), Agrawal and Heady (1968), McInerney (1969), Hazell (1970), and Kawaguchi and Maruyama (1972) and Hazell (2001).

The concept of game theory has led to agricultural economists reevaluate their views about management decisions and strategies that will be followed by farmers; Nevertheless, there are a number of agricultural issues that can be solved with the application of game theory. Game theory has been implemented in the production and marketing issues, the relationship between land owners and lease of land in rented farms, too.

So far, Iran has had a less interest in farm management and planning. In general, the linear programming method has been used in agricultural planning. Linear programming will determine the most profit based on given information and does not consider risk-taking and uncertainty in computing. For this reason, this study is important for two reasons: First, the first research on the study area (ie, Khorasan Razavi province) is the application of game theory in farm planning. Second, game theory has been used to account for risk-taking and uncertainty in field programming. The first objective of this study is to demonstrate the relationship between game theory and linear programming. The second goal is determination of the highest expected income of expected outcome earned from studied products in the worst conditions. To achieve this goal, game theory has been tested for the most important products of Khorasan Razavi province, including irrigated wheat, rain fed wheat, irrigated barley, rain fed barley, irrigated sugar beet and irrigated cotton.

Ozkan and Akcaoz (2001) in his article as game theory and its use for crops in Antalya province utilized the "Wald" decision-making criterion between 1980 and 1999. They concluded that peanut and cotton are the riskiest crops for the evaluated area and have the highest expected revenue under the worst conditions And enter the optimal crop plan.

Goodarzi and Homayoun Far (1385) in their research as the application of game theory on crops grown in Fars province utilized the "Wald" decision-making criterion in the game theory to determine the highest levels of income in the worst condition. The results showed that potato and paddy-paddy field were the riskiest yields in Fars province for the period 1383-1363 and this product had the highest expected revenue in the worst conditions; hence, they are included in the optimization program.

Karbasi, Rostamian and Yaghoubi (1390) showed in a research as the application of game theory on the legumes cultivation of Kohgiluyeh Boyer Ahmad province between 1361 to 1387 that white beans have not suggested in any cropping pattern of 26 implemented field programs and in contrast, most cultivation is by red beans.

MATERIALS AND METHODS

In this study, gross product value data, including price and yield of major products such as: Irrigated wheat, rain fed wheat, irrigated barley, rain fed barley, irrigated sugar beet and irrigated cotton for 1388 - 1379 which were prepared from Agriculture Organization of Khorasan Razavi province. It considered that effects of climate, prices and other factors that was about last year, would be valid for subsequent years, in this model [16]. Gross production value was calculated by multiplying crop yield and prices received by farmers. To estimate the optimal cropping pattern of the products using game theory, we implemented QM for Windows software.

Games generally are classified according to two criteria: 1) the number of participants in the game and 2) the net outcome of the game. The first criterion is the number of participants with conflicting interests. The second criterion makes it possible to distinguish between the zero-sum games and non-zero sum games. A zero-sum game is a game in which the algebraic sum outcomes for all participants and for all possible combinations of strategy are equal to zero. Farmers are working in a situation combined with risk and uncertainty. Uncertainty of future price and crops yield will cause uncertainty in farmer's income. Thus, the entering of risk in agricultural planning is essential. So all the risks and uncertainty facing a farmer can be summarized in the form of a combination of natural ingredients and Farmer in front of nature are considered as actors in two-person zero-sum game that largely nature may be ineffective Decision of a farmer in selected his farm financial programs randomly [10]. In this situation, there are different decision-making criterions to help select a farm program. Four classic criteria in this regard included: Wald's criterion (maximin), Laplace's criterion, Hurwiz's criterion and Savage's regret criterion. In this study, we implemented Wald's criterion (maximin) in game theory model. Criteria, which based on it the farmer, select the best (highest) income under the worst (lowest) state of nature [10].

Game theory rests on postulate the behavior of participants and may make possible to achieve balance in these conditions. The first actor is afraid of the second player will recognize his chosen strategy; accordingly, prediction

of his behavior for his rival would be easy. If the first player has m strategy and second player has n strategy, the possible outcomes of game can be shown by the following benefit matrix.

[a ₁₁	<i>a</i> ₁₂		<i>a</i> _{1n}]	
a ₂₁ :	a ₂₂ :	 、	a_{2n} :	Profit matrix
a_{m1}	a_{m2}		a_{mn}	

In this matrix, a_{ij} is the first actor's profit, when he uses his i-th strategy and the second actor uses his j-th strategy. If the first player chooses the i-th strategy, minimum his benefit (ie maximum rival's profit) is determined with the smallest element in row i of the benefit matrix min_ja_{ij} . Also, the first player wishes to maximized at least his expected profits, therefore the first player chooses strategy i and min_ja_{ij} is the highest for his, as a result $max_imin_ja_{ij}$ is his desired outcome. He cannot achieve less benefit and may even take more profits. The second player is afraid of the first player to be aware of his information and behavior. If the second player uses his j-th strategy, his fear is that the first player chooses strategy that be related to column j's greatest element of the profit matrix (i.e. max_ia_{ij}). So, he chooses j strategy, and max_ia_{ij} is smallest for his and his expected profit is equal to $-min_imax_ia_{ij}$. Two players' decisions are in the balance, when:

 $max_{i}min_{j}a_{ij} = min_{j}max_{i}a_{ij}$

In the most games, a player will choose strategy that is not predicted by the rival players. Obviously, in these games there are no players who want to accurately his choice be predicted by rival player. Therefore, he chooses a strategy with p probability. Such strategies called a mixed strategy. The strategy that is made with one probability in a choice is called "pure strategy". If R is a set of pure strategies available to player A, the set of mixed strategies for player A is the set of all available probability distributions in the R domain. "Probabilities are calculated based on The number of observed frequency."

Probability of playing strategy r in R for player A is equal to P_1 . Also, probability of playing strategy c by player B will be equal to P_c . For solve this game, we should find a set of mixed strategies (P_c, P_r) which are somewhat in balance.

Suppose that each player has a probability subjective belief (subjective probabilities) about the rival actor's strategies and each player will choose strategy that can maximize his expected outcome. For example, suppose that player A and B play r and c respectively. Therefore, expected outcome of the player A is equal to $I_r(r,c)$. Suppose that actor A has a subjective probability distribution on player B's choices and is shown by Π_c . Π_c is subjective probability distribution on the player B will play it. Also, player B has subjective probability distribution on the player A's choices and is shown with Π_r [20].

Since player A makes his choice without knowing the player B's choice, the possibility player A in the occurred pure outcome (r,c) is equal to $P_r\Pi_c$. This probability equals the probability that player A will play the strategy r multiplied by the player A's subjective probability about player B plays the strategy c. Hence, the aim of player A to select the probability distribution (P_r) is to maximize the following function.

Expected revenue from player A = $\sum_{r} \sum_{c} P_r \prod_{c} I_r(r, c) \ge E$

From one side, the player B is willing to minimize his expected losses.

Expected loss from player B = $\sum_{c} \sum_{r} P_{c} \Pi_{r} I_{c}(r, c) \leq E$

If both players use their optimal probabilities, the expected outcome for both players would be identical and equal to the game value. If A player uses his optimal probability, his expected revenue cannot be less regardless what strategy player B select than the value of E games. The player A's expected income would be greater than E when player B uses of non-optimal probabilities. In this study, we are using game theory and measure of Wald's criterion (with a zero-sum game) can obtain the highest income in the worst natural conditions for farmers. The Wald's criterion, E is the expected income of expected outcome. X_1 to X_n are production activities, a_{ij} (technical coefficients) is the gross product value of productions per hectare.

E: Expected income X_1, X_2, \dots, X_n Rate of production activities $a_{11}, a_{21}, \dots, a_{m1}$ $a_{12}, a_{22}, \dots, a_{m2}$ $\dots \dots \dots \dots a_{1n}, a_{2n}, \dots, a_{mn}$ Gross product values per hectare of crops

An optimal solution

One game can be stated by converting to a linear programming problem [7]. Linear programming problems must have three elements: physical functioning, limitations and non-negative conditions. These three elements are as well as in a two-person game where it's total score is zero. So between game theory and linear programming, there are common elements (Linear objective function, linear side constraints, the non-negative conditions and primal/ dual relationship) so a two-person zero-sum game can be converted into an equivalent linear programming problem. In a two-person zero-sum game, each player's goal is to maximize the amount of his acquired points, whereas the rival player tries to minimize his lost points. In other words, the players` aim in game theory is to maximize their consequences or minimize the outcome of the opponent (maximum for themselves and minimum for competitor). Bierman and others (1973) have made a linear programming of the game problem. It is supposed that the game has two players, A and B. Player A has mixed and pure strategies ($a_1, a_2, ..., a_n$) and player B has strategies ($b_1, b_2, ..., b_n$). The player A's expected outcome when use strategy a_i and player B use strategy b_j , is equal to a_{ij} . The payment function A, ie mathematical expectancy A, is defined as follows:

$$E(X,Y) = XAY = \sum_{i=1}^{m} \sum_{j=1}^{n} x_i a_{ij} y_j$$

Where, $X = (x_1, x_2, ..., x_m)$ and $Y = (y_1, y_2, ..., y_n)$ are strategies for A and B respectively. Reply to a game is a pair of mixed strategies,

$$\begin{split} \bar{X} &= (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_m) \\ \bar{Y} &= (\bar{y}_1, \bar{y}_2, \dots, \bar{y}_n) \end{split}$$

And E is a real number such that:

$E(\overline{X}, j)$	j = 1, 2,, n	for pure strategies
$E(i, \overline{Y})$	i = 1, 2,, m	for pure strategies

Here \overline{X} and \overline{Y} is called optimal strategies and the E number is called game value.

For example, if player B chooses b₁ strategy, strategy of player A should be such that:

 $a_{11}X_1 + a_{21}X_2 + a_{31}X_3 + \dots + a_{m1}X_m = >E$

The same way, if player B applies strategy b₂, player A must act in order to ensure to obtain the value E:

 $a_{12}X_1+a_{22}X_2+a_{32}X_3+\ldots+a_{m2}X_m => E$ The situation would be similar for each adopted strategy by a

The situation would be similar for each adopted strategy by actor B. Thus, the problem of linear programming for actor A would be as follows:

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\begin{array}{l} MAX: E\\ a_{11}X_1 + a_{21}X_2 + a_{31}X_3 + \ldots + a_{m1}X_m - E => 0\\ a_{12}X_1 + a_{22}X_2 + a_{32}X_3 + \ldots + a_{m2}X_m - E => 0\\ \ldots\\ a_{1n}X_1 + a_{2n}X_2 + a_{3n}X_3 + \ldots + a_{mn}X_m - E => 0\\ X_1 + X_2 + X_3 + \ldots + X_m = 1\\ X_1 => 0\\ X_2 => 0\\ X_2 => 0\\ X_3 => 0\\ \ldots\\ X_m => 0 \end{array}
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Relationship $X_1 + X_2 + X_3 + ... + X_m = 1$ ensures that the total probability would be equal to one. The problem's solution puts is a weighted combination of the strategies available to player A (ie, X_1 , X_2 , X_3 , ..., X_m); In addition, it gives him the E game value.

In this study, game theory model has become the linear programming that is given in Table 1.

Table 1. Linear Progran	ıming Equivalent	Game Theory Model
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Max: V					
$2473899.06 X_1 + 445701.271 X_2 + 2192438.392 X_3 + 371015.079 X_4 + 6945613.602 X_5 + 5772839.701 X_6 - V >= 0$					
$4229494.039 X_1 + 886557.39 X_2 + 2914724.518 X_3 + 633024.997 X_4 + 9500317.863 X_5 + 7465700.613 X_6 - V >= 0$					
$5302691.523 X_1 + 1403244.8 X_2 + 3735724.474 X_3 + 897625.5 X_4 + 11710118.88 X_5 + 10249026.24 X_6 - V >= 0$					
$5892848.184 X_1 + 1421803.966 X_2 + 4586959.709 X_3 + 1049907.156 X_4 + 12045087.14 X_5 + 9303064.169 X_6 - V >= 0$					
$6480237.022 X_1 + 1155086.9 X_2 + 51924865.86 X_3 + 901617.442 X_4 + 13713374.01 X_5 + 10000493.95 X_6 - V \ge 0$					
$6177453.732 X_1 + 978226.507 X_2 + 5220717.888 X_3 + 853862.711 X_4 + 15293390.64 X_5 + 12203014.45 X_6 - V >= 0$					
$7807339.428 X_1 + 1288617.046 X_2 + 7665851.213 X_3 + 1090882.01 X_4 + 16.378882.61 X_5 + 15341900.57 X_6 - V > = 0$					
$6946471.314 X_1 + 953413.947 X_2 + 6173728.171 X_3 + 1301077.862 X_4 + 17036925.35 X_5 + 14037833.15 X_6 - V >= 0$					
$11228398.79 X_1 + 1904516.881 X_2 + 8553860.33 X_3 + 1616382.399 X_4 + 19965602.4 X_5 + 14200551.34 X_6 - V >= 0$					
$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 100$					
$6282092.565 X_1 + 1159685.412 X_2 + 5137387.92 X_3 + 968377.24 X_4 + 13621034.72 X_5 + 10952713.8 X_6 = 65206070 X_1 + 1159685.412 X_2 + 5137387.92 X_3 + 968377.24 X_4 + 13621034.72 X_5 + 10952713.8 X_6 = 65206070 X_1 + 10000 X_2 + 10000 X_1 + 10000 X_2 + $					
X_1 : irrigated wheat X_2 : rain fed wheat X_3 : irrigated barley					
X_4 : rain fed barley X_5 : irrigated sugar beet X_6 : irrigated cotton					

Source: The result of research

RESULTS AND DISCUSSION

In this study, Wald's criterion (maximin) was used. According to the maximin criterion the farmer tries to choose "the best of the worst". This means that the farmer selects the combination of activities which will maximize his minimum income. This strategy gives the farmer maximum security. Life of farmer may be dependent to his farm's income such that, if he loses all his farm income, he can't prepare Essential goods. If the farmer pursues the maximin strategy he can be regarded a pessimist or as ultra careful [3]. In result, the farmers that their life dependent to their farm incomes, prefer strategies which have less income and with more secure, to risky strategies whit major incomes. Given the level of Expected income and the degree of risk, farmers can choose their desired programs. The results of game theory are given in Table 2. Based on the gross product value of the investigated crops, 41 different farm plans were inducted into model In order to determine the appropriate pattern. The results of the game theory model showed that if expected income is higher than 13621040 rial/ha or less than 968377.3 rial/ha a solution is not feasible. When the expected income is approximately 13621040 rial/ha the optimum solution is found at plan 1. As the expected income increases after this level the average lowest income decreases. If expected income decreases after the optimum solution (plan 1), the lowest income decreases also.

	E (11	T / T	Crop Patterns (%)				<u> </u>	
Farm	Expected Income	Lowest Income	Irrigated	Rain fed	Irrigated	Rain fed	Irrigated Sugar	Irrigated
Plan	(11a1/11a)	(mai/ma)	Wheat	Wheat	Barley	Barley	beet	Cotton
1	13621040	6945613	0	0	0	0	100	0
2	13304720	6806588	0	0	0	0	88/1455	11/8545
3	12988410	6667562	0	0	0	0	76/2911	23/7089
4	12672090	6528536	0	0	0	0	64/4366	35/5634
5	12355770	6389509	0	0	0	0	52/5821	47/4179
6	12039460	6250483	0	0	0	0	40/7276	59/2724
7	11723140	6111457	0	0	0	0	28/8732	71/1268
8	11406820	5972430	0	0	0	0	17/0186	82/9814
9	11090510	5833404	0	0	0	0	5/1642	94/8358
10	10952710	5772840	0	0	0	0	0	100
11	10774190	5676256	0	0	0	1/7880	0	98/2120
12	10457870	5505119	0	0	0	4/9561	0	95/0439
13	10141560	5333982	0	0	0	8/1243	0	91/8757
14	9825241	5162845	0	0	0	11/2924	0	88/7076
15	9508925	4991709	0	0	0	14/4605	0	85/5395
16	9192608	4820572	0	0	0	17/6286	0	82/3714
17	8876291	4649435	0	0	0	20/7968	0	79/2032
18	8559975	4478298	0	0	0	23/9649	0	76/0351
19	8243658	4307161	0	0	0	27/1330	0	72/8670
20	7927342	4136025	0	0	0	30/3012	0	69/6988
21	7611025	3964888	0	0	0	33/4693	0	66/5307
22	7294708	3793751	0	0	0	36/6374	0	63/3626
23	6978392	3622614	0	0	0	39/8055	0	60/1945
24	6662076	3451477	0	0	0	42/9737	0	57/0263
25	6029443	3109204	0	0	0	49/3099	0	50/6901
26	5713126	2938067	0	0	0	52/4781	0	47/5219
27	5396809	2766930	0	0	0	55/6462	0	44/3538
28	5080493	2595793	0	0	0	58/8143	0	41/1857

Table 2. Game Theory Results

29	4764176	2424657	0	0	0	61/9824	0	38/0176
30	4447860	2253520	0	0	0	65/1506	0	34/8494
31	4131543	2082383	0	0	0	68/3187	0	31/6813
32	3815227	1911246	0	0	0	71/4868	0	28/5132
33	3498910	1740109	0	0	0	74/6550	0	25/3450
34	3182593	1568973	0	0	0	77/8231	0	22/1769
35	2866277	1397836	0	0	0	80/9912	0	19/0088
36	2549960	1226699	0	0	0	84/1594	0	15/8406
37	2233644	1055562	0	0	0	87/3275	0	12/6725
38	1917327	884425/5	0	0	0	90/4956	0	9/5044
39	1601010	713288/6	0	0	0	93/6637	0	6/3363
40	1284694	542151/9	0	0	0	96/8319	0	3/1681
41	968377/3	371015/1	0	0	0	100	0	0
Source: The result of research								

Source: The result of research

It is seen from Table 2 that while expected income is 13621040 rial/ha, the average lowest income is 6945613 rial/ha. In plan irrigated sugar beet is (100%) part of the plan (plan 1). When the expected income is level 13304720 rial/ha, irrigated cotton is included in the farm plan (plan 2). In this plan, irrigated sugar beet about (88%) and irrigated cotton about (12%) are used. If the expected income level is 10952710 rial/ha, the average lowest income is approximately 5772840 rial/ha and Irrigated cotton is included all of (100%) plan (plan 10). When the expected income is level 10774190 rial/ha, rain fed barley will also be included in the farm plan (plan 11). In this plan, rain fed barley about (2%) and Irrigated cotton about (98%) are used. In this case the average lowest income is 5676256 rial/ha. If the expected income level is 4447860 rial/ha, the average lowest income is approximately 2253520 rial/ha and Irrigated cotton about (35%) and rain fed barley about (65%) are used (plan 30). The average lowest income is approximately 371015.1 rial/ha, if the expected income level be 968377.3 rial/ha. In this plan, Irrigated cotton is removed from the plan and rain fed barley is only included in the farm plan (plan 41).

With the game theory model, when expected income is at the highest level for the examined region, irrigated sugar beet is only included in the farm plan. As the expected income decreases, the share of the irrigated sugar beet decrease, and irrigated cotton and rain fed barley are respectively included in the farm plans. The plan with the lowest expected income level, only rain fed barley, which is less risky crops, is in the farm plan.

CONCLUSION

In this study, the results indicate that Irrigated sugar beet was the most risky crops and rain fed barley was less risky crops. If expected income is more than 13621040 rial/ha or less than 968377.3 rial/ha, a solution is not feasible. When the average lowest income is at highest level (maximin), just Irrigated sugar beet is in the farm plan and this case expected income level is 13621040 rial/ha (plan1).

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