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ESTIMATION OF THE PARAMETERS OF THE GENERALIZED EXPONENTIAL DISTRIBUTION IN THE PRESENCE OF OUTLIERS GENERATED FROM UNIFORM DISTRIBUTION

Parviz Nasiri and Mehdi Jabbari Nooghabi

ABSTRACT: *Gupta and Kundu (1999) defined the cumulative distribution function of the generalized exponential (GE). It has many properties that are quite similar to those of the gamma distribution. This paper deals with the estimation of the parameters of the generalized exponential distribution in the presence of outliers generated from uniform distribution. The maximum likelihood, moment and mixture of the estimators are derived. These estimators are compared empirically when all of the parameters are unknown. Their bias and mean square error (MSE) are investigated with help of numerical techniques.*

Keywords: *Generalized Exponential Distribution, Uniform Distribution, Outliers, Maximum Likelihood, Moment and Mixture Estimators*

1. INTRODUCTION

Recently the two-parameter generalized exponential (GE) distribution has been proposed by the authors. It has been studied extensively by Gupta and Kundu (1999, 2001a, 2001b, 2002, 2003a,2003b, 2004), Raqab (2002), Raqab and Ahsanullah (2001) and Zheng (2002). Note that the generalized exponential distribution is a sub-model of the exponentiated weibull distribution introduced by Mudholkar and Srivastava (1993) and later studied by Mudholkar, Srivastara and Freimer (1995) and Mudholkar and Huston (1996). Dixit, Moore and Barnett (1996), assumed that a set of random variables $(X_1, X_2, \ldots, X_n)$ repre-sent the distance of an infected sampled plant from a plant from a plot of plants inoculated with a virus. Some of the observations are derived from the airborne dispersal of the spores and are distributed according to the exponential distribution. The other observations out of n random variables (say k) are present because aphids which are know to be carriers of barley yellow mosaic dwarf virus (BYMDV) have passed the virus into the plants when the aphids feed on the sap. Dixit and Nasiri (2001) considered estimation of the param-eters of the exponential distribution in the presence of outliers generated from uniform distribution. In this paper, we consider generalized exponential distribution in presence of outlier generated from uniform distribution.

The two-parameter GE distribution has the following density function and the distribution function

$$f(x;\alpha)=\alpha e^{-x}\left(1-e^{-x}\right)^{\alpha-1},\ x>0,\alpha>0, \tag{1}$$

$$f(x;\alpha)=\left(1-e^{-x}\right)^{\alpha-1},\ x>0,\alpha>0, \tag{2}$$

Let the random variables $(X_1,X_2,\ldots,X_n)$ are such that k of them are distributed with pdf $g(x,\alpha,\theta)$

$$g(x;\alpha,\theta)=\left|\frac{1}{\alpha\theta}I(0,\alpha\theta)(x),\ \alpha>0,\theta>0,\right. \tag{3}$$

and remaining $(n-k)$ random variables are distributed with pdf

$$f(x;\alpha)=\alpha e^{-x}\left(1-e^{-x}\right)^{\alpha-1},\ x>0,\alpha>0 \tag{4}$$

where I is an indicator function.

Therefore the joint distribution of $(X_1, X_2, \ldots, X_n)$ is

$$f(x_1,x_2,\ldots,x_n;\alpha,\theta)=\frac{k!(n-k)!}{n!}\prod_{i=1}^{n}f(x_i;\alpha)\sum^{\bullet}\prod_{j=1}^{k}\frac{g\left(x_{A_j}\right)}{f\left(x_{A_j}\right)}, \tag{5}$$

where

$$\sum^{\bullet}=\sum_{A_1=1}^{n}\ \sum_{A_2=A_1+1}^{n-1}\cdots\sum_{A_k=A_{k-1}+1}^{n-k}.$$

For $g(x;\alpha,\theta)$and $f(x;\alpha)$are given in (3) and (4), $f(x_1, x_2,\ldots,x_n)$ is

$$f(x_1,x_2,\ldots,x_n;\alpha,\theta)=\frac{k!(n-k)!}{n!}\alpha^n e^{-\sum_{i=1}^{n}x_i}\prod_{i=1}^{n}\left(1-e^{-x_i}\right)^{\alpha-1}\sum^{\bullet}\prod_{j=1}^{k}\frac{\frac{1}{\alpha\theta}I_{(0,\alpha\theta)}\left(x_{A_j}\right)}{\alpha e^{-x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{\alpha-1}}$$

$$\frac{k!(n-k)!}{n!}\frac{\alpha^{n-1}e^{-n\bar{x}}}{(\alpha\theta)^k}\prod_{i=1}^{n}\left(1-e^{-x_i}\right)^{\alpha-1}\sum^{\bullet}\frac{\prod_{j=1}^{k}I_{(0,\alpha\theta)}\left(x_{A_j}\right)}{\prod_{j=1}^{k}e^{-x_{A_j}}\left(1-e^{-x_{A_j}}\right)\alpha-1} \tag{6}$$

It is easy to show that the marginal distribution of X is

$$f(x;\alpha,\theta)=\frac{k}{n}\frac{1}{\alpha\theta}I_{(0,\alpha\theta)}(x)+\frac{n-k}{n}\alpha e^{-x}\left(1-e^{-x}\right)^{\alpha-1},x>0,\alpha>0,\theta>0. \quad (7)$$

The paper is organized as follows:

Section 2, 3 and 4 discusses the methods of moment (MM), maximum likelihood (ML) andmixture estimators (Mix). The bias and MSE will be investigated with help of numericaltechnique in section 5.

2. METHOD OF MOMENT

The r^{th} moments of X may be determined direct or using the moment of generating function. Here we consider the moment generating function.

$$M(t)=E\left(e^{tX}\right)=\int_0^\infty e^{tx}f(x;\alpha,\theta)\,dx$$

$$=\int_0^\infty e^{tx}\left[\frac{k}{n}\frac{1}{\alpha\theta}I_{(0,\alpha\theta)}(x)+\frac{n-k}{n}\alpha e^{-x}\left(1-e^{-x}\right)\alpha^{\alpha-1}\right]dx$$

$$\frac{k}{n}\int_0^{\alpha\theta}\frac{e^{tx}}{\alpha\theta}dx+\frac{n-k}{n}\int_0^\infty\alpha e^{(t-1)x}\left(1-e^{-x}\right)^{\alpha-1}dx$$

$$\frac{k}{n}\frac{e^{\alpha\theta t}-1}{\alpha\theta t}+\frac{n-k}{n}\int_0^\infty\alpha e^{(t-1)x}\left(1-e^{-x}\right)^{\alpha-1}dx.$$

Let $y=e^{-x}$ then $x=ln(y),dx=\frac{-dy}{y}$, and $M(t)$ is given by

$$M(t)=\frac{k}{n}\frac{e^{\alpha\theta t}-1}{\alpha\theta t}+\frac{n-k}{n}\int_0^\infty\alpha y^{-t}\left(1-y\right)^{\alpha-1}dx$$

$$\frac{k}{n}\frac{e^{\alpha\theta t}-1}{\alpha\theta t}+\frac{n-k}{n}\frac{\Gamma(\alpha+1)\Gamma(1-t)}{\Gamma(\alpha-t+1)}, \quad (8)$$

where $\Gamma(.)$ is the gamma function. Differentiating $M(t)$ and evaluating at $t=0$, we get the $E(X)$ and $E(X^2)$ as

$$M't=\frac{k}{n\alpha\theta}\left[\frac{\alpha\theta e^{\alpha\theta t}}{t}-\frac{\alpha\theta e^{\alpha\theta t}-1}{t^2}\right]$$

$$+\frac{n+k}{n}\left[\frac{\Gamma(\alpha+1)\Gamma(1-t)\Gamma'(\alpha-t)}{\Gamma^2(\alpha-t+1)}-\frac{\Gamma(\alpha+1)\Gamma'(1-t)}{\Gamma(\alpha-t+1)}\right], \quad (9)$$

where $\Gamma'(.)$ is the derivative of the gamma function.

$$Mt''=\frac{k}{n\alpha\theta}\left[\frac{(\alpha\theta)^2 e^{\alpha\theta t}}{t}-\frac{2\alpha\theta e^{\alpha\theta t}}{t^2}+\frac{2(e^{\alpha\theta t}-1)}{t^3}\right]+\frac{n-k}{n}\left[\frac{\Gamma'(\alpha-t+1)}{\Gamma^2(\alpha-t+1)}\right.$$

$$\left.-\frac{\Gamma(\alpha+1)(\Gamma'(1-t)\Gamma'(\alpha-t+1)+\Gamma(1-t)\Gamma''(\alpha-t+1))}{\Gamma^2(\alpha-t+1)}\right] \quad (10)$$

where $\Gamma''(.)$ is derivative of $\Gamma'(.)$.

From the equations (9) and (10), $E(X)$ and $E(X^2)$ are given by

$$E(X)=M'(0)=\frac{k}{n}\frac{\alpha\theta}{2}+\frac{n-k}{n}\left[\frac{\Gamma'(\alpha+1)}{\Gamma(\alpha+1)}-\frac{\Gamma'(1)}{\Gamma(1)}\right], \quad (11)$$

and

$$E(X^2)=M''(0)=\frac{k}{n}\frac{(\alpha\theta)^2}{3}+\frac{n-k}{n}\left[\frac{2\Gamma'(\alpha+1)}{\Gamma^2(\alpha+1)}-\frac{\Gamma'(1)\Gamma'(\alpha+1)+\Gamma''(\alpha+1)}{\Gamma(\alpha+1)}\right]. \quad (12)$$

One should note that for $k = 0$, $E(X)$ and $E(X^2)$ are given by Gupta and Kundu (1999). Let A and B be a function of α as

$$A=\frac{n-k}{n}\left[\frac{\Gamma'(\alpha+1)}{(\alpha+1)}-\frac{\Gamma'(1)}{\Gamma(1)}\right], \quad (13)$$

and

$$B=\frac{n-k}{n}\left[\frac{2\Gamma'(\alpha+1)}{\Gamma^2(\alpha+1)}-\frac{\Gamma'(1)\Gamma'(\alpha+1)+\Gamma''(\alpha+1)}{\Gamma(\alpha+1)}\right] \quad (14)$$

Then

$$m_1=\frac{1}{n}\sum_{i=1}^{n}x_i=\frac{k}{n}\frac{\hat{\alpha}\hat{\theta}}{2}+A, \quad (15)$$

$$m_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 = \frac{k}{n}\frac{\left(\bar{\alpha}\bar{\theta}\right)^2}{3} + B, \tag{16}$$

Equations (15) and (16) imply that

$$m_2 - B - \frac{4n}{3k}\left(m_1 - A\right)^2 = 0 \tag{17}$$

We can solve (17) by Newton -Raphson method. Hence solution of the equation is

$$\alpha_{i+1} = \alpha_i - \frac{g(\alpha_i)}{g'(\alpha_i)}, \tag{18}$$

where

$$g(\alpha_i) = m_2 - B(\alpha_i) - \frac{4n}{3k}\left(m_1 - A(\alpha_i)^2\right),$$

$$g'(\alpha_i) = -B'(\alpha_i) - \frac{8n}{3k}A'(\alpha_i)\left(m_1 - A(\alpha_i)\right),$$

$$A'(\alpha_i) = \frac{n-k}{n}\left[\frac{\Gamma''(\alpha_i+1)}{\Gamma'(\alpha_i+1)} - \frac{\Gamma'^2(\alpha_i+1)}{\Gamma^2(\alpha_i+1)}\right],$$

and

$$B'(\alpha_i) = \frac{n-k}{n}\left[\frac{2\Gamma''(\alpha_i+1)}{\Gamma^2(\alpha_i+1)} - \frac{4\Gamma'^2(\alpha_i+1)}{\Gamma^3(\alpha_i+1)}\right]$$

$$-\frac{n-k}{n}\left[\frac{\Gamma'(1)\Gamma''(\alpha_i+1)+\Gamma'''(\alpha_i+1)}{\Gamma(\alpha_i+1)} - \frac{\Gamma'(1)\Gamma'^2(\alpha_i+1)+\Gamma'(\alpha_i+1)\Gamma''(\alpha_i+1)}{\Gamma^2(\alpha_i+1)}\right]$$

Also $\bar{\theta}$ obtains as

$$\bar{\theta} = \frac{1}{\bar{\alpha}}\frac{3\left(m_2 - \bar{B}\right)}{2\left(m_1 - \bar{A}\right)}$$

3. METHOD OF MAXIMUM LIKELIHOOD

Proceeding with the method of maximum likelihood, the likelihood function for a sample of size $n,(X_1,X_2,...,X_n)$, is given by

$$L(\alpha,\theta)=\frac{k!(n-k)!}{n!}\alpha^n e^{-\sum_{i=1}^{n} x_i}\prod_{i=1}^{n}\left(1-e^{-x_i}\right)^{\alpha-1}\sum^{*}\prod_{j=1}^{k}\frac{\frac{1}{\alpha\theta}I_{(0,\alpha\theta)}(x)}{\alpha e^{-x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{\alpha-1}}$$

$$=\frac{k!(n-k)!}{n!}\frac{\alpha^{n-1}e^{-n\bar{x}}}{(\alpha\theta)^k}\prod_{i=1}^{n}\left(1-e^{-x_i}\right)^{\alpha-1}\sum^{*}\frac{\prod_{j=1}^{k}I_{(0,\alpha\theta)}\left(x_{A_j}\right)}{\prod_{j=1}^{k}e^{-x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{\alpha-1}} \quad (20)$$

Then

$$L(\alpha,\theta)\simeq\frac{\alpha^{n-1}e^{-n\bar{x}}}{x_{(n)}^{k}}\prod_{i=1}^{n}\left(1-e^{-x_i}\right)^{\alpha-1}\sum\prod_{j=1}^{k}\frac{1}{e^{-x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{\alpha-1}},$$

where

$$\hat{\alpha\theta}=x(n)=\max\left(X_1,X_2,...,X_n\right).$$

Hence

$$\hat{\theta}=\frac{x_n}{\alpha}, \quad (22)$$

To estimate α, we consider $\ln(L(\alpha,\theta))$as

$$In\left(L(\alpha,\theta)\right)\simeq(n-1)In(\alpha)-n\bar{x}-k\,In\left(x_{(n)}\right)+(\alpha-1)\sum_{i=1}^{n}In\left(1-e^{-x_i}\right)$$

$$+In\left(\sum^{*}\prod_{j=1}^{k}\frac{1}{e^{-x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{\alpha-1}}\right). \quad (23)$$

Taking the derivative with respect to α and equating to 0, we obtain the normal equation as

$$\frac{d\,InL(\alpha)}{d\alpha}\simeq\frac{n-1}{\alpha}+\sum_{i=1}^{n}In\left(1-e^{-x_i}\right)$$

$$-\frac{\Sigma^{*}\prod_{j=1}^{k}e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha}In\left(1-e^{-x_{A_j}}\right)}{\Sigma^{*}\prod_{j=1}^{k}e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha}} \quad (24)$$

Since $\frac{d\,In\,L(\alpha)}{d\theta}=0$, hence

$$\frac{n-1}{\alpha}+\sum_{i=1}^{n} In\left(1-e^{-x_i}\right)\simeq\frac{\Sigma^*\prod_{j=1}^{k} e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha} In\left(1-e^{-x_{A_j}}\right)}{\Sigma^*\prod_{j=1}^{k} e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha}} \tag{25}$$

For $k = 0$, it is given by Gupta and Kundu (1999).Here, we need to use either the scoring algorithm or the Newton -Raphson method tosolve the non-linear equation.

Here, We solve (25) by Newton -Raphson method. Hence solution of the equation is

$$\alpha_{i+1}=\alpha_i-\frac{g(\alpha_i)}{g'(\alpha_i)} \tag{26}$$

Where

$$g(\alpha)\simeq\frac{n-1}{\alpha}+\sum_{i=1}^{n} In\left(1-e^{-x_i}\right)$$

$$-\frac{\Sigma^*\prod_{j=1}^{k} e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha} In\left(1-e^{-x_{A_j}}\right)}{\Sigma^*\prod_{j=1}^{k} e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha}} \tag{27}$$

and

$$g'(\alpha)\simeq-\frac{n-1}{\alpha^2}+\frac{\Sigma^*\prod_{j=1}^{k} e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha}\left(In\left(1-e^{-x_{A_j}}\right)\right)^2}{\Sigma^*\prod_{j=1}^{k} e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha}}$$

$$-\left[\frac{\Sigma^*\prod_{j=1}^{k} e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha}\left(In\left(1-e^{-x_{A_j}}\right)\right)}{\Sigma^*\prod_{j=1}^{k} e^{x_{A_j}}\left(1-e^{-x_{A_j}}\right)^{1-\alpha}}\right]^2 \tag{28}$$

4. MIXTURE OF METHOD OF MOMENT AND MAXIMUM LIKE-LIHOOD

Read (1981) proposed the methods, which avoid the difficulty of complicated equations.According to Read (1981), replacement of some, but not all of the equations in the systemof likelihood may make it more manageable.

One sees from (24) the ML estimator for the parameter α of the GE distribution withpresence of outlier can not be obtained in closed forms and therefore that is little pointin considering the method any further. So from (19)

$$\hat{\theta} = \frac{1}{\hat{\alpha}} \frac{3(m_2 - B)}{2(m_1 - A)} \tag{29}$$

and

$$\hat{\alpha} = \frac{x(n)}{\hat{\theta}}. \tag{30}$$

So we can easily find the mixture estimators of α and θ.

5. NUMERICAL EXPERIMENTS AND DISCUSSIONS

In order to have some idea about Bias and Mean Square Error (MSE) of methods of moment, MLE and mixture, we perform sampling experiments using a *R* software. The result of bias of the estimators are given in Table 1 and 3 for $\alpha = 0.5$, $\theta = 5$, $k = 1$, 2, respectively. Also Table 2 and 4 show the MSE of the estimators for $\alpha = 0.5$, $\theta = 5$, $k = 1,2$, respectively.

According to Table 1 and 3, the bias of mixture estimators of α and θ are less than the others. Table 2 and 4 are shown that the MSE of mixture estimators of α and θ are less than the MSE of the other estimators for all values of n and k. So the mixture estimators of α and θ are more efficient than the others.

Table 1
Bias for $k = 1$, $\alpha = 0.5$ and $\alpha = 5$.

n	*MME* Bias of $\hat{\alpha}$	*MLE* Bias of $\hat{\alpha}$	*Mix* Bias of $\hat{\alpha}$	*MME* Bias of $\hat{\theta}$	*MLE* Bias of $\hat{\theta}$	*Mix* Bias of $\hat{\theta}$
3	0.1730	1.0905	0.3162	−3.8681	−4.7750	−3.8681
4	−0.1303	0.0102	0.0125	−4.1626	−4.7946	−4.1626
5	−0.1713	0.0361	0.0300	−3.8274	−4.9343	−3.8274
6	−0.1415	−0.0240	0.0124	−3.9011	−4.7122	−3.9011
7	0.2095	0.3820	0.1077	−4.4908	−4.9378	−4.4908
8	−0.0800	0.1259	0.0221	−4.3359	−4.9765	−4.3359
9	−0.3624	−0.2787	0.0251	−4.1475	−4.9035	−4.1475
10	−0.2934	−0.2196	0.0324	−4.2438	−4.9126	−4.2438
15	0.1159	−0.1312	0.0436	−4.3907	−4.9280	−4.3907
20	−0.0579	−0.1928	0.0264	−4.2572	−4.9362	−4.2572
25	−0.1534	−0.2070	0.0196	−4.1124	−4.9405	−4.1124
30	−0.0432	−0.1595	0.0366	−4.1456	−4.9082	−4.1456

Table 2
MSE for k = 1, $\alpha = 0.5$ and $\theta = 5$.

n	*MME* MSE of $\hat{\alpha}$	*MLE* MSE of $\hat{\alpha}$	*Mix* MSE of $\hat{\alpha}$	*MME* MSE of $\hat{\theta}$	*MLE* MSE of $\hat{\theta}$	*Mix* MSE of $\hat{\theta}$
3	1.3887	8.7781	0.3337	18.8058	22.9525	18.8058
4	0.7004	1.3014	0.2188	20.8334	23.1991	20.8334
5	0.5695	1.4385	0.2254	21.5238	24.3689	21.5238
6	0.6625	1.1337	0.2186	21.2566	22.6192	21.2566
7	3.5681	5.5920	0.2351	21.9822	24.4093	21.9822
8	1.2414	2.7580	0.2318	21.8872	24.7697	21.8872
9	0.3018	0.5185	0.2312	23.7426	24.1279	23.7426
10	0.4702	0.7559	0.2281	23.1561	24.2027	23.1561
15	3.4269	1.2414	0.2254	22.6195	24.3316	22.6195
20	1.7628	0.8866	0.2306	23.0897	24.4026	23.0897
25	1.1045	0.8157	0.2342	24.0026	24.4407	24.0026
30	1.8795	1.0689	0.2268	23.7563	24.1662	23.7563

Table 3
Bias for $k = 2$, $\alpha = 0.5$ and $\theta = 5$.

n	*MME* Bias of $\hat{\alpha}$	*MLE* Bias of $\hat{\alpha}$	*Mix* Bias of $\hat{\alpha}$	*MME* Bias of $\hat{\theta}$	*MLE* Bias of $\hat{\theta}$	*Mix* Bias of $\hat{\theta}$
3	0.1902	0.5898	0.4566	− 4.2162	− 4.6716	− 4.2162
4	− 0.2724	− 0.1378	− 0.1359	− 3.6365	− 4.4882	− 3.6365
5	− 0.3155	− 0.0969	0.0334	− 4.1828	− 4.9324	− 4.1828
6	− 0.2425	− 0.0065	0.0782	− 4.4319	− 4.9100	− 4.4319
7	− 0.2575	− 0.2402	0.0896	− 4.3047	− 4.7603	− 4.3047
8	− 0.3740	− 0.2595	0.0211	− 4.4535	− 4.9521	− 4.4535
9	− 0.3148	− 0.2634	0.0507	− 4.5624	− 4.9062	− 4.5624
10	− 0.3534	− 0.2405	0.0544	− 4.4864	− 4.8923	− 4.4864
15	− 0.3274	− 0.2401	0.0275	− 4.2045	− 4.9159	− 4.2045
20	− 0.1770	− 0.2373	0.0510	− 4.1272	− 4.8304	− 4.1272
25	0.1147	0.0606	0.0437	− 3.4915	− 4.8824	− 3.4915
30	− 0.0901	− 0.1113	0.0288	− 3.0563	− 4.8560	− 3.0563

Table 4
MSE for $k = 2$, $\alpha = 0.5$ and $\theta = 5$.

n	*MME* MSE of $\hat{\alpha}$	*MLE* MSE of $\hat{\alpha}$	*Mix* MSE of $\hat{\alpha}$	*MME* MSE of $\hat{\theta}$	*MLE* MSE of $\hat{\theta}$	*Mix* MSE of $\hat{\theta}$
3	1.4653	3.9106	0.6273	19.6194	22.1475	19.6194
4	0.2814	0.5437	0.2065	20.6607	21.1917	20.6607
5	0.3037	0.9844	0.2244	21.5027	24.3558	21.5027

contd...

6	0.5230	1.7050	0.2207	21.9009	24.1649	21.9009
7	0.4781	0.5302	0.2246	21.9146	23.0626	21.9146
8	0.2828	0.5880	0.2334	22.5216	24.5435	22.5216
9	0.4420	0.6293	0.2276	22.7304	24.1584	22.7304
10	0.3399	0.7311	0.2282	22.7656	24.0502	22.7656
15	0.3454	0.5981	0.2293	22.7404	24.2225	22.7404
20	0.8659	0.6084	0.2224	23.1280	23.5629	23.1280
25	1.9023	1.5752	0.2178	23.5681	23.9069	23.5681
30	0.8482	0.7676	0.2262	22.2315	23.6843	22.2315

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Parviz Nasiri
Department of Statistics,
Tehran Payame Noor University,
Fallahpour St., Nejatollahi St., Tehran, Iran

Mehdi Jabbari Nooghabi
Department of Statistics,
Ferdowsi University of Mashhad, Iran