

# Application of PSO for Selective Harmonic Elimination in a PWM AC/AC Voltage Regulator

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**Abstract**— In this paper the particle swarm optimization (PSO) technique is used to find an optimal solution for the selective harmonic elimination (SHE) problem in PWM AC/AC voltage regulators. For this kind of voltage controllers, SHE leads to nonlinear and transcendental equations; as a result, solving them by conventional numerical methods highly depends on proper selection of initial values. The proposed PSO based algorithm affectively computes the required switching instances to eliminate pre-specified harmonics from the output voltage. Problem formulation, objective function and obtained switching angles trajectories for eliminating different sets of harmonics are presented. Simulation results using Matlab/Simulink are presented to confirm the theoretical results.

**Keywords**- Selective harmonic elimination (SHE); particle swarm optimization (PSO); PWM; AC voltage regulator.

## I. INTRODUCTION

AC voltage regulators have been used in various applications such as industrial heating, lighting control, induction motor soft starter and speed controller for fans and pumps. The phase angle control and integral cycle control strategies have been widely used for these requirements [1-2]. Although they have many advantages such as simplicity and low cost, but delayed firing angles cause discontinuity as well as considerable low order harmonics in load current and a lagging power factor at the source side. These problems can be mitigated by pulse width modulated (PWM) AC choppers [3-4]. Selective harmonic elimination (SHE) is a long-established method for PWM control of inverters and AC choppers. This technique still receives remarkable attentions due to significant advantages of digital signal processing tools utilized for realization of SHE algorithm [5]. The main challenge associated with SHE technique is to solve a set of nonlinear transcendental equations which commonly has several answers [6-7]. Usually numerical methods such as Newton's method are used, but results of these methods highly depend on the selection of initial values. In some cases, if the initial guess is not chosen properly, the number of iterations may either increase or even not converge at all. During the recent years many evolutionary algorithms (EA) have found many intentions due to their high global search capability. EAs are population-based search techniques inspired by natural mechanisms or phenomenon; among them, Genetic

Algorithms (GA) and particle swarm optimization (PSO) have been proposed and applied to many real-world optimization problems. PSO is a robust stochastic optimization technique based on the movement and intelligence of swarms. PSO uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the best solution. Unlike GA and many heuristic algorithms, PSO has a simple organization, clear concept, flexibility to control the balance between the global and local exploration of search space, and more important, PSO has relatively easy implementation.

In this paper, a method based on PSO technique is proposed to eliminate the pre-specified harmonics from the output voltage of PWM controlled AC voltage regulators. The objective function derived from the SHE problem is minimized using PSO algorithm to compute the switching angles without going for the multiple solutions of the set of nonlinear equations. Extensive simulations conducted in MATLAB/Simulink verify the superiority of the proposed strategy in finding the optimal solution of SHE problem for AC voltage regulators.

## II. PROBLEM FORMULATION

### A. Selective harmonic elimination

Bidirectional power transfer in the AC/AC voltage controllers requires the utilization of switches capable of blocking voltages of either polarity, conduction of current in both directions, as well as the turn off capability. The PWM AC voltage regulator circuit which is used in this work is depicted in Fig. 1. There are some other techniques of realizing such bi-directional switches [8]. Considering the switching pattern given in Fig. 2(a), the output voltage waveform of the AC chopper is shown in Fig. 2(b), where the odd and half-wave symmetries are obvious.

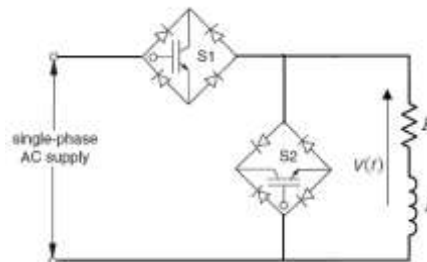


Figure 1. PWM AC/AC voltage regulator (AC chopper)

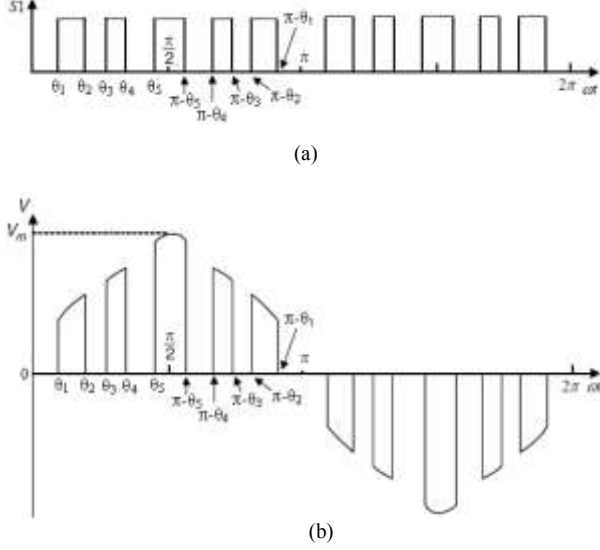


Figure 2. (a) Switching pattern and (b) output voltage waveform of AC chopper

The Fourier series expansion of the output voltage is as follows:

$$V(\omega t) = \sum_{n=1}^{\infty} A_n \cos(n\omega t) + B_n \sin(n\omega t) \quad (1)$$

As a result of odd symmetry, for all values of  $n$  we have  $A_n = 0$ . Furthermore,  $B_2, B_4, B_6 \dots$  i.e. even order harmonics are absent in the output voltage (as a result of half-wave symmetry). So, Fourier series will be simplified to (2), where the amplitude of the  $n$ -th order harmonic is given by (3).

$$V(\omega t) = \sum_{n=3,5,\dots}^{\infty} B_n \sin(n\omega t) \quad (2)$$

$$B_n = \frac{2V_m}{\pi} \sum_{i=1}^k (-1)^i \left[ \frac{\sin(n-1)\theta_i}{n-1} - \frac{\sin(n+1)\theta_i}{n+1} \right]; n=3,5,7,\dots \quad (3)$$

where,  $V_m$ ,  $\theta_i$  and  $k$  are the peak value of the input sine wave, the  $i$ -th switching angle, and the number of switching angles per quarter cycle, respectively. It is obvious that by  $k$  switchings per quarter cycle,  $k-1$  number of harmonics can be eliminated. The amplitude of fundamental component is computed as:

$$B_1 = V_m \left( 1 + \frac{2}{\pi} \sum_{i=1}^k (-1)^i \left[ \theta_i - \frac{\sin 2\theta_i}{2} \right] \right) \quad (4)$$

In conventional SHE technique, nonlinear transcendental equations are formed by assigning the desired value for the fundamental component and zero for the pre-specified harmonics to be eliminated. These equations contain trigonometric terms and therefore have multiple solutions. Usually, numerical methods such as Newton-raphson have been used for finding the solutions. The convergence of these

techniques, in most cases, depends on the initial values selected for the switching angles.

### B. Proposed PSO method

As an optimization problem, SHE can be formulated as follows:

$$\text{Min} \{g(\Theta) = |B_1 - m_a V_m| + |B_3| + \dots + |B_{k-1}|\} \quad (5)$$

subject to:  $0 < \theta_1 < \theta_2 < \theta_3 < \dots < \theta_k < \frac{\pi}{2}$ .

The decision vector of above optimization problem is the switching angles vector  $\Theta = [\theta_1 \theta_2 \dots \theta_k]^T$  and  $m_a = B_1/V_m$  is called the modulation index. The output voltage amplitude is controlled up to  $V_m$  by controlling the modulation index over the range (0, 1).

In this work, PSO algorithm is adopted to solve the optimization problem introduced in (5). The particle swarm optimization (PSO) technique is a population based search algorithm, which uses social rules to search in the decision space by control of a set of independent particles (agents). The position of each particle represents a particular solution of the problem, i.e. the switching angles vector. These particles are used to compute the value of fitness (objective) function to be optimized.

PSO method begins with randomly initialization of particle positions in the search space. Every particle may change its position and consequently may explore the solution space. The best position (switching angles) reached by all the particles during their paths is called *gbest* (global best solution) and best position that the agent itself has reached during its search is called *pbest*.

At any iteration of algorithm the particles move around according to their velocity and position; then the objective function is evaluated for each particle in order to rank the current location. The new search points are then stochastically updated regarding the *pbest* and *gbest* solutions according to the following equations [9]:

$$V_i^{k+1} = \omega V_i^k + c_1 \text{rand}() (pbest_i^k - x_i^k) + c_2 \text{rand}() (gbest_i^k - x_i^k) \quad (6)$$

$$x_i^{k+1} = x_i^k + V_i^{k+1} \quad (7)$$

where,  $\omega$ ,  $c_j$  and  $V_i$  are inertia weight, scaling factors (constants) and particles velocity, respectively and *rand()* is the random positive number with a uniform distribution over the range (0, 1).

In fact, the main PSO operator is the velocity update (equation (6)), that takes into account the *pbest* and *gbest* which results in a migration of the entire swarm towards the global optimum. The values of *pbest* and *gbest* are updated after each iteration. This procedure continues until the convergence is obtained.

### III. NUMERICAL RESULTS & SWITCHING ANGLE TRAJECTORIES

The proposed algorithm for SHE problem in PWM controlled AC voltage regulators has been realized by writing m-file codes in Matlab. The results are verified through simulations of the circuit in Matlab/Simulink. The optimization program is executed for different values of modulation indices and the associated switching angle vectors are calculated for each  $m_a$ . To verify the effectiveness of the resulting switching angle vectors, the AC chopper circuit is controlled according to these proposed vectors in Matlab/Simulink. The results are presented in the following sections.

#### A. Eliminating 3<sup>rd</sup> and 5<sup>th</sup> harmonics

As stated before, three switching instances per quarter cycle are needed to eliminate two individual harmonics. Switching angle trajectories will be obtained by plotting the calculated angles over the modulation index ( $m_a$ ). Fig. 3 shows these trajectories for the case of elimination of 3<sup>rd</sup> and 5<sup>th</sup> order harmonics.

These trajectories have been used as look-up tables to generate the switching signals of an AC chopper circuit. The chopper circuit parameters are summarized in Table I. The output voltage waveform and its harmonics spectrum for  $m_a = 0.7$  are given in Fig. 4 and 5, respectively. Fig. 5 clearly shows that the 3<sup>rd</sup> and 5<sup>th</sup> order harmonics are effectively eliminated from the output voltage spectrum. Variations of the output voltage THD (%) with modulation index are illustrated in Fig. 6.

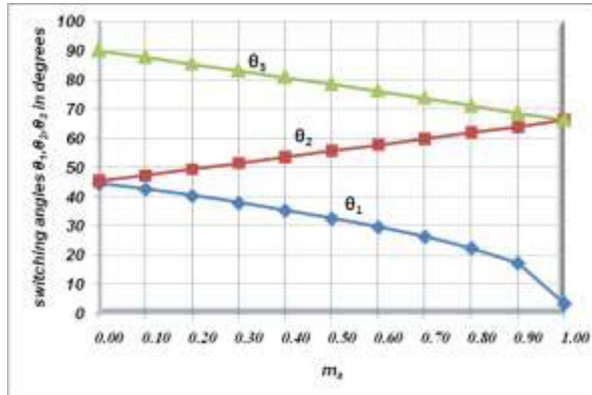


Figure 3. Switching angle trajectories to eliminate harmonics {3,5}

TABLE I. CHOPPER CIRCUIT PARAMETERS

Parameter	Value
Supply voltage amplitude	220 V
Load resistance	10 $\Omega$
Load inductance	6.5 mH

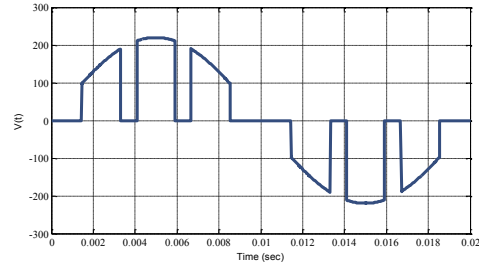


Figure 4. Output voltage waveform for  $m_a = 0.7$  with 3<sup>rd</sup> and 5<sup>th</sup> harmonics eliminated

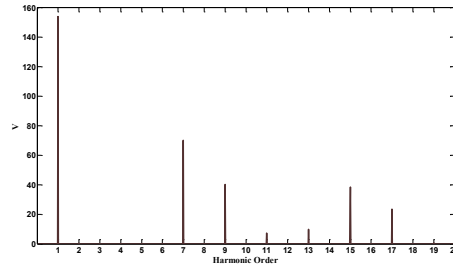


Figure 5. Output voltage spectrum for  $m_a = 0.7$  with 3<sup>rd</sup> and 5<sup>th</sup> harmonics eliminated

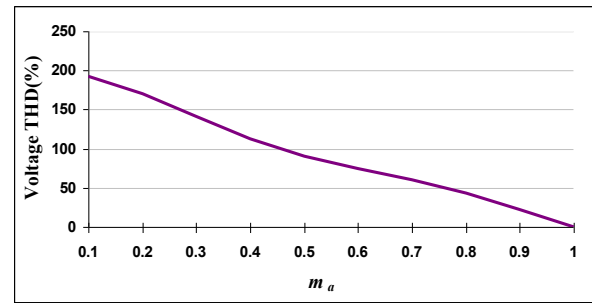


Figure 6. Output voltage THD versus modulation index with 3<sup>rd</sup> and 5<sup>th</sup> harmonics eliminated

#### B. Eliminating 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup> and 9<sup>th</sup> harmonics

In this case, switching angles are calculated in order to eliminate {3, 5, 7, 9} harmonics from the output voltage waveform. Fig. 7 shows the switching angle trajectories. The output voltage waveform for  $m_a = 0.8$  is shown in Fig. 8. The harmonics spectrum of output voltage is given in Fig. 9 which confirms the successful elimination of desired harmonics. Variations of the output voltage THD (%) versus modulation index are presented in Fig. 10.

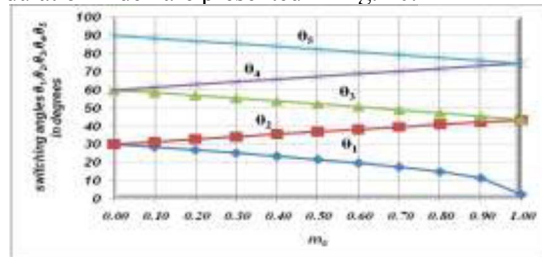


Figure 7. Switching angle trajectories to eliminate harmonics {3,5,7,9}

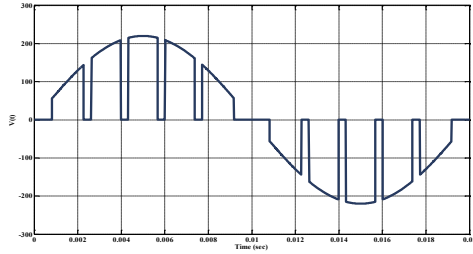


Figure 8. Output voltage waveform for  $m_a = 0.8$  with 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, and 9<sup>th</sup> harmonics eliminated

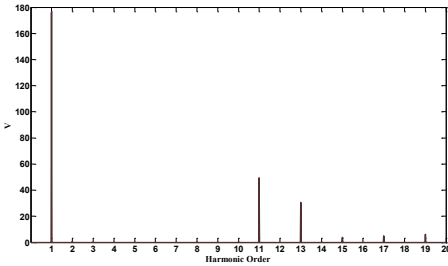


Figure 9. Output voltage spectrum for  $m_a = 0.8$  with 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, and 9<sup>th</sup> harmonics eliminated

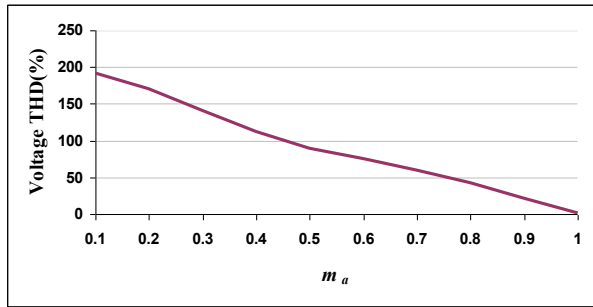


Figure 10. Output voltage THD versus modulation index with 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, and 9<sup>th</sup> harmonics eliminated

From Fig. 5 and Fig. 9 it is obvious that applying PSO algorithm to SHE problem effectively eliminates the desired harmonics from output voltage waveform.

On the other hand, Fig. 3 and 7 show that the variations of switching angles over the modulation index, especially around  $m_a = 1$ , are smooth. This feature is important in practical applications where steep and/or fluctuating variations of  $\theta$  values lead to ineffective interpolation between two adjacent stored values.

#### IV. CONCLUSION

In this paper a simple and computationally tractable solving strategy for eliminating selected harmonics and achieving a desired fundamental component is proposed. The particle swarm optimization (PSO) technique is used to solve the non-linear transcendental equations resulted from SHE technique. Switching angle trajectories for different cases are

presented. Various simulations using Matlab/Simulink confirm the theoretical results.

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