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ORIGINAL ARTICLE

Cylindrical and spherical ion acoustic solitary waves in electron-positron-ion plasmas with superthermal electrons

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Abstract Propagation of cylindrical and spherical ion acoustic solitary waves in plasmas consisting of cold ions, superthermal electrons and thermal positrons are investigated. It is shown that cylindrical/spherical Korteweg-de-Vries equation governs the dynamics of ion-acoustic solitons. The effects of nonplanar geometry and also superthermal electrons on the characteristics of solitary wave structures are studied using numerical simulations. Obtained results are compared with the results of the other published papers and errors in the results of some papers are pointed.

Keywords Cylindrical and spherical ion acoustic solitary waves · e-p-i plasma · Superthermal electrons

1 Introduction

The dynamics of ion-acoustic waves, which is one of the basic wave processes in plasmas, has been studied for several decades theoretically and also experimentally. The first experimental observation of ion-acoustic solitons has been made by Ikezi et al. (1970). Electron-positron (e-p) pairs exist in the plasmas emanating from the pulsars and also inner region of the accretion disks surrounding the central black holes in the active galactic nuclei (Miller and Witta 1987; Goldreich and Julian 1969; Michel 1982; Schlickeiser and Shukla 2003). Such pairs can also exist in the Van Allen radiation belts and near the polar cap of fast rotating neutron stars (Lightman 1982, 1987; Burns and Lovelace 1982; Zdziarski 1987) in semiconductor plasmas (Shukla et al. 1986) intense laser fields (Berezhiani et al. 1992) tokamaks (Helander and Ward 2003) as well as in the solar atmosphere (Tandberg-Hansen and Emslie 1988). It has been observed that the electron-positron plasmas behave differently as opposed to typical electron-ion (e-i) plasmas (Rizatto 1988; Yu 1985). Since in many astrophysical environments there exist a small number of ions along with the electrons and positrons. Therefore it is important to study linear and nonlinear behaviour of plasma waves in electron-positron-ion (e-p-i) plasmas. A lot of researches have been carried out to study the (e-p) and (e-p-i) plasmas in the past few years (Nejoh 1996; Verheest et al. 1995; Mushtaq and Shah 2005; Kourakis et al. 2007). For instance, Nejoh (1996) investigated the effect of ion temperature on the large amplitude ion-acoustic waves in (e-p-i) plasmas and observed that the ion temperature decreases the amplitude and increases the maximum Mach number of the ion acoustic waves. In the other hand, space plasma observations indicate clearly the presence of ion and electron populations which are far away from their thermodynamic equilibrium (Shukla et al. 1986; Ghosh and Bharuthram 2008; Pakzad 2009a, 2010). Numerous observations of space plasmas (Feldman et al. 1973; Formisano et al. 1973; Scudder et al. 1981; Marsch et al. 1982) clearly indicate the presence of superthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. The latter may arise due to the effect of external forces acting on the natural space environmental plasmas or to the wave-particle interaction which ultimately leads to kappa-like distributions. As a consequence, a high-energy tail appears in the distribution function of the particles. Many authors have investigated ion acoustic waves in e-p-i plasmas with electrons which evolve far away from their thermodynamic equilibrium (Ghosh and Bharuthram 2008; Pakzad 2009b, 2009c; Renyi 1955; Tsallis 1988). Recently, N. Boubakour (2009) has studied

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ion acoustic solitary waves in e-p-i plasmas with superthermal electrons. He showed that lumps of ion-acoustic solitons can be appeared in the presence of superthermal electrons.

However, all these investigations are limited to one-dimensional (planar) geometry which may not be a realistic situation in space and laboratory devices, since the waves observed in space (laboratory devices) are not one dimensional. It has been proved that the solitons in the nonplanar cylindrical and spherical geometries are expanding or contracting shell-like nonlinear waves which exhibit general soliton properties with the exception that their amplitudes vary with the radial distance (Ze et al. 1980). On the other side, cylindrical and spherical symmetric solitons have been observed in plasmas (Tsukabayashi et al. 1981; Mamun and Shukla 2001; Jehan et al. 2007).

In these studies electrons were assumed to be isothermal. But superthermality plays an important role in determining the nature of solitary waves. Existence of (e-p-i) plasma with superthermal electrons in astrophysical and space environments is a logical motivation for investigating planar and nonplanar (cylindrical and spherical) ion-acoustic envelope solitary waves in a three species unmagnetized, collisionless plasmas which are composed of cold ion fluid, superthermal electrons and thermal positrons.

In fact, these kinds of plasmas are frequently encountered in space and can result for example from the interaction of the solar wind which contains superthermal electrons (Gaelzer et al. 2008) with space plasma comprising electrons, positrons and ions in thermal equilibrium. Under certain circumstances, this plasma can survive or be sustained on a time scale that the ion-acoustic wave can be realized. Furthermore, do not look at e-p-i plasma as a pair (e-p) in a matter. An e-p-i plasma system with two different temperatures for the electrons is another interesting situation. For instance, outflows of electron-positron plasma from pulsars entering interstellar cold, low-density electron-ion plasma form two temperature electron-positron-ion plasmas. In this kind of plasmas, electrons having two different temperatures and higher density than positrons have a great chance to be in resonance with nonlinear ion-acoustic wave and be trapped by it. On the other hand, positrons having much higher temperature cannot be trapped by the wave, so they stay free. These hot free positrons in the weakly nonlinear wave limit are isothermal (Chatterjee et al. 2010).

Small amplitude cylindrically or spherically symmetric ion-acoustic waves were discussed by several authors. Such these situations have been observed in some astrophysical plasmas as well as laboratory experiments (for example see, Tsukabayashi et al. 1981; Nakamura and Ogino 1982). Only few investigations have been reported on the study of nonplanar ion acoustic solitary waves in e-p-i plasmas (Jehan et al. 2007; Sahu and Roychoudhury 2005; Sabry et al. 2009). Also nonplanar dust acoustic solitary waves in dusty plasmas with superthermal ions and electrons have been investigated before (Eslami et al. 2011).

Motivated by the above situation, existence of IAWs in e-p-i plasmas consisting of cold ions, Maxwell-Boltzmann distributed positrons and superthermal electrons is studied in presented paper. We show here how the ion acoustic solitary waves in cylindrical and spherical geometries differ qualitatively from that in one-dimensional planar geometry and how superthermal electrons affect on them. The plan of the paper is as follows: In Sect. 2, we will give a short definition of the model. Cylindrical and spherical modified KdV equations are derived in Sect. 3. The numerical solutions of cylindrical and spherical modified KdV equations and the effect of superthermality on the characteristics of these solitons will be discussed in Sect. 4 and finally, conclusion and some remarks are presented in Sect. 5.

2 Formulation of the model

We focus on cylindrical and spherical ion acoustic solitary waves in plasmas containing cold ion fluid, superthermal electrons and thermal positrons. In equilibrium, the charge neutrality condition is $n_{e0} = n_{i0} + n_{p0}$, where n_{i0}, n_{p0} and n_{e0} are the unperturbed number densities of the ion, positron and electron, respectively. The nonlinear dynamics of IA waves in cylindrical and spherical geometries is governed by

$$\begin{aligned}
&\frac{\partial n_i}{\partial t} + \frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m n_i u_i\right) = 0 \\
&\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial r} = -\frac{\partial \phi}{\partial r} \\
&\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m \frac{\partial \phi}{\partial r}\right) = n_e - n_p - n_i
\end{aligned} \tag{1}$$

where $m = 0$, for one-dimensional flat geometry and $m = 1, 2$ for cylindrical and spherical geometries, respectively. The ion density (n_i) and the ion velocity (u_i) are normalized by the ion equilibrium density n_{i0} and the ion-acoustic speed $C_s = \sqrt{\frac{T_e}{m_i}}$ respectively, where m_i is the ion mass. ϕ is electrostatic potential which is normalized by $\frac{T_e}{e}$ where T_e is the electron temperature and "e" is the absolute value of electron charge. In the equation set (1), the densities of the plasma species are normalized by the unperturbed electron density n_{e0}, space variable is normalized by the electron Debye length $\lambda_D = \sqrt{\frac{T_e}{4\pi n_{e0} e^2}}$ and time variable is normalized by the electron plasma period $T = \sqrt{\frac{m_e}{4\pi n_{e0} e^2}}$. One dimensional, kappa distribution commonly is given by

$$f_e(x, v_x) = \frac{n_{e0}}{\sqrt{2\pi\kappa\theta^2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa+\frac{1}{2})}\left(1 - \frac{e\psi}{\kappa m_e\theta^2} + \frac{v_x^2}{2\kappa\theta^2}\right)^{-\kappa-1} \tag{2}$$

in which the parameter κ shapes the superthermal tail of the distribution and Γ is standard gamma function and $\theta = \sqrt{\frac{T_e}{m_e}\frac{\kappa-\frac{1}{2}}{\kappa}}$. T_e and m_e are electron temperature and its mass respectively. Using definition $\phi = \frac{e\psi}{T_e}$ and integrating (2) over the velocity space, for suitable normalization factor one takes (Tribeche and Boubakour 2009)

$$n_e = \frac{1}{1-p}\left(1 - \frac{\phi}{\kappa - \frac{1}{2}}\right)^{-\kappa-\frac{1}{2}} \tag{3}$$

where $p = \frac{n_{p0}}{n_{e0}}$ is the ratio of unperturbed positron density to unperturbed electron density. There are several representations for superthermal distribution which are different in normalization terms or definition of θ. The parameter κ shapes predominantly the superthermal tail of the distribution (Boubakour et al. 2009) and the normalization has been provided for any values of the $\kappa > \frac{1}{2}$ (Tribeche and Boubakour 2009). In the limit $\kappa \to \infty$, superthermal distribution reduces to the Maxwell–Boltzmann distribution. Thermal positrons following Maxwellian distribution

$$n_p = \frac{p}{1-p}e^{-\sigma\phi} \tag{4}$$

where $\sigma = \frac{T_e}{T_p}$ and T_p is the positron temperature.

3 Derivation of the cylindrical/spherical KdV equation

In order to investigating the nonlinear solutions of Eq. (1) we have employed the standard reductive perturbation technique to derive the modified KdV equation. Localized wave solution is a function of $\mathrm{Exp}(i\theta)$ with $\theta = kr - \omega(k)t$ in which r is measured in the direction of the wave vector. The dispersion relation characterizes by $\omega(k)$ and k is the wave number. For small values of k (long wavelengths) one can use $k - \epsilon^p K$ in which ϵ is a small parameter and K is new (small) wave number. Parameter p is an unknown number to be determined later. Now $\omega(k) = \omega(\epsilon^p K)$ can be expanded using a Taylor series. Therefore we have $\theta = K\epsilon^p(r - \omega'(0)t) - K^3\epsilon^{3p}\omega'''(0)t$ where derivatives are constant values. It may further be noted that this prescription is closely related to validity of hyperbolic approximation and similarity transformation. It often turns out that when KdV equation occurs, the p usually takes the value of $1/2$. According to this method we introduce the stretched coordinates ξ and τ as follows (Maxon and Viecelli 1974)

$$\tau = \epsilon^{\frac{3}{2}}t, \qquad \xi = -\epsilon^{\frac{1}{2}}(r + \lambda t) \tag{5}$$

where ϵ is a small parameter and λ is the wave phase velocity. The dependent variables are expanded as

$$\begin{aligned} n_i &= 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \epsilon^3 n_i^{(3)} + \cdots \\ u_i &= \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \epsilon^3 u_i^{(3)} + \cdots \\ \phi &= \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \epsilon^3\phi^{(3)} + \cdots \end{aligned} \tag{6}$$

Substituting the stretching (5) and the expansions (6) into the basic equations (1) and (3–4), we obtain to the lowest-order in ϵ

$$\begin{aligned} &\lambda n^{(1)} = -u^{(1)} \\ &-\lambda u^{(1)} = \phi^{(1)} \end{aligned} \tag{7}$$

where

$$\lambda = \sqrt{\frac{(1-p)}{p\sigma + (\kappa+\frac{1}{2})/(\kappa-\frac{1}{2})}} \tag{8}$$

To next higher order in ϵ, we obtain the following set of equations:

$$\begin{aligned} &\lambda\frac{\partial n^{(2)}}{\partial\xi} - \frac{\partial n^{(1)}}{\partial\tau} + \frac{\partial}{\partial\xi}\left(n^{(1)}u^{(1)}\right) + \frac{\partial u^{(2)}}{\partial\xi} + \frac{mu^{(1)}}{\lambda\tau} = 0 \\ &\lambda\frac{\partial u^{(2)}}{\partial\xi} - \frac{\partial u^{(1)}}{\partial\tau} + u^{(1)}\frac{\partial u^{(1)}}{\partial\xi} = -\frac{\partial\phi^{(1)}}{\partial\xi} \\ &\frac{\partial^2\phi^{(1)}}{\partial\xi^2} = \left[\frac{(\kappa+\frac{1}{2}) + p\sigma(\kappa-\frac{1}{2})}{(1-p)(\kappa-\frac{1}{2})}\right]\phi^{(2)} \\ &\qquad + \left[\frac{(\kappa+\frac{1}{2})(\kappa+\frac{3}{2})}{(1-p)(\kappa-\frac{1}{2})^2} - \frac{p\sigma}{1-p}\right]\frac{\phi^{(2)}}{2} - n_1^{(2)} \end{aligned} \tag{9}$$

Combining above equations and making use of the first-order results, we obtain a modified KdV equation

$$\frac{\partial\phi^{(1)}}{\partial\tau} + \frac{m\phi^{(1)}}{2\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0 \tag{10}$$

where

$$\begin{aligned} A &= \frac{1}{2}\left\{\frac{3}{\lambda} + \frac{\lambda^3}{1-p}\left(p\sigma^2 - \frac{(2\kappa+1)(2\kappa+3)}{(2\kappa-1)^2}\right)\right\} \\ B &= \frac{\lambda^3}{2} \end{aligned} \tag{11}$$

Equation (10) is the cylindrical/spherical KdV equation describing the nonlinear propagation of the ion acoustic solitary waves in plasmas with superthermal electrons. In this equation A and B are the coefficients of nonlinear and dispersive terms.

The above results are comparable with the results of Boubakour et al. (2009) which deals with same plasmas but in flat geometry. The terms λ and B as well as the KdV equation (in the limit $m = 0$) in our calculations are as same as the results of Boubakour et al. (2009). But our calculated parameter "A" is different from Boubakour et al. (2009). To control the validity of our results we have compared our results with the results of Shah and Saeed (2011). Plasmas consisting of superthermal electrons and positrons with relativistic ions in flat geometry have been investigated in Shah and Saeed (2011). They have shown that shock profiles are created in this situation. Our results in the limit of thermal positrons and non-relativistic ions are exactly equal to the results of this paper. It is interesting that the presence of superthermal positrons beside the superthermal electrons and relativistic ions destroys the solitary wave profile and changes it into the shock structures (Shah and Saeed 2011). Our results are also in agreement with the results of Choi et al. (2011). Plasmas containing kappa distributed electrons have been considered in this paper. Therefore our results are comparable with the results of Choi et al. (2011) when we take $p = 0$ in (11). Our calculated coefficients "A" and "B" in this limit are equal to the results of Choi et al. (2011) too. Creation of both negative and positive solitary waves has been predicted in Choi et al. (2011). But it seems that there exists a mistake in the description of wave phase velocity in this paper. They have taken this quantity independent of plasma parameters. Our results show that only positive solitons can be propagated in such these plasmas which will be discussed in the next section.

4 Numerical results and discussion

When the geometrical effect is taken into account ($m \neq 0$), an exact analytical solution for the modified KdV equation (10) is not available. Therefore, we have numerically solved Eq. (10) and have studied the effects of superthermal electrons and also cylindrical/spherical geometry on the propagation of ion acoustic solitary waves. In the numerical procedure the modified KdV equation has been solved in time with a standard fourth-order Runge-Kutta scheme (Press et al. 1992) with a time step of 10^{-4}. The spatial derivatives were approximated with centred finite difference approximations using spatial grid spacing of 0.1 (Sabry et al. 2009; Maxon and Viecelli 1974). The used initial condition in all numerical solutions is the form of the stationary solution of Eq. (10) for $m = 0$.

Equation (10) contains the geometrical term $\frac{m\phi^{(1)}}{2\tau}$ which is singular at $\tau \to 0$. Therefore we can study the behaviour of solitary profiles near the singularity. It is clear that Eq. (10) is symmetrical respect to time and therefore we can (numerically) solve Eq. (10) in the left limit ($\tau < 0$) or right limit ($\tau > 0$) of the singularity. Indeed the evolution of a solitary wave which moves toward the singularity ($\tau < 0$) is the same as its evolution when it goes far away from the singularity ($\tau > 0$). Almost in all the papers the solitary wave evolution has been considered when it goes toward the singularity (Mamun and Shukla 2001; Jehan et al. 2007; Sahu and Roychoudhury 2005; Sabry et al. 2009; Mamun and Shukla 2002) as well as evolution of shock profiles in such this situation (Masood and Rizvi 2009).

At great values of time ($|\tau| \gg 1$) the geometry effects are weak so we can take this stage as the initial stage of evolution i.e.

$$\phi^{(1)} = \phi_0 \mathrm{sech}^2\left(\frac{\chi}{w}\right), \qquad \chi = \xi - u\tau \tag{12}$$

where $\phi_0 = \frac{3u}{A}$ is soliton amplitude and $w = \frac{\sqrt{4B}}{u}$ is its width. Here "u" is constant velocity of the solitary wave.

Polarity of created soliton in the media is depending on the sign of the parameter "A". "A" is a complicated function of the medium parameters therefore an analytical studying is not helpful. We scanned all the area of the medium parameters numerically in order to find negative solitons. But our numerical calculations show that only positive solitary waves can be propagated in this plasma. In the limit of $\kappa \to \infty$ one can easily show that $A \to \frac{1}{2\lambda}(3 + \frac{(1-p)(p\sigma^2-1)}{(1+p\sigma)^2})$ which is positive (Note that all the appeared plasma parameters in this situation are smaller than one). In the limit $k \to \frac{1}{2}$ we have $\lambda \to 0$ and $A \to \infty$. However the possibility of negative soliton propagation have been predicted in Choi et al. (2011) for such these medium, but it seems this result is not right.

Numerical solutions of Eq. (10) reveals that for sufficiently large values of $|\tau|$ (e.g., $\tau = -14.0$) the spherical and cylindrical solitary waves reduces to one dimensional solitary waves. This is due to a large value of $|\tau|$ which causes the term $\frac{m}{2\tau}\phi^{(1)}$, is no longer dominant. However, as the value of $|\tau|$ decreases, the term $\frac{m}{2\tau}\phi^{(1)}$ becomes dominant and both spherical and cylindrical solitary waves will be different from one dimensional solitary wave. The results of numerical solutions of (10) (using (12) as initial condition at large $|\tau|$) are displayed in Figs. 1 to 5. These figures show how the superthermal parameters (κ) affect on ion acoustic solitary waves in cylindrical and spherical geometries.

Figure 1 presents evolution of a rarefactive solitary wave from $\tau = -14$ to $\tau = -3$ in cylindrical geometry. Solitary solution of flat space has been taken as initial condition at $\tau = -14$ where geometric effects are sufficiently small. Soliton amplitude becomes larger when time reaches smaller $|\tau|$ while its width decreases. On the other hand Fig. 1 clearly shows that soliton amplitude and its width increases when superthermal parameter "κ" increases. It is expectable, because the soliton energy increases when population of superthermal electrons increases.

Let us compare the position of solitons with different values of "κ " in time $\tau = -3.3$. Created soliton in a medium

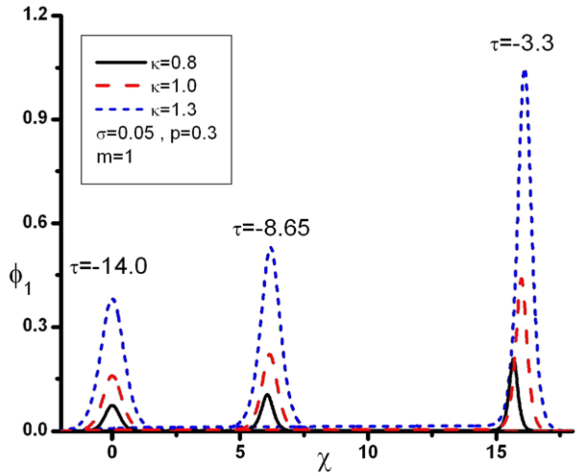

Fig. 1 Time evolution of cylindrical solitary waves ($m=1$), $\phi^{(1)}$ versus spatial coordinate χ at times $\tau=-14.0$, $\tau=-8.65$ and $\tau=-3.3$ for $\sigma=0.05$, $p=0.3$ with different values of κ

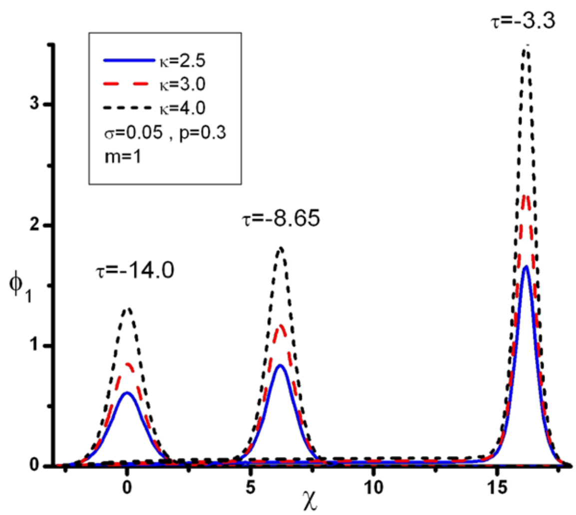

Fig. 2 Time evolution of cylindrical solitary waves ($m=1$), $\phi^{(1)}$ versus spatial coordinate χ at times $\tau=-14.0$, $\tau=-8.65$ and $\tau=-3.3$ for $\sigma=0.05$, $p=0.3$ with different values of κ

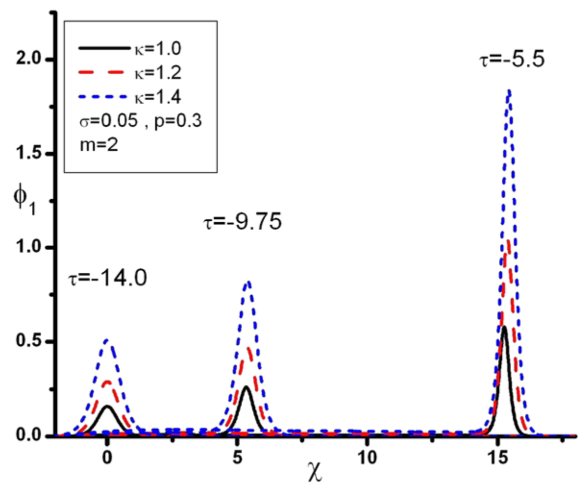

Fig. 3 Time evolution of cylindrical solitary waves ($m=2$), $\phi^{(1)}$ versus spatial coordinate χ at times $\tau=-14.0$, $\tau=-9.75$ and $\tau=-5.5$ for $\sigma=0.05$, $p=0.3$ with different values of κ

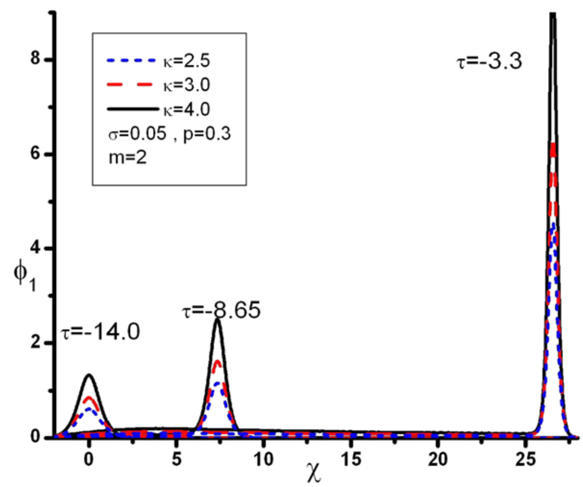

Fig. 4 Time evolution of cylindrical solitary waves ($m=2$), $\phi^{(1)}$ versus spatial coordinate χ at times $\tau=-14.0$, $\tau=-8.65$ and $\tau=-3.3$ for $\sigma=0.05$, $p=0.3$ with different values of κ

with bigger "κ" goes faster than solitons "living" in a media which has smaller population of superthermal electrons.

Figure 2 presents the evolution of compressive solitary waves. All the features are the same as Fig. 1. But the increasing rate of the soliton velocity is not very sensible in larger values of "κ". Anyway, solitary waves have smaller velocities in the media with smaller "κ ".

Evolution of spherical solitary waves has been presented in Figs. 3 and 4. These figures also indicate that amplitude (width) of the solitary waves becomes larger (smaller) when the geometrical effects become dominant. On the other hand solitons in media with bigger "κ" have greater amplitude and width. Increasing the soliton velocity respect in time evolution also has been observed in this situation too.

Figure 5 provides a qualitative comparison between the geometrical effects on the evolution of a solitary wave. This figure clearly shows that solitons have larger velocities in spherical geometry. Note that the changing rate of the soliton velocity is not constant. Therefore we can conclude that the soliton moves under the influence of a kind of force on geometrical situations. Comparison of the soliton amplitude in different geometries clearly indicates that solitons in spherical geometry have greater energy. Geometrical effects come from the term $\frac{m}{2\tau}\phi^{(1)}$ in (10) and this term has its greater effects with bigger values of m.

Our results also can be compared with very recent published paper (Ghosh et al. 2012). Nonplanar ion acoustic solitary waves with superthermal electrons and positrons have been investigated in this paper.

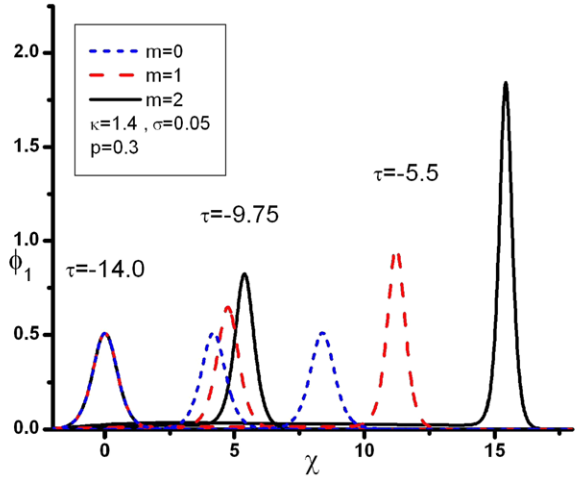

Fig. 5 Time evolution of solitary waves in different geometries, $\phi^{(1)}$ versus spatial coordinate χ at times $\tau = -14.0$, $\tau = -9.75$ and $\tau = -5.5$ for $\sigma = 0.05$, $p = 0.3$ with different values of $\kappa = 1.40$

5 Conclusions and remarks

Nonlinear analysis has been carried out to derive the appropriate Korteweg- de Vries equation for cylindrical and spherical ion-acoustic waves in an unmagnetized e-p-i plasmas comprised of cold ions, thermal positrons and superthermal electrons. The effects of nonplanar geometry and superthermal distributed electrons on the amplitude and width of the ion acoustic solitary waves are investigated numerically. Soliton amplitude (width) increases (decreases) during the evolution when time reaches smaller values. Soliton velocity also increases during the time evolution toward the smaller values of time. Beside this, soliton amplitude and also its width increases when superthermal parameter increases. Therefore soliton energy increases when "κ" increases. The cylindrical and spherical solitary waves with greater values of "κ" move faster in comparison with solitons propagate in a media with smaller values of "κ". Also Soliton acceleration in spherical geometry is greater than that in cylindrical geometry. There are several features which needs further investigations. For example, the effects of high relativistic particles and QED effects in such these plasmas have not been studies completely. Presented investigation might help us to find a better perspective of cylindrical and spherical ion acoustic solitary waves which occur in astrophysical plasmas. Also comparing the results of plasma systems with different distribution of constituents in spherical/cylindrical geometries also can help us to find better knowledge in plasma physics.

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