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Research

Mathematical Knowledge of Architecture in the Works of Kâshânî

Abstract. Ghiyâth al-Dîn Jamshîd Kâshânî (al-Kâshî) is the prominent mathematician and astronomer of ninth/fifteenth century of Islamic civilization who founded the scientific methodology in practical and theoretical knowledge of mathematics in architecture. He played a significant role in the interaction and concurrence among mathematicians and architects, by addressing the relation between architecture and mathematics in two areas of knowledge, theoretical and practical, the latter itself including theoretical practice and practical practice. This paper addresses the question of how Kâshânî's research has led to the foundation of the first theoretical basis for the application of mathematics in architecture in the form of easily practicable solutions for drawing, and measuring different types of ceilings, arches, vaults, domes, and ornaments, as well as estimating required materials. Further, we will discuss how his knowledge in mathematics and astronomy were utilized in management, design and construction of Samarkand observatory and its astronomical instruments. By exploring how the mathematical knowledge of a Persian scientist was utilized in architecture and craftsmanship, we will shed some light on the hidden layers of Kâshânî's architectural life, who until now has been considered only a mathematician and astronomer.

Introduction

The written resources are very rare regarding theoretical fundaments of Islamic architecture compared with the amount of its executed buildings. Despite this fact, most of architectural theoretical thoughts and related branches have remained undiscovered among some scientific (including mathematics[1]) texts. On the other hand, the collaboration between mathematicians and architects in the past has provided the context for including practical mathematics texts in architecture treatises. Among them, Kâshânî is a key figure in the relationship of mathematics and architecture. He is considered as one of those scientists whose precise examination of architecture and handcrafting in the light of mathematics is outstanding among some of his works such as *Key of Arithmetic* and *Letters*. Although, there have been some valuable investigations explaining and interpreting his works, most of his architectural activities and internal thoughts have remained untouched.

Kâshânî was a mathematician, astronomer, efficient author, designer and innovator of astronomical instruments. Not much information is available on his life in Kâshân and only some parts of his life and activities in Samarkand has been reflected in historical texts, particularly his *Letters* to his father. Kâshânî was born in 790/1387 in Kâshân, where he spent most of his first period of life. He probably had explored some other cities in Iran, before departure for Samarkand in 824/1421, when Ulûgh Beg (1394-1449), Tamerlane's grandson, invited him. There, he received the membership of Ulûgh Beg's scientific circle.

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He died in 832/1429 beside the Samarkand observatory which had been designed and directed by himself.

This paper, based on descriptive and analytical research, analyzes Kâshânî's practical and theoretical works, and aims at achieving a more comprehensive grasp of his accomplishments in connecting mathematics, astronomy, and architecture. Here we will:

1. Address the relationship between mathematics and architecture from his point of view. This relationship involves many aspects of architecture from details to the whole and leads to special interactions among mathematicians, architects, masons and tradesmen. Then, we will explain:
2. How this interaction reaches its perfection in design and construction of some architectural details, the Samarkand observatory, and its astronomical instruments.

Through this approach we will see how Kâshânî not only created new patterns and developed other patterns of his time, but also had taken long steps towards establishing a framework to connect architecture and mathematics theoretically.

The Bounds between Mathematics and Architecture in Kâshânî's Works

Kâshânî's activities can be categorized into three classes due to his ideas on the areas of knowledge presented in diagram 1.[2] In this classification, he believed in an intermediate area connecting the two purely theoretical and practical areas; in other words it could be assumed as knowing the precise techniques and contrivances necessary for putting scientific (i.e. mathematical) concepts into practice. In this domain, despite being aware of required methodologies to respond practical questions, the individual has not yet experienced the third area. For example, some of mathematical texts which have been written for practical intentions such as Abu'l-Wafâ's (940-998) *Geometrical Constructions* and Kâshânî's *Key of Arithmetic* are categorized into the second area: theoretical practice; and the performance of these books which is supposed to be accomplished by tradesmen is categorized as pure practice.

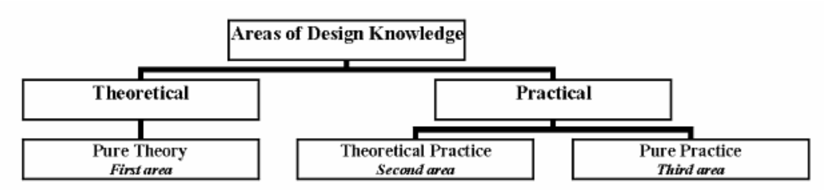

Diagram 1. Different area of knowledge in Kâshânî's point of view

Among the significant works which Kâshânî has written, *Key of Arithmetic* has gained the most reputation in the history of the relationship between mathematics with handcrafting and architecture. This book, in fact, is an introductory glossary on mathematics which has been written based on practical requirements of calculators, tradesmen, architects, etc. In addition to *Key of Arithmetic*, his *Letters* has not still been considered in the light of architecture, though the author had been engaged with architectural concerns.

The key question is that what inspired Kâshânî to write a basic document on the application of mathematics in architecture and handcrafting. In fact, amongst Kâshânî's works, particularly his *Letters,* there exist some descriptions of architecture and its relevant activities. The assignment of a comprehensive part of one of his books in mathematics, to these descriptions of handcrafting and architecture, makes it reasonable to infer that he had been deeply concerned with practical and theoretical affairs of those disciplines, and had reached a particular level of skill in them. This point of view, together with investigating other documents on Kâshânî's life, and also the architectural discourses that he had put forward, reveals some so far hidden layers of his life. Therefore, we will first take a look at the fourth chapter (*maqâla*) of *Key of Arithmetic,* and then we will investigate the *letters.*

The Fourth Chapter of *Key of Arithmetic.* This chapter is on the measurement of surfaces and volumes of geometric forms, and their applications in handcrafting and architecture. Here, first Kâshânî has defined several types of geometric forms and measured their surfaces and volumes [Ghorbani 1988: 97, 109]. Then, in the last section (*bâb*), "Measurement of Structures and Buildings", after the definition of arches, vaults, domes, and muqarnases, just as mathematical axioms, he again measured each of them. Also, he has presented some tables to calculate the surface of equilateral polygons in terms of multiplication of a constant value and the square value of one edge, in an easily accessible manner; and for those equilateral polygons that were widely in use, he presented formulas defining edges, surface, and the radius of in-circle, in terms of each other [Ghorbani 1988: 102].

It is believed that geometric elements forming architecture in the view of mathematics have not been studied by mathematicians comprehensively and scientifically in the Islamic world in pre-Kâshânî era [Memarian 1988: 406]. Reminding the lack of these studies, at the introduction of the ninth section Kâshânî proclaims that:

> The associated specialists restrictedly write on the volume of an arch, and vault in an incomplete way in their books, though I mention all of these measurement calculations as it is required, since the need to measure the volume and area of buildings is more than any other volume [al-Kâshî 1977: 353].

Beforehand, it is necessary to remind that the present paper is not trying to probe into Kâshânî's method of measuring geometric shapes and buildings,[3] but seeks to explore how mathematical knowledge of architecture has been organized in his point of view.

There have been different opinions on Kâshânî's purpose in writing this paper, particularly the ninth section, and who his audiences were [Necipoglu 1995; Golombek and Wilber 1988]. However, in the introduction of *Key of Arithmetic*, he explicitly asserts that this book is to satisfy calculators and geometers by providing them with easy and practicable solutions [al-Kâshî 1977: 37]. For example, Kâshânî has described practical methods (in designing, drawing and implementing) while elaborating muqarnas construction and calculations [al-Kâshî 1977: 387], which made it possible for mathematicians and architects to establish a common language.

Kâshânî's account of geometers is not exactly clear. The geometricians, addressed by Kâshânî, may include two groups. First, the architects whose designs are based on geometry and Kâshânî has considered their precision through preparation of simple tables for drawing and calculating geometric forms in an easy though meticulous way. The second group may be those people whose duty is preparation of geometric and architectural plans.

However, yet there is no evidence implying on construction plans being prepared by efficient people in geometry.

Nevertheless, through a closer look at the fourth chapter and disregarding its purpose, this chapter in itself is the most important source for theoretical edition of geometric components forming architecture in the view of mathematics. In fact, before presenting his methods of measuring the buildings constituents, Kâshânî has first decomposed geometric forms and established a system of precisely defined components, including their classification, type of function, method of drawing and implementation. By so doing he in fact founded the theoretical structure of form's components and their combinatorial language that is briefly sketched out as follows.

I: *The first seven sections of the fourth chapter*. Kâshânî presents a complete division for measuring all geometric shapes and their volumes. In parts of his classification he chooses to use the names which were common between craftsmen and architects of his era, instead of their prevalent geometric terms. These sections are an introduction to understand and use the ninth section. In diagram 2, I have tried to extract a general image out of these sections. Based on the general divisions, this diagram could be expanded to encompass all the subdivisions which Kâshânî has also classified.

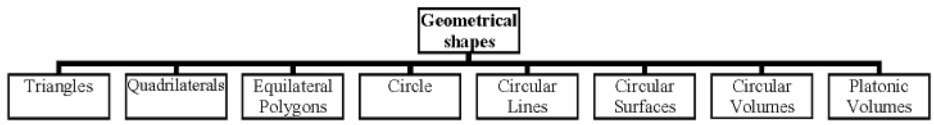

Diagram 2. Kâshânî's classification of geometric shapes

II: *The ninth section of the fourth chapter*. After the general approach which he had put forward in classifying geometric forms, Kâshânî has presented a division containing elements of architecture based on geometry of forms and different kinds of their application in order to measure the building's volumes. Although it seems that this classification does not include all kinds of arches, domes and muqarnases up until Timurid era, the following diagram could be considered as a basic structure for classifying two and three dimensional geometric shapes in architecture.

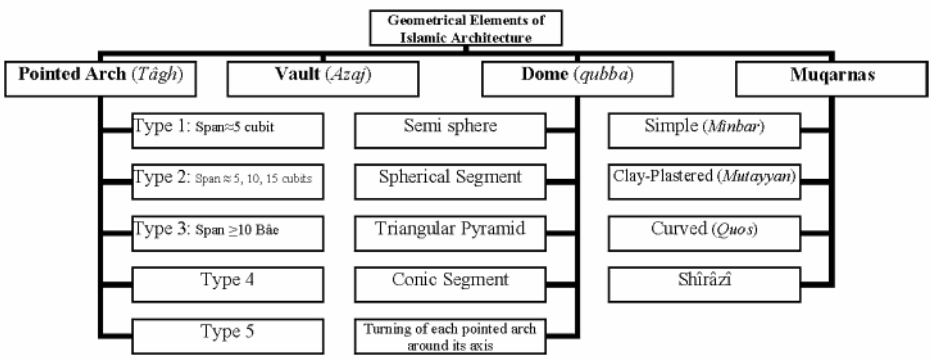

Diagram 3. Kâshânî's classification of geometric elements forming architecture

Here, some significant points should be mentioned: First, contrary to some investigators' ideas [Memarian 1988: 63], Kâshânî has not examined only one vault. Based on his definition, which says if the depth of each kind of arch or each of pointed arches, be greater than its span, it will be included in vault definition, it is wrong to assume that vault in Kâshânî's book is restricted to barrel vault. Second, while knowing that elliptical profiles are in use in arches and domes of Islamic architecture and circular profiles are not suitable arches for curved coverings [Dold-Samplonius 2003: 247], it is surprising that he has not even considered the ellipses separately, though his attitude to combine circular sectors to estimate and draw ellipses is still a common and acceptable technique. Finally, Kâshânî has presented the basis for drawing other kinds of arches through the division of a circle using the first and second methods into 7, 9, 10, etc., parts; also other divisions of the diameter of a circle in his third and fourth methods will lead to newer types of arches.

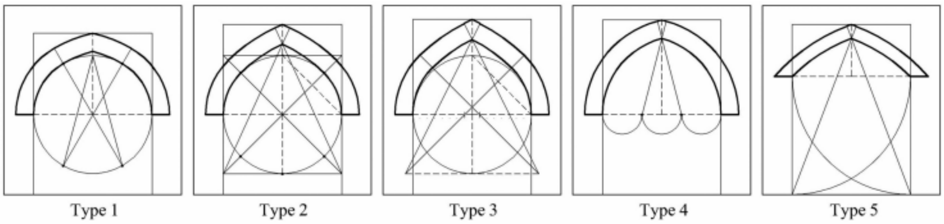

Fig. 1. Five methods for designing and drawing different types of pointed arches from Kâshânî's *Key of Arithmetic*

Letters. Kâshânî's *Letters* are samples of the most important documents of science history that reflects some characteristics of Ulûgh Beg's scientific circle. Furthermore, Kâshânî's two letters and the appendix of his second letter are authentic records implying on his precise knowledge and view to architecture. In his *Letters* Kâshânî in addition to describing the design and constructing the Samarkand observatory, points to the usage of mathematical and astronomical knowledge in responding to two practical questions. His first inquiry was to design and build a sundial on the surface of a marbled wall of unknown (i.e., arbitrary) azimuth at the royal palace [Bagheri 1997: 244]. The second question was to design and place a single slot in the wall of a *mihrâb* (prayer niche in a mosque) which during the whole year, would make it possible for a light beam to enter the mosque [Bagheri 1997: 244] exactly at the afternoon praying time (*azân*) according to Abû-Hanîfa's Sunnite sect. Here I will present a solution which I believe Kâshânî should have been aware of, once he was asked to do so. Although his account of the solution could have been completely of his own, the presented solution proves his depth of knowledge (fig. 2).

In Abû-Hanîfa's sect, when the length of a vertical gnomon's shadow on a level surface reaches twice the height of that gnomon, it is the time of afternoon prayer; in other words, the height angle of sunshine should reach to 26.6º. The locus of sun lights passing through the end point of the gnomon and making the given angle is an empty circular cone (figs. 2.1-2).[4] In the stereographic diagram[5] of Samarkand, in all afternoons of the year, the azimuth angle of the sun varies within the 65º between 220º and 285º (fig. 2.3). Therefore a section from the thin surface of this cone is the locus that includes all the required extensions of sun lines; and gathers the lines all in one vertex (fig. 2.4). If we subtract this thin volume from a one meter thick wall, (which is perpendicular to Kiblah extension), the required crack would be achieved (figs. 2.5-6). Kennedy [1960; cf. Bagheri 1997: 252-253] has assumed the vertex of this cone was located at the outer side of the wall; but due to Kennedy's belief, contrary to what was needed, there would be a slot at the outer side, and a hyperbolic crack in the inner side to the *mihrâb*. On the other hand, the cone vertex could not be located at the outer surface of the wall. It seems that Kâshânî has first made this cone-shape section with other materials, like metal, and then set it up in the wall.

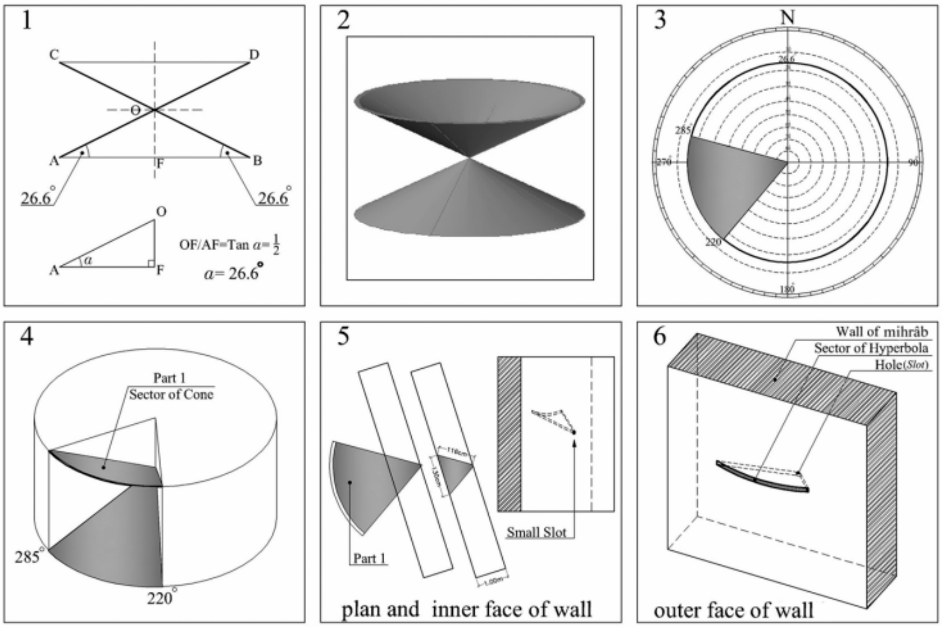

Fig. 2. The process of designing a slot which leads the sun line in the mihrâb in the time of afternoon praying

Kâshânî and Samarkand Observatory

Ulûgh Beg was an efficient mathematician and astronomer. To prepare a new Zîj (set of astronomical tables) he decided to build an observatory. At that moment he asked Qâżîzâda-i Rûmî (1364-1436) to found a scientific organization comprising of skilled and experienced figures of his time. Qâżîzâda, passing from Kâshân, came in contact with Kâshânî in an astrolabe studio. Once arrived in Samarkand, Qâżîzâda purposed Kâshânî's invitation to Ulûgh Beg and added that the erection of the new observatory is not feasible without Kâshânî's collaboration [Kalântar Zarrâbî 1977: 410]. Ulûgh Beg did not trust on any one without passing a tight process of qualification in theoretical and practical respects; and Kâshânî was known as an eligible person whom could be in charge of this unique project of Islamic world, after he participated in many scientific circles and proved his superiority over other scientists. This provided the necessary arrangements to accredit him as the director to design, and build the observatory and its astronomical instruments.[6]

Date and Characteristics of the Observatory. There is little and incomplete information about the building of the Samarkand observatory. In *Great Soviet Encyclopedia* it is mentioned that in 1908 J.L.Vjatkin succeeded in discovering the remains of this observatory, (i.e., meridian arc,[7] in fig. 3). Later excavation led to the discovery of the foundation of a circular wall approximately 46 m diameter (without a marbled covering), that sextant's instrument (or probably a quadrant) were based on its center in 1948 (fig. 4). The radius of sextant was 40.04 m. The whole sextant was located in a hole, with approximately 2 m width, and 11 m depth, up on a stone hill and only some parts of it were erected upper the ground.[8] The hill's surface was about 21m up the ground. This cylindrical building with an estimated height about 30 to 33 meters was built based on the pattern of the Marâgha observatory. It was developed by marbled facades (probably combined with brick) [Sayili 1960: 274-275]. Two parallel brick curves covered with marble, with 117 cm height and 30 cm width and 51 cm distance from each other, were found in the middle part of sextant [Varjavand 2005: 385]. Beside the two outer sides of these walls, there are some stairs in order to get access to angles above sextant.

Fig. 3. Meridian arc of the Samarkand observatory. Photo from http://travel.webshots.com/photo/1037307920026227552HqOLEM, uploaded by Sealevel5

Fig. 4. The remains of the Samarkand observatory. Photo from http://travel.webshots.com/photo/1197239572047825426TdtaYZ, uploaded by Stephenshephard

The tower of the observatory was built in three storeys. There were some rooms for staff and servicemen in the first storey, and the stars were observed from upper storeys that were covered by thorough arches.[9] According to historians, the walls inside the observatory were covered with elegant pictures of ten celestial spheres; the shape of seven orbiting planets' circle; fixed stars' position; and also earth, with its subdivisions, seven climates, mountains, deserts, and seas [Sayili 1960: 282]. According to Samarkandi, there were some gardens, palaces and rooms near the observatory for resting, examining, and supervising [Samarkandi 1964: 45-46]: In the *Letters* it is also pointed that in his meetings with Kâshânî, Ulûgh Beg monitored and approved the construction process in building and making each astronomical instrument [Bagheri 1996: 73]. The architectural description of Kâshânî in the appendix of the second letter apparently refers to these buildings, and as he has stated, he was a resident of one of these lodges.

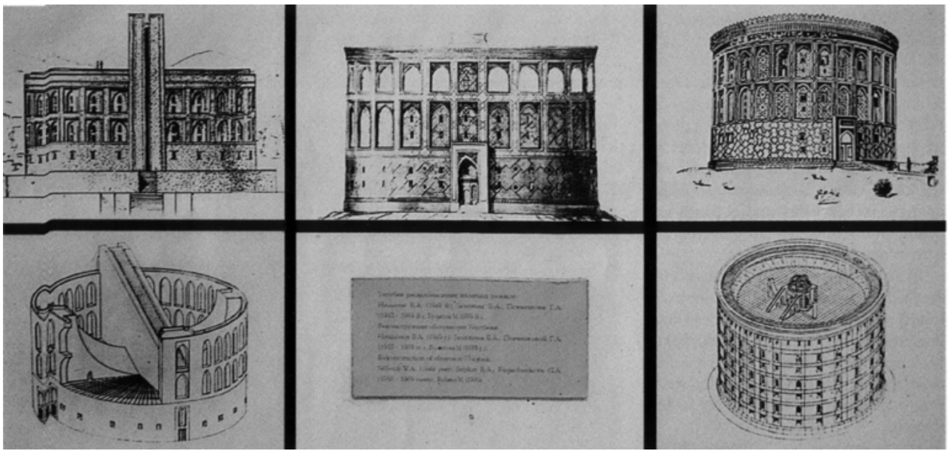

Fig. 5. Some drawings of the Samarkand observatory, recreated by Soviet Union scientists. From [Du Mont 2002: 42]

Kâshânî and the Observatory's Design and Construction Process. In accordance with *Letters* and historians, the design of observatory's building was accomplished according to:

- Kâshânî's descriptions on meridian arc (*geometric pulpit*) of the Marâgha observatory.
- The changes which were made by Kâshânî due to a change in the overall scale of design and its astronomical instruments.

Kâshânî reminds us about the executional requirements and the problems which were encountered in the design of the Samarkand observatory. Since brick in Samarkand was instable compared to Marâgha, and due to the enormous size of sextant curve, the building if implemented would become too tall, it would be probable to collapse. Therefore, about half of sextant should have been located under the ground. In addition to shorten its height this contrivance made the use of material, and construction time economic and also prevented the development of unnecessary spaces (fig. 6). Also, he indicates that the observatory's roof were managed to be flat so that some astronomical instruments could be placed on it [Bagheri 1997: 246]; hence the building probably incorporated stairs in order to give the people and instruments suitable access to the roof.

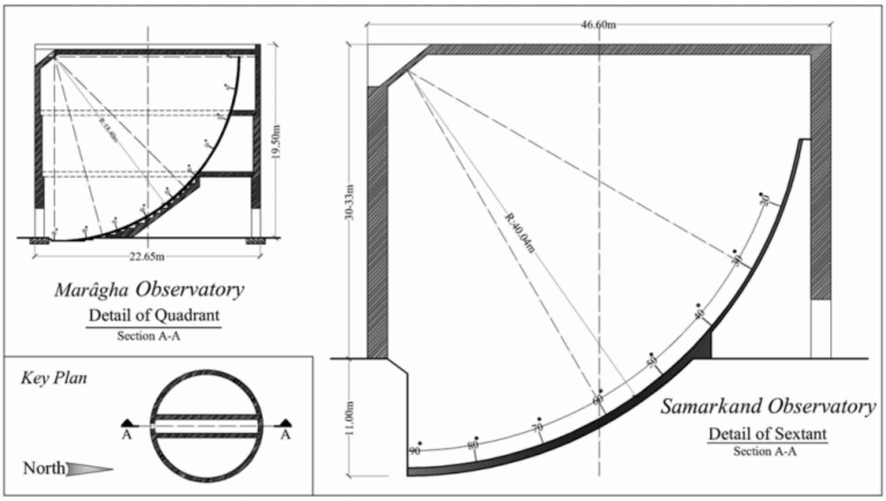

Fig. 6. A comparison between the scales in Marâgha and the Samarkand observatory. Based on [Varjavand 2005: 194, 385]

One of the most basic questions in building observatories was to determine the meridian line of the place of observation. While pointing out this question and giving a descriptive response, Kâshânî says: when they went to test the evenness of the surface, he saw that the masons had made a triangle which each of its edges was 4 cubits (~240.4 cm) long. Afterwards, master Ismâîl's son claims that we shall ascertain that at least two edges of the triangle are equal. And during a long argument Kâshânî demonstrates that this condition is not necessary for the triangle [Bagheri 1996: 67]. Although his proposed method (fig. 7.2) was not practically advantageous comparing the common method used by the masons (fig. 7.1), it is evident that Kâshânî's mathematical knowledge let him to think about other alternatives not discovered before. This argument is also a sign showing Kâshân's close interaction with architects and masons during the observatory's construction process.

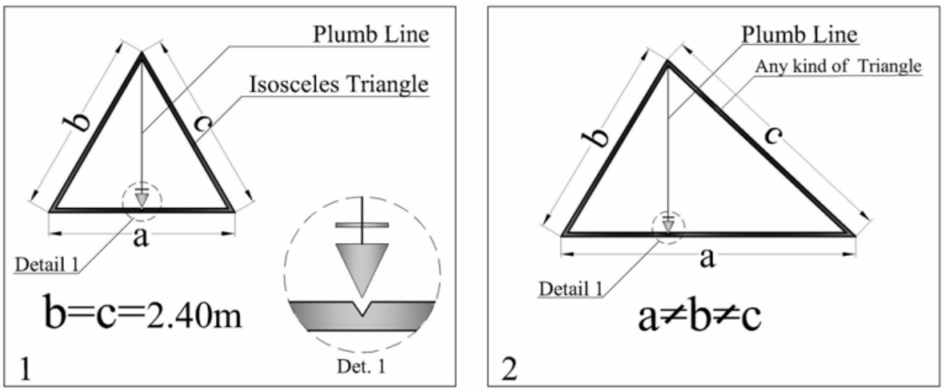

Fig. 7. To test the evenness of the land, masons traditionally used a triangle with two equal edges, but Kâshânî did not think it necessary

Also, before Kâshânî's arrival in Samarkand, astronomers who didn't know about the quality of the Marâgha observatory tried to prepare an armillary sphere (fig. 8). In order to do this, they had built some metal rings with 3 meters diameter and 4 cm thickness. Because the rings were too narrow and big they could not stand against torsion. Consequently Kâshânî ordered to destroy them, and new instruments were built according to his design [Bagheri 1997: 245-246]. In addition, Kâshânî points to his survey on observatory's construction material (i.e. brick and limestone) [Bagheri 1996: 71, 74].

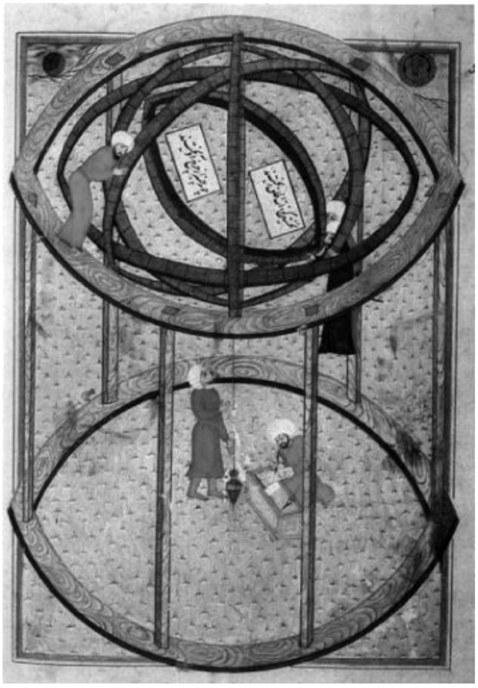

Fig. 8. An Armillary sphere (Zhât al-khalak) located at the Istanbul observatory, Istanbul University Library, F. 1404 (cf. [Sayili 2007])

Kâshânî, while being a skilled mathematician who offered methods to draw plans for architectural and handcrafts elements, was well aware of the techniques necessary to implement his theoretical knowledge; so proficient that he had no need to any plan. He indicates his collaboration with blacksmith master with no need of a draft [Bagheri 1996: 68] while observatory's building operation was in parallel in progress; and says: "…here, they ordered Master Ibrâhîm Saffâr to come to my lodge; and under my supervision he completed an armillary sphere" [Bagheri 1996: 68].

Finally, it is worth noticing that Kâshânî's intention in his *Letters* was not to make scientific texts. Nevertheless, these documents shed some light on valuable information which reflects some part of a mathematician's practical effort in the design and construction of the biggest and outstanding observatory in the Islamic world.

Conclusion

We saw through the text that Kâshânî was an efficient mathematician also familiar with architectural affaires. It was his dominance over these two different extremes that enabled him to found a form language of architectural elements, improve traditional patterns, and invent new patterns of his own – what he has theoretically tried to establish by classifying the design knowledge in three different areas.

It seems that the unity between knowledge and the art of form-making in Islamic civilization is the key to identify part of underlying thoughts and theoretical basis of Islamic architecture and related handcrafting; however, it hides under numerous mathematics treatises and books, and has not been considered deeply so far.

In academic discourses there are complaints about architecture and design having unscientific or arbitrary foundation, from scholars inside and outside this discipline. Although it is sometimes claimed that the conscious scientific movement in architecture has initiated in recent decades, and the intricate aspects of traditional architecture is usually referred to unconsciousness of their developers (see, for example, [Alexander 1964: 46-54]), there are evidences implying that in Islamic-Persian architecture there had been some figures who did appreciably try to found a scientific architecture. This paper supported this claim by introducing the thoughts of Kâshânî and illustrating how his ideas were reflected in his works.

Acknowledgments

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Notes

1. Mathematics means different branches of calculation, algebra, geometry, trigonometry, astronomy and music in this paper which in some scientific divisions made in the past architecture was subsumed under it.
2. [Bagheri 1996: 72]; for the second letter refer to [Kennedy 1960; 1983: 722-277].
3. Regarding Kâshânî's method in measuring building, there have been some studies. For example, see [Dold-Samplonius 2003].
4. Bagheri has also inserted this point in his editions on behalf of Kennedy in [Bagheri 1997: 252-253].
5. It is probable that Kâshânî used an astrolabe to define this azimuth angle.
6. In addition to suggestion and emphases which were made by Kâshânî in letters on his role in observatory, some historians' remarks are noticeable regarding it. For example, Khwând Mir says, "Ulugh Beg ordered efficient masters to set up an observatory in Firdaus (lit. "Garden") and Ghyâth al-Din and Moeîno al-Din Kâshânî tried to do it" [Khwând Mir 1954: 21].
7. A sextant is usually 1/6 of a circumference. If the curve is 1/4 of a circumference, it is called "mural quadrant".
8. These explanations are inserted in the editions which Bagheri has added to Kâshânî's *Letters* in [Bagheri 1996: 121].
9. These explanations are inserted in the editions which Bagheri has added to Kâshânî 's *Letters* in [Bagheri 1996: 122].

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