ECG Signal Compression Using Compressed Sensing with Nonuniform Binary Matrices

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Abstract—**Wearable ECG sensors can assist in prolonged monitoring of cardiac patients. Compression of ECG signals is pursued as a means to minimize the energy consumed during transmission of information from a portable ECG sensor to a server. In this paper, compressed sensing is employed in ECG compression. To increase compression ratio and reduce distortion of the ECG signal, a non-uniform binary sensing matrix is proposed and evaluated.**

Keywords - Signal compression; Electrocardigram; Compressed sensing

I. INTRODUCTION

Based on recent World Health Organization statistics, cardiovascular diseases are among the most common cause of death [1]. Electrocardiogram (ECG) is an effective method for diagnosis and monitoring of cardiac diseases. Heart attack victims usually seek medical assistance two or more hours after the onset of symptoms, which is sometimes too late. Therefore it is highly beneficial to detect signs of a heart attack immediately to seek prompt medical attention. Real-time online monitoring of patients by means of wearable ECG sensors is a means to achieve that goal. In such applications, the amount of power consumed in transmitting information from the sensor to a server needs to be minimized by means of signal compression.

In diagnosing some cardiac diseases, a portable ECG Holter monitor should be carried by patients to record and store their ECG signals for long periods of time (e.g., up to two weeks). Compression of ECG signals is necessary to minimize storage and power requirements.

Although there are many papers addressing the problem of ECG compression [10-19], only a few studies have been published in the specific area of ECG signal compression using CS. Allstot *et al.* [10] apply a thresholding operation first and then apply CS on ECG signal exploiting sparsity of ECG signal in the time domain. A significant drawback of this technique is that thresholding in time domain removes most of the basic features of the ECG signal. Hong-xin *et al.* [11] present a combination of wavelet and CS to compress ECG and EEG signals. Their method achieves high compression ratio with good reconstruction quality, but at the cost of increased complexity. L. Polania *et al.* [12] use distributed CS for adjacent beats of ECG. They perform a preprocessing first to detect the QRS complex and generating constant periods, then distributed CS is applied to compress the signal. Mamaghanian *et al.* [4] compare CS to wavelet compression combined with thresholding on ECG signals. They implemented the two algorithms on a Shimmer™ board with TI's MSP430 micro-controller, and obtained CR, PRD, run time and power consumption. This work shows that, at a given PRD level, CS algorithm performs better than DWT. It also shows that the complexity of a Gaussian sensing matrix is too high for it to be practical. For a sparse binary sensing matrix (such that each row has constant number of nonzero elements), ECG compression results are barely acceptable. In summary, previous research shows that if a straight-forward CS without preprocessing is used, CR and PRD will not be acceptable.

In this paper, we employ the compressed sensing (CS) method [2,3] to compress ECG signals. Compressed sensing works well when the signal of interest is sparse or very nearly sparse. Since sparsity of ECG signals is not high, the compression ratio of CS is rather poor and requires improvement. The objective of this paper is to propose and assess the use of a novel non-uniform sensing matrix that improves CS performance on ECG signals.

The performance of compression algorithms is usually measured in terms of two percentage indices [4]:

1) Compression Ratio (CR): This index is defined as

$$CR = \frac{b_{orig} - b_{comp}}{b_{orig}} \times 100 \tag{1}$$

where b_{orig} and b_{comp} are the number of bits required for the original and compressed signals, respectively.

2) Percentage Root-mean-square Distortion (PRD): This index quantifies distortion, the error between the original signal and the reconstructed one. It is defined as:

$$PRD = \frac{\left\| x_{orig} - x_{rec} \right\|}{\left\| x_{orig} \right\|} \times 100 \tag{2}$$

where x_{orig} is the original signal and x_{rec} is the reconstructed signal.

Signal compression in body-area sensor network not only should have low PRD and high CR but also it should present low complexity. Compressed sensing is a new method that offers reasonable CR and PRD at low complexity on the sensor side.

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This paper is organized as follows: In Section II, we briefly describe the theory of compressed sensing. In Section III, we review the recent literature on CS-based ECG compression. Then in Section IV, we present our proposed method. In Section V, our simulation results are presented. At the end, in Section VI, the contribution of this paper is summarized.

II. COMPRESSED SENSING

A fundamental principle of today's digital signal processing is the Shannon sampling theory: If a signal with a bandwidth of Ω is sampled at the rate of 2Ω samples per second (the Nyquist rate), the signal can be reconstructed reliably and without errors. The recently-developed compressed sensing (CS) theory [2,3] states that "sparse" signals can be reconstructed from a smaller number of samples than required by Nyquist rate. This method can be suitable for implementing low-resource sensor applications [4], as it reduces the amount of samples required in processing or storage.

Now, we introduce briefly the CS theory. Let x be a real-valued, finite-length, one-dimensional, discrete-time signal, which is viewed as an N×1 column vector (we call it an *input frame* of size N). Each signal in R^N can be represented as a superposition of an orthonormal basis of N×1 vectors $\{\psi_i\}_{i=1}^{N}$. Defining $\Psi = [\psi_1 | \psi_2 | \ldots | \psi_N]$ such that vectors ψ_i are the columns of Ψ, then x can be stated as:

$$x = \sum_{i=1}^{n} \alpha_i \psi_i = \Psi\alpha \tag{3}$$

where α is the N×1 column vector of weighting coefficients.

If α vector has $N - K$ zero or near zero values then signal x is called *K-sparse* in Ψ domain. The compressed sensing theorem [2,3] proves that x can be reconstructed from linear superposition of x samples, *i.e.*, instead of reconstructing from x, the signal can be reconstructed from y which is a vector of M linear projections of x onto another basis Φ ($M<N$).

$$y = \Phi x = \Phi\Psi\alpha = \Theta\alpha \tag{4}$$

where Φ is called the *sensing matrix*. We call y, the *output frame* of size M. Since Φ is non-square and hence irreversible, the signal x must be reconstructed by solving the convex optimization problem:

$$\min \|\alpha\|_{l_1} \quad \textit{subject to} \quad \Theta\alpha = y \tag{5}$$

Numerous algorithms have been presented to solve this [5]-[9]. However, two distinct conditions must be satisfied when applying convex optimization to signal reconstruction:

a) $$M > K\log(\frac{N}{K}) \tag{6}$$

b) For all K-sparse α vectors, Φ should satisfy the RIP condition:

$$(1-\delta_k)\|\alpha\|_2 \le \|\Phi\Psi\alpha\|_2 \le (1+\delta_k)\|\alpha\|_2 \tag{7}$$

where the constant δ_k must not be too close to 1.

It is well-know that random sensing matrices with independent identically distributed (i.i.d) elements nearly satisfy the RIP condition [2].

III. PROPOSED METHOD

We propose to use a CS algorithm that employs a special non-uniform binary sensing matrix to improve ECG compression performance. This matrix is described in this section. The CS algorithm operates on input frames of uncompressed samples of length N, and produces output frame of compressed samples of length M. Hence the size of a sensing matrix Φ is M×N. Note that in CS, we have CR=100× (N–M)/N. If N is set to a fixed value, M should be minimized in order to maximize the compression ratio. Now, we describe the uniform and non-uniform binary sensing matrices.

A. *Uniform Binary Sensing Matrix*

A matrix is called uniform binary if all its entries belong to the binary set {0,1}, and the probability distribution of its entries is i.i.d Bernoulli with p being the probability of 1. This matrix nearly satisfies the RIP condition for large N. The binary nature of the matrix elements makes matrix multiplication easy to implement.

B. *Non-uniform Binary Sensing Matrix*

Our proposed non-uniform binary matrix is loosely dependent on the ECG waveform to be sampled. First note that in each heartbeat, the QRS complex is our "region of interest" (ROI) in an ECG waveform. One would like to acquire and preserve more information from this part of the waveform to enhance *PRD*. Therefore in our algorithm at the sensor side (before performing CS on an input frame), we first detects ROI in that frame. This is done by detecting the location of QRS peak (called n_p) in the input frame and then setting a window of size W symmetrically around the peak time. The ROI will be [n_p–W/2 , n_p+W/2] within the frame. Given that the QRS complex duration is between 70ms and 130ms, for a sampling frequency of 360 Hz (which is the sampling frequency of our ECG waveforms), the size of this window is normally between 26 to 46 samples. The size of ROI window (W) is determined and fixed for each person. But the location of ROI within the frame varies from one frame to the next; hence it must be determined for each input frame. This location, i.e. n_p, is an extra piece of information that must be sent to the receiver (i.e., the reconstruction algorithm).

Our non-uniform binary sensing matrix **Φ** is a block matrix constructed as follows:

$$\Phi = \left[\begin{array}{c|c} \Phi 1 & 0 \\ \hline \multicolumn{2}{c}{\Phi 2} \end{array}\right] \tag{8}$$

where **0** is a block matrix of size $M_1\times(N-W)$ consisting of all zeros, $\mathbf{\Phi}_1$ is a block matrix of size $M_1\times W$, and $\mathbf{\Phi}_2$ is a block matrix of size $M_2\times N$. In addition, $\mathbf{\Phi}_1$ is a uniform binary matrix with probability p_1, and $\mathbf{\Phi}_2$ is a uniform binary matrix with probability p_2. The matrix $\mathbf{\Phi}_1$ will be shifted horizontally in the upper part of $\mathbf{\Phi}$ so that it is aligned to ROI in an input frame. (The horizontal shift operation causes a circular rotation of rows the upper part of $\mathbf{\Phi}$.) Fig. 1 shows an example of a non-uniform binary sensing matrix below an ECG waveform. The black dots represent 1's and blank parts represent 0's of the matrix.

Note that when performing CS, $\mathbf{\Phi}_2$ is multiplied into an entire input frame of ECG samples, performing a CS on the whole frame. On the other hand, $\mathbf{\Phi}_1$ is multiplied to the ROI of an input frame, taking additional samples from the interesting part of ECG. We define a parameter q defined as

$$q = M_1 / M \qquad (9)$$

This parameter along with M, W, p_1 and p_2 are to be determined for best performance. It should be noted that, although the location of peak of QRS complex changes in each input frame, it is not necessary to regenerate matrix $\mathbf{\Phi}$ for each new frame. Our CS algorithm is summarized as follows:

1) Use a sample of person's ECG to determine best M, W, q, p_2, p_2. Generate matrix $\mathbf{\Phi}$**.** Send $\mathbf{\Phi}$ to receiver.

2) For each ECG input frame x, do:

   a) Determine n_p (peak time of QRS) in x.

   b) Shift $\mathbf{\Phi}_1$ in $\mathbf{\Phi}$ to align it with ROI in x.

   c) Determine $y = \mathbf{\Phi}x$.

   d) Send y and n_p to the receiver.

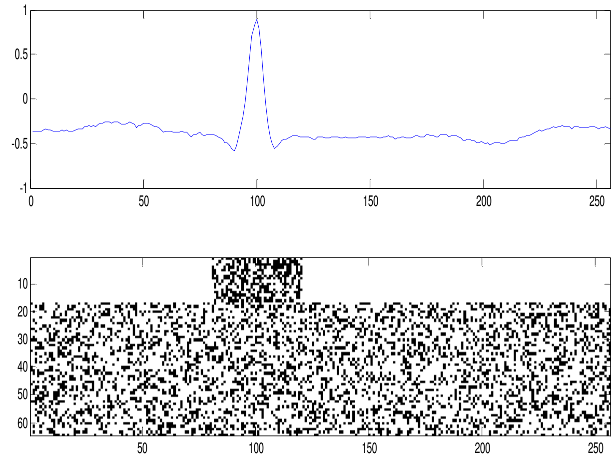

Figure 1. Top) an input ECG signal; Bottom) a binary non-uniform sensing matrix with p_1=0.5, p_2=0.4, w=40, q=0.25

IV. Simulation Results

In this section, we evaluate our proposed CS algorithm and compare its performance (by simulation) to that of the CS algorithm using uniform binary sensing matrices.

To reconstruct the ECG signal, the sparsification matrix $\mathbf{\Psi}$ is required to be known (in addition to $\mathbf{\Phi}$) at the receiver. The sparser the signal x in the $\mathbf{\Psi}$ domain, the closer the reconstructed signal will be to the original signal. Therefore, a good choice of $\mathbf{\Psi}$ affects the signal reconstruction and PRD significantly. In our research, we compared various discrete wavelets which have been shown to produce good results on ECG waveforms. The good candidates are the Daubechies wavelets (db2,db3,db4,…,db10), symlet4 and biorthogonal4.4. A comparison of the results shows that db10 and bior4.4 perform best. Fig. 2 shows PRD vs. CR for the two wavelets. We finally adopted biorothogonal4.4 as it shows superior performance over db10. We also evaluated the impact of four parameters p_1, p_2, q, w to determine the best values for CS compression. As an example, PRD versus q for CR=80% is plotted in Fig. 3 for different values of W.

We have also simulated PRD by simultaneously altering W and q for fixed values of CR=66%, 75% and 80%. The results are shown in Table I. In this table, p_1 and p_2 are set to 0.5 and 0.3, respectively. The best values for W and q can be found in Table I for best performance. For example, for CR=80%, the values W=50 and q=0.33 result in the lowest PRD.

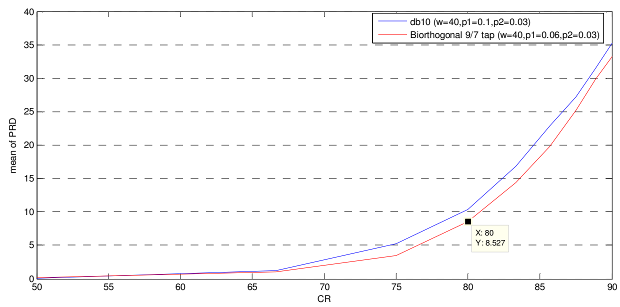

Figure 2. PRD vs CR for two wavelets: db10 and bior4.4

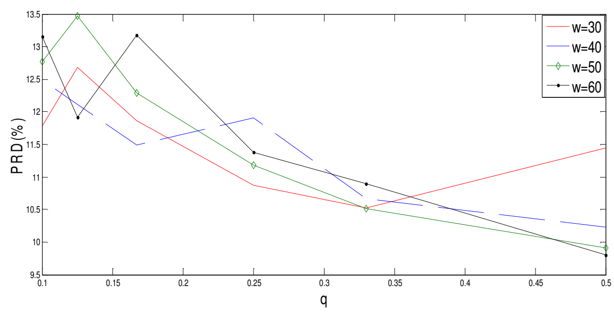

Figure 3. PRD vs. q for different value of W and CR=80%

TABLE I. PRD vs. w and q for CR=80%

CR=66%					
	***q*=0.5**	***q*=0.33**	***q*=0.25**	***q*=0.167**	***q*=0.125**
***W*=40**	4.97	5.25	5.26	5.64	5.69
***W*=50**	5.02	5.25	5.40	5.74	5.80
***W*=60**	5.09	5.11	5.35	5.64	5.76
CR=75%					
	***q*=0.5**	***q*=0.33**	***q*=0.25**	***q*=0.167**	***q*=0.125**
***W*=40**	7.44	7.52	8.36	8.64	9.24
***W*=50**	7.59	7.75	8.95	8.62	8.87
***W*=60**	7.59	8.24	8.95	9.53	9.58
CR=80%					
	***q*=0.5**	***q*=0.33**	***q*=0.25**	***q*=0.167**	***q*=0.125**
***W*=40**	10.83	11	12.3	12.44	12.47
***W*=50**	10.18	11.13	12.2	13.49	12.89
***W*=60**	11.46	11.55	11.79	13.75	12.61

Table II summarizes the simulated PRD values vs. p_1 and p_2, in order to show the impact of these parameters on signal compression. In Table II, W and q are considered 40 and 0.33 respectively. The best values for p_1 and p_2 can be found in this table for best performance. As shown in Table II, p_1=0.4 and p_2=0.5 result in the lowest PRD. Also the worst (highest) PRDs are obtained when $p_1 = p_2$. In the proposed method, CR depends only on M, the number of rows in matrix Φ, but PRD for a given CR depends on p_1, p_2, q and W. In other words, to increase CR, one should decrease the number of sensing matrix rows. Changing p_1, p_2, q and W will not affect CR.

In Fig. 4, compression performance has been plotted for 2 types of matrices: (1) Uniform binary matrix with p=0. 3. (2) Non-uniform binary matrix with p_1=0.3, p_2=0.4, q=0.5 and W=50. As seen in Fig. 4, our proposed method has reduced PRD (especially at high CR) compared to the uniform CS method.

TABLE II. PRD vs. p_1 and p_2

CR=66%					
	p_2=0.1	**p_2=0.2**	**p_2=0.3**	**p_2=0.4**	**p_2=0.5**
p_1=0.3	5.02	4.94	6.68	4.9	4.84
p_1=0.4	4.99	4.89	4.89	6.5	4.82
p_1=0.5	4.95	4.89	4.93	4.89	6.63
CR=75%					
	p_2=0.1	**p_2=0.2**	**p_2=0.3**	**p_2=0.4**	**p_2=0.5**
p_1=0.3	7.62	7.53	10.37	7.44	7.59
p_1=0.4	7.86	7.39	7.35	10.26	7.35
p_1=0.5	7.41	7.42	7.41	7.41	11.02
CR=80%					
	p_2=0.1	**p_2=0.2**	**p_2=0.3**	**p_2=0.4**	**p_2=0.5**
p_1=0.3	11.18	10.71	14.95	10.67	10.73
p_1=0.4	11.04	11.03	10.88	14.52	10.46
p_1=0.5	11.1	10.49	10.85	10.74	15.51

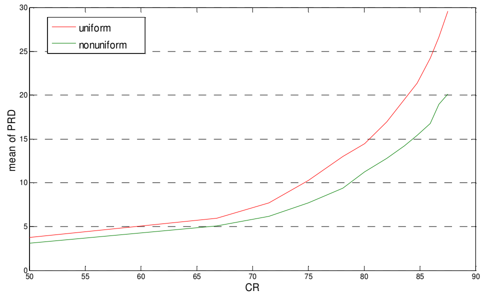

Figure 4. PRD vs CR for uniform and non-uniform sensing matrices

V. CONCLUSION

This paper proposed a CS-based compression algorithm for ECG signals using a special class of non-uniform sensing matrices. This matrix is designed to take into account the region of interest (the QRS complex) and to increase the overall PRD. We evaluated this scheme against CS-based compression with uniform binary matrices. Simulation results shows that, at the same compression ratio, PRD of this method is improved over that of the compressed sensing with uniform binary matrices.

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