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A Method for Solving Dynamic Equations of a 3-PRR Parallel Robot

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Abstract. In this paper, kinematic relationships for a 3-PRR planar parallel robot are first presented. The robot dynamics equations are formulated using Lagrange equations of first kind. The derived equations are a mixed set of differential and algebraic constraint equations, DAE, which must be satisfied simultaneously. In order to solve the robot dynamic equations, a new method is presented in which the dynamics equation is first partitioned into two parts. The constraint equations and the dependent coordinates are next eliminated. This reduces the dynamic equations to a set of differential equations as a function of three independent coordinates. Finally, a trajectory for the robot end-effector is specified and PD controller which follows the desired trajectory is implemented. The proposed method significantly simplifies the solution of the dynamics equations.

Introduction

Among advantages of parallel robots over its serial counterparts are improved accuracy, higher stiffness and higher load to weight ratio. The main weakness of these robots is limited and more complex kinematics analysis which can lead to challenges in robot's path control. To control the robot, it is necessary to have an appropriate dynamic model. A number of researchers have controlled robot by using neural networks and without considering the robot dynamics [1]. The inverse dynamics method [2] is used to control the path of a parallel robot with translational motion. Numerical solution may also be used to solve the robot dynamics equations. Although numerical solutions may be sufficient to investigate robot dynamic behavior, it cannot be used directly to control the robot. Therefore attempts have been made to find an analytical solution. One of the problems in formulating parallel robots dynamic is that the number of generalized coordinates is larger than system's degrees of freedom. This leads to differential algebraic equations (DAE) that is in fact a combination of differential equations and algebraic equations due to dependence between generalized coordinates. One way to solve these equations is to use numerical methods which may lead to instability [3]. Staicu [4] solved inverse dynamics problem of a 3-PRR parallel robot by using virtual work method. Kordjazi and Akbarzadeh [5] investigated inverse dynamics of a triangle 3PRR parallel manipulator using natural orthogonal complement. Robot path control also requires the solution to its kinematics. Kamali and Akbarzadeh [6] presented a method for a general solution to the direct kinematics problem of parallel manipulators in trajectory following by introducing a new concept based on basic regions. Also, Enferadi and Akbarzadeh [7] presented a novel approach for forward position analysis of a double-triangle spherical parallel manipulator.

Kinematics and Dynamics model of the 3-PRR Parallel Robot

Kinematics Model. The 3-PRR robot is comprised of three closed kinematic chains. See Fig. 1. Each kinematic chain consists of one prismatic joint and two successive revolute joints. The direction of the three Prismatic joints are star-shaped with 120 degrees angle. Additionally, the end-effector is in form of an equilateral triangle connected by three revolute joints.

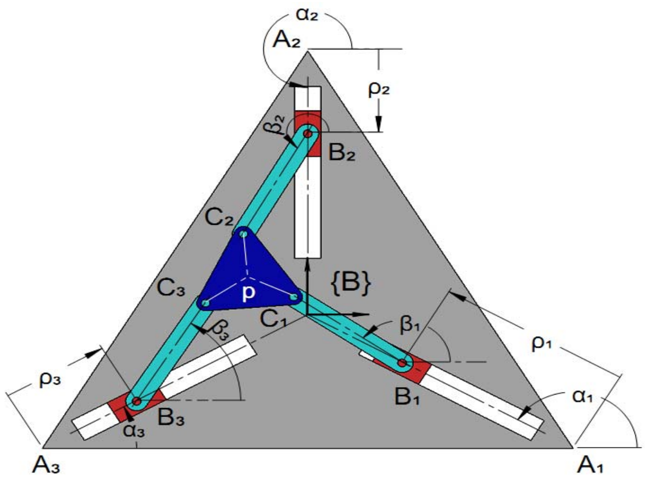

Fig. 1: Schematic view of 3-PRR parallel robot

The structural kinematics and dynamics parameters of the robot are supplied in Table 1.

Table 1: kinematics and dynamics parameters of 3-PRR parallel robot

Triangle Platform	100 mm	Mass of Sliders	0.2 Kg
Mass of Platform	0.2 Kg	Mass of Middle Link	0.02 Kg
Size of Angles α_i	120º, 270º, 30º	Dimention of Middle Link	200×25×1.5 mm

The origin of the fixed base reference coordinate system XYZ is located at intersection of sliders axis. Generalized coordinates for the robot ($\boldsymbol{\rho}$, $\boldsymbol{\beta}$, $\mathbf{X_p}$) are shown in Fig. 1. The matrix $\boldsymbol{\rho}$ is comprised of distance from points A_i to sliders position, $\boldsymbol{\rho}=[\rho_1 \quad \rho_2 \quad \rho_3]^T$. The matrix $\boldsymbol{\beta}=[\beta_1 \quad \beta_2 \quad \beta_3]^T$ represent angles between X axis of the base frame and $\overline{B_iC_i}$. Finally, position and orientation of platform's center of mass expressed in base frame is $\mathbf{X_p}=\left[x_p \quad y_p \quad \varphi\right]^T$. The goal of the inverse kinematics is to obtain $\boldsymbol{\rho}$ using platform's position and orientation, $\mathbf{X_p}$. Therefore [8],

$$\rho_i = M_i \pm \sqrt{l^2 - S_i^2} \quad ; \quad \begin{cases} M_i = \left(x_{C_i} - x_{A_i}\right)\cos\alpha_i + \left(y_{C_i} - y_{A_i}\right)\sin\alpha_i \\ S_i = \left(x_{C_i} - x_{A_i}\right)\sin\alpha_i - \left(y_{C_i} - y_{A_i}\right)\cos\alpha_i \end{cases} \tag{1}$$

where l is length of the middle links, $\overline{B_iC_i}$. Additionally, α_i represents angles between X axis of the base frame with $\overline{A_iB_i}$. In order to solve the robot dynamic equations, it is necessary to calculate velocity and acceleration values of the generalized coordinates system. We can then write [8],

$$\dot{\rho}_i = \frac{1}{\vec{a}_i.\vec{b}_i}\left[b_{ix} \quad b_{iy} \quad e_{ix}b_{iy} - e_{iy}b_{ix}\right]\left[\dot{x}_p \quad \dot{y}_p \quad \dot{\varphi}\right]^T = \mathbf{J_{pi}}\dot{\mathbf{X}}_\mathbf{p} \tag{2}$$

$$\dot{\beta}_i = \frac{1}{l^2}\left\{\left[-b_{iy} \quad b_{ix} \quad e_{ix}b_{ix} + e_{iy}b_{iy}\right] - \left(\vec{b}_i \times \vec{a}_i\right)J_{pi}\right\}\dot{\mathbf{X}}_\mathbf{p} \tag{3}$$

$$\ddot{\rho}_i = \frac{1}{\vec{a}_i.\vec{b}_i}\left\{\left[b_{ix} \quad b_{iy} \quad e_{ix}b_{iy} - e_{iy}b_{ix}\right]\ddot{\mathbf{X}}_\mathbf{p} - \vec{b}_i.\vec{e}_i\dot{\varphi}^2 + \dot{\beta}_i^2 l^2\right\} \tag{4}$$

$$\ddot{\beta}_i = \frac{1}{l^2}\left\{\left[-b_{iy} \quad b_{ix} \quad e_{ix}b_{ix} + e_{iy}b_{iy}\right]\ddot{\mathbf{X}}_\mathbf{p} - \left(\vec{b}_i \times \vec{e}_i\right)\dot{\varphi}^2 - \left(\vec{b}_i \times \vec{e}_i\right)\ddot{\rho}_i\right\} \tag{5}$$

where $\vec{e}_i$ is the positional vector from P to C_i, $\vec{a}_i$ is unit vector along axis of i^{th} slider and $\vec{b}_i$ is the positional vector from B_i to C_i.

Dynamics of the 3-PRR Parallel Robot. Robot kinematic relationships are expressed according to matrix $\mathbf{X} = \begin{bmatrix} \boldsymbol{\rho} & \boldsymbol{\beta} & \mathbf{X_p} \end{bmatrix}^T \in R^9$. This matrix has nine components. However, since the robot has three degrees of freedom, there are six constraint equations among the proposed general coordinates. Constraint equations regarding geometry of robot are expressed as,

$$\begin{cases} \Gamma_i : \quad x_{A_i} + \rho_i \cos\alpha_i + l\cos\beta_i = x'_{C_j}\cos\varphi - y'_{C_i}\sin\varphi + x_p & i = 1,2,3 \\ \Gamma_{j+3} : \; y_{A_j} + \rho_j \sin\alpha_j + l\sin\beta_j = x'_{C_j}\sin\varphi + y'_{C_j}\cos\varphi + y_p & j = 1,2,3 \end{cases} \tag{6}$$

Where x'_{C_i} and y'_{C_i} represent x and y coordinates of points C_i, respectively, and are measured from the mass center of the platform, P, when φ is zero. Next, to obtain the robot dynamics equations, Lagrange equations of first kind [8] are used as,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial (T - V)}{\partial q_i} = Q_i + \sum_{k=1}^{m} \lambda_k \frac{\partial \Gamma_k}{\partial q_i} \tag{7}$$

Where T is kinetic energy, V is potential energy, q_i are generalized coordinates, m is the number of constraint equations, λ_k is constraint equations coefficients and Γ_k is k^{th} constraint equation.

Upon obtaining the kinetics and potential energy of the system and placing them in Lagrange equation, Eq. 7, the final robot dynamic equation can be written as [8],

$$\begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{0} \\ \mathbf{M}_{21} & \mathbf{M}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{33} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{\rho}} \\ \ddot{\boldsymbol{\beta}} \\ \ddot{\mathbf{X}}_\mathbf{p} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ \mathbf{J}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_6 \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{\boldsymbol{\rho}} \\ \mathbf{0} \\ \mathbf{F}_{\mathbf{ext}} \end{bmatrix} \tag{8}$$

Where $\mathbf{F}_{\boldsymbol{\rho}}$ is force on sliders and $\mathbf{F}_{\mathbf{ext}}$ is external force on the platform. Additionally, matrix $\mathbf{V}$ includes non-linear terms due to centrifugal and Coriolis accelerations. The dynamic equations obtained in this section are differential algebraic equations (DAE) as they are a mixed set of differential and algebraic constraint equations.

Solution of Dynamic Equations

As stated before, the 3-PRR robot has 3 degrees of freedom. Dynamic equations of the robot are derived using the nine generalized coordinates which are related through 6 algebraic constraint equations. To solve system of differential equations, first matrix of Lagrange multipliers, $\boldsymbol{\lambda}$, is eliminated. To do this, Eq. 8 is separated into two smaller equations using matrix partitioning as,

$$\begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{0} \\ \mathbf{M}_{21} & \mathbf{M}_{22} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{\rho}} \\ \ddot{\boldsymbol{\beta}} \\ \ddot{\mathbf{X}}_\mathbf{p} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_6 \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{\boldsymbol{\rho}} \\ \mathbf{0} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{M}_{33} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{\rho}} \\ \ddot{\boldsymbol{\beta}} \\ \ddot{\mathbf{X}}_\mathbf{p} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{J}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_6 \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{\mathbf{ext}} \end{bmatrix} \tag{10}$$

By evaluating the matrix of Lagrange multipliers, $\boldsymbol{\lambda}$, from Eq. 9 and substituting it in Eq. 10 we eliminated the Lagrange multipliers and obtain a single equation,

$$\left(\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{M}_{33} \end{bmatrix} - \begin{bmatrix} \mathbf{J}_3 \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{0} \\ \mathbf{M}_{21} & \mathbf{M}_{22} & \mathbf{0} \end{bmatrix} \right) \begin{bmatrix} \ddot{\boldsymbol{\rho}} \\ \ddot{\boldsymbol{\beta}} \\ \ddot{\mathbf{X}}_\mathbf{p} \end{bmatrix} - \begin{bmatrix} \mathbf{J}_3 \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \end{bmatrix}^{-1} \left(\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} - \begin{bmatrix} \mathbf{F}_{\boldsymbol{\rho}} \\ \mathbf{0} \end{bmatrix} \right) = \begin{bmatrix} \mathbf{F}_{\mathbf{ext}} \end{bmatrix} \tag{11}$$

Eq. 11 can be written in a more compact form as,

$$\begin{bmatrix} \hat{\mathbf{M}}_{11} & \hat{\mathbf{M}}_{12} & \hat{\mathbf{M}}_{13} \end{bmatrix}_{3\times9} \begin{bmatrix} \ddot{\boldsymbol{\rho}} \\ \ddot{\boldsymbol{\beta}} \\ \ddot{\mathbf{X}}_{\mathbf{p}} \end{bmatrix}_{9\times1} + \begin{bmatrix} \hat{\mathbf{G}} \end{bmatrix}_{3\times1} = \begin{bmatrix} \mathbf{F}_{\mathbf{ext}} \end{bmatrix}_{3\times1} \tag{12}$$

In Eq. 12, $\hat{\mathbf{M}}_{11}$ through $\hat{\mathbf{M}}_{13}$ are 3×3 square matrices and can be written in expanded form as,

$$\hat{\mathbf{M}}_{11}\ddot{\boldsymbol{\rho}} + \hat{\mathbf{M}}_{12}\ddot{\boldsymbol{\beta}} + \hat{\mathbf{M}}_{13}\ddot{\mathbf{X}}_{\mathbf{p}} + \hat{\mathbf{G}} = \mathbf{F}_{\mathbf{ext}} \tag{13}$$

The next goal is to express $\ddot{\rho}$ and $\ddot{\beta}$ in terms of $\ddot{\mathbf{X}}_{\mathbf{p}}$. This replacement will result in an equation written as a function of the independent coordinates $\mathbf{X}_{\mathbf{p}}$ which, describes motion of the moving platform. To do this, we note that the dependent variables (ρ, β) and their derivatives are related to $\mathbf{X}_{\mathbf{p}}$ through solution of inverse kinematics of the robot, Eq. 4 and Eq. 5. Therefore,

$$\ddot{\boldsymbol{\rho}} = \mathbf{J}_{\boldsymbol{\rho}}\ddot{\mathbf{X}}_{\mathbf{p}} + \mathbf{K}_{\boldsymbol{\rho}}, \quad \ddot{\boldsymbol{\beta}} = \mathbf{J}_{\boldsymbol{\beta}}\ddot{\mathbf{X}}_{\mathbf{p}} + \mathbf{K}_{\boldsymbol{\beta}} \tag{14}$$

By substituting Eq. 14 in Eq. 13, dynamic equation of robot as function of independent coordinates is obtained. Next, dynamic equations of system are transformed into state space representation. Considering $\mathbf{Y} = \begin{bmatrix} x_p & y_p & \varphi & \dot{x}_p & \dot{y}_p & \dot{\varphi} \end{bmatrix}^T$ as state space of the system, we have,

$$\begin{bmatrix} \mathbf{I}_3 & \mathbf{0} \\ \mathbf{0}_3 & \hat{\mathbf{M}}\ \mathbf{J} + \hat{\mathbf{M}}\ \mathbf{J} + \mathbf{M} \end{bmatrix} \begin{bmatrix} \dot{Y}_1 \\ \dot{Y}_2 \\ \vdots \\ \dot{Y}_6 \end{bmatrix} = \begin{bmatrix} Y_4 \\ Y_5 \\ Y_6 \\ -\hat{\mathbf{M}}_{11}\mathbf{K}_{\boldsymbol{\rho}} - \hat{\mathbf{M}}_{12}\mathbf{K}_{\boldsymbol{\beta}} - \hat{\mathbf{G}} + \mathbf{F}_{\mathbf{ext}} \end{bmatrix} \tag{15}$$

Control and simulation of robot

In this section, motion of the robot following a specified path is discussed. A proportional derivative, PD, controller is designed which uses dynamic model of the robot to simulate it's motion in each simulation loop. The PD control law is,

$$\mathbf{F}_{\boldsymbol{\rho}} = -\mathbf{k}_{\mathbf{p}}\left(\boldsymbol{\rho} - \boldsymbol{\rho}_{\mathbf{dis}}\right) - \mathbf{k}_{\mathbf{v}}\left(\dot{\boldsymbol{\rho}} - \dot{\boldsymbol{\rho}}_{\mathbf{dis}}\right) \tag{16}$$

Where, $\mathbf{k}_{\mathbf{p}}$ and $\mathbf{k}_{\mathbf{v}}$ are controller gains and ρ, $\rho_{\mathbf{des}}$ are actual and desired positions of the sliders, respectively. Next, a desired path for the end-effector, position and orientation of platform's center of mass, is specified. To do this, x_p, y_p coordinates and φ orientation are selected as,

$$x_p = \frac{x_f}{t_f}t - \frac{x_f}{2\pi}\sin\left(\frac{2\pi}{t_f}t\right), \; y_p = o \;, \; \varphi = 0 \tag{17}$$

Where $x_f = 2mm$, $t_f = 10ms$ are final position and duration of motion, respectively. Fig. 2 shows the block diagram of the controller.

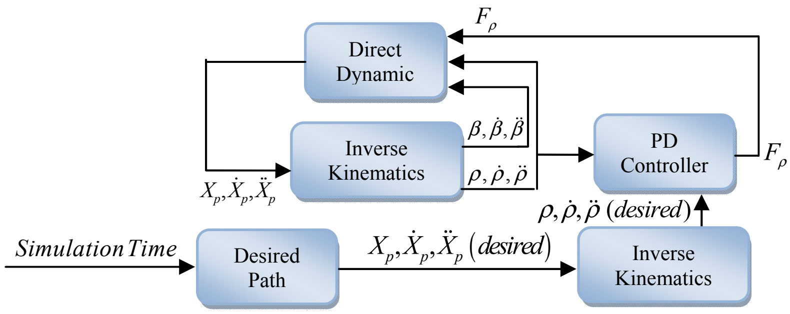

Fig 2. Block diagram of controller

The desired and actual, controlled, position of x_p during motion after implementing the designed controller is shown in Fig. 3. The controller gains are $k_p = 4000$, $k_v = 40$.

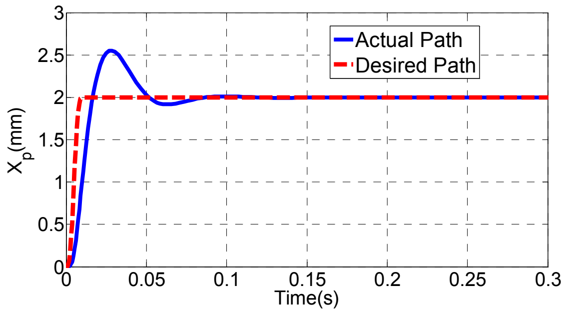

Fig 3. Actual and desired position

Summary

In this paper, a method is presented where dynamics equations of a 3-PRR parallel robot are solved. First kinematics and dynamics modeling of the 3-PRR parallel robot is briefly presented. Using Lagrange equations of first kind, dynamic equations of robot are derived leading into a mixed set of differential and algebraic equations. By separating and eliminating the constraint equations, dynamic equations of robot reduce to a set of differential equations as a function of 3 independent coordinates. Next, the dynamics equations are coded in Matlab software. A PD controller along with the dynamics model is both used in a trajectory following example. It is shown that the robot successfully follows the desired trajectory. The presented solution method significantly simplifies the solution of the dynamics equations.

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