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Using Inverse Laplace Transform for the Solution of a Flood Routing Problem

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Abstract. The inverse Laplace transform is of great importance in mathematical sciences when an analytical solution exists in Laplace domain. A new solution of the linearized St. Venant equations (LSVE) has been obtained for flood routing. The LSVE has been previously used by many researchers. In the formulation, the linearized form of the Manning formula is combined with the LSVE to get a Laplace transformable, simplified set of equations. There are different simplifications in the literature to get an analytical solution in time domain, not applicable to complicated form of equations. The results of discharge and depth predictions show that improved De Hoog algorithm provides a solution with very small error, when applied to the LSVE. Moreover, the model solution is compared against the numerical solution of the LSVE using the well-known Preissmann implicit finite difference scheme. The model outputs indicate a very good agreement with the numerical solution. It is notable that in the results reported by Litrico and Fromion [1], the LSVE was limited to maximum variation of 5% in discharge, however in the current paper the range of the variation is reached to 50% of the initial discharge.

Keywords: Inverse Laplace transform, Mathematical modeling, St. Venant equations, Flood routing.
PACS: 47.10.A-

INTRODUCTION

Floods can cause some damage in areas near rivers. In order to prevent flood damage, it is very important to estimate floods as well as the effects of stream channels on floods. For flood routing problems, one-dimensional St. Venant equations describe the flood propagation. In order to obtain an analytical solution for flood routing in a channel section, various approximations to the St. Venant equations have been proposed. One of the simplified models is the linearized St. Venant equations (LSVE) [1, 2]. The accuracy of these approximations to the full nonlinear St. Venant equations is discussed in detail in many studies [3, 4]. In the current paper, the Laplace domain analytical solution obtained by Litrico and Fromion [1] for general form of geometry is used to numerically transform the solution into the time domain. Also, De Hoog algorithm [5] is used for Laplace inversion as previously implemented to solve diffusion wave model [6] or to predict water quality in rivers [7]. The accuracy of the model is compared against the numerical solution of the LSVE through the well-known Preissmann implicit scheme for general cross-section. Unlike other methods such as numerical schemes, this procedure will result in a very short computation time and provides a great convergence, very accurate response for flood routing problem. Two synthetic examples are assessed to evaluate the effect of different hydraulic conditions. The results show that the predicted discharge and water level using the linearized boundary conditions are associated with a very small error of <1%.

MATHEMATICAL FORMULATION

The matrix form of the LSVE in Laplace domain is given by [1]:

$$\frac{\partial \kappa}{\partial t} + A\,\frac{\partial \kappa}{\partial x} + B\,\kappa = 0 \tag{1}$$

where κ is the transfer matrix includes fluctuations of the discharge $q(x,t)$ and the depth $y(x,t)$ around the reference values (Q_0, Y_0). In this new formulation, the linearized form of the Manning formula as downstream boundary condition in Laplace domain is obtained as follows:

$$\hat{q}\left(L,s\right) = k_v\,\hat{y}\left(L,s\right) \tag{2}$$

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where $k_v = \partial Q / \partial Y$.

De Hoog et al. [5] have introduced an improved method for Laplace inversion, which provides a great convergence, with a very accurate response. The introduced formula has the following form:

$$y(x,t) = \frac{\exp(\gamma_0 t)}{T}\left[\frac{\hat{y}(x,\gamma_0)}{2} + \sum_{m=1}^{\infty} Re\left\{\hat{y}\left(x,\gamma_0 + \frac{jm\pi}{T}\right)exp\left(\frac{jm\pi}{T}\right)\right\}\right] \tag{3}$$

where *2T* is the period of the Fourier series approximating the inverse on the interval [*0,2T*], *Re* denotes the real part of $\hat{y}(x,s)$ and $j = \sqrt{-1}$. Also, $s = \gamma_0 + j\omega$ in which γ_0 is such that the contour of inverse Laplace integration is to the right of any singularities of $\hat{y}(x,s)$ [5].

CHANNEL DESCRIPTION

Two cases have been considered in a rectangular cross-section. The hypothetical tests are listed in Table 1, where S_b is the bed slope, *L* is the channel length, *B* is the bed width and *n* is the Manning coefficient. The considered amount of bed slopes is common for natural rivers and also, generally, the wider the channel the gentler the bed slope. So, wide-steep or narrow-flat cases are omitted [2, 8]. Case 1 has a greater width and smaller slope than case 2.

TABLE 1. Specification of the tests.

	L (*m*)	**B (*m*)**	**n**	$\mathbf{S_b}$	$\mathbf{Q_0}(m^3/s)$	$\mathbf{Y_0}(m)$
Case 1	3500	25	0.03	0.0001	20	1.78
Case 2	6500	10	0.03	0.0005	20	2.08

RESULTS AND DISCUSSION

The numerical solution of the LSVE through the Preissmann scheme and inverse Laplace transform algorithm of the De Hoog have been shown in Figs. 1 and 2 for the two cases. A triangular input hydrograph is considered for flood routing plotted in Figs. 1(a) and 2(a). It is clear that the solutions of the De Hoog algorithm and Preissmann scheme are almost in complete agreement for both the downstream predicted water level and discharge.

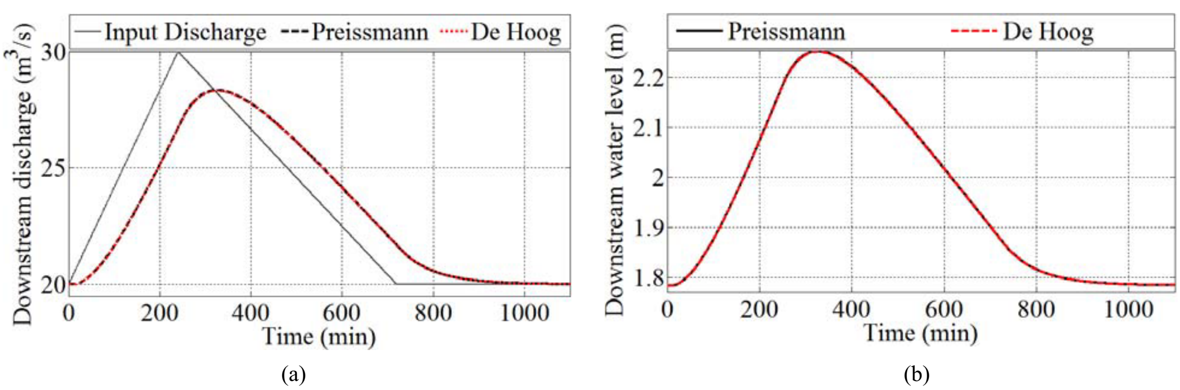
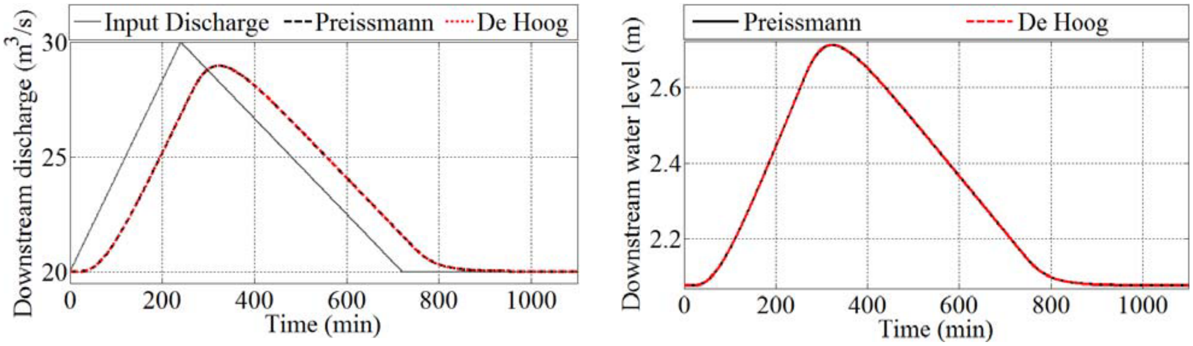


FIGURE 1. The LSVE solution through the Preissmann scheme and the De Hoog algorithm for (a) downstream discharge as well as input hydrograph and (b) downstream water level, case 1.



(a) (b)

FIGURE 2. The LSVE solution through the Preissmann scheme and the De Hoog algorithm for (a) downstream discharge as well as input hydrograph and (b) downstream water level, case 2.

Table 2 lists the peak flow points for the downstream predicted water level and discharge.

TABLE 2. Model outputs: downstream discharge and water level and maximum percentage errors.

	$\mathbf{Y_{max}}\ (m)$			$\mathbf{Q_{max}}\ (m^3/s)$		
	Preissmann	**De Hoog**	**$\mathbf{Error_{max}}$(%)**	**Preissmann**	**De Hoog**	**$\mathbf{Error_{max}}$(%)**
Case 1	2.253	2.252	0.06	28.332	28.313	0.08
Case 2	2.711	2.711	0.01	28.966	28.97	0.02

The maximum percentage errors between the numerical solution through the Preissmann scheme and the De Hoog algorithm are small for steep channel (case 2). Unlike case 1, case 2 is a long sloping channel which its slope is 5 times case 1 slope. So, this large slope would cause the channel behave more linearly because the downstream boundary effect is felt less, even though the more channel length would increase the amount of error. This matter causes the better performance of the solution for case 2.

The continuity equation is checked for both cases; the errors between the total inflow and predicted outflow volume are less than 0.04% and very close to zero, for the cases 1 and 2, respectively. The time to peak is equal in both the De Hoog algorithm and the Preissmann scheme for both cases. This shows that the model has a very good capability in capturing the peak flow point value and peak time. In the De Hoog algorithm the times to peak are 328 min for case 1 and 324 min for case 2.

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