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Hamid Moeenfard, Ali Darvishian & Mohammad Taghi Ahmadian

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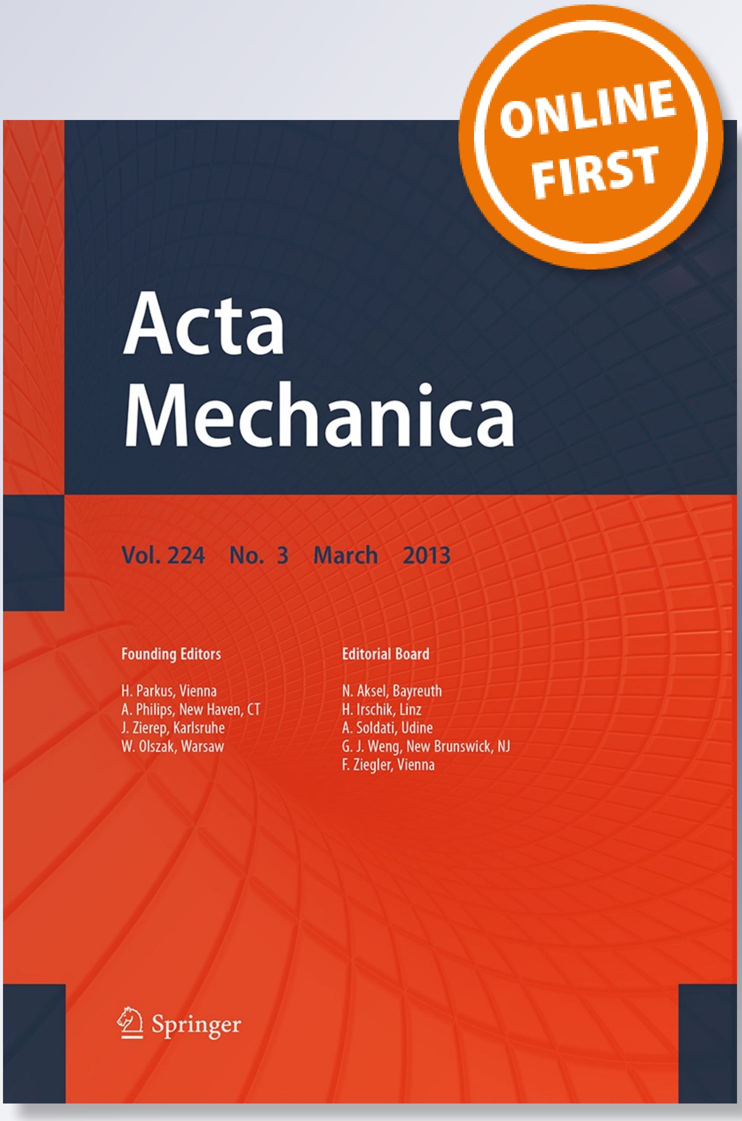

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Hamid Moeenfard · Ali Darvishian ·
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Abstract In the current paper, a coupled two degree of freedom model which considers both bending and torsion of the supporting torsion beams is presented for electrostatically actuated torsional nano/micro-actuators under the effect of van der Waals (vdW) force. Newton's second law is utilized for finding the normalized equations governing the static behavior of the actuator. The implict function theorem is then utilized for finding the equations governing the pull-in state of the actuator. The related results show that torsion model considerably overestimates the pull-in parameters of the nano/micro-actuator. The concept of the instability mode is introduced, and it is shown that when the ratio of the bending stiffness to the torsion stiffness of the supporting torsion beams is relatively low, the dominant instability mode of the actuator would be the bending mode and otherwise the dominant instability mode would be the torsion mode. It is also observed that the presence of the vdW force can significantly reduce the pull-in angle and pull-in deflection of the nano/micro-actuator. The presented results also show that the vdW force can lead to considerable reduction in the pull-in voltage of the actuator. The equilibrium behavior of the actuator is studied, and it is observed that the vdW force and also bending of the supporting torsion beams greatly reduce the maximum allowable voltage which can be applied to the actuator. Results of this paper can be used for successful design of electrostatically actuated torsional nano/micro-actuators where the size of the actuator is sufficiently small, and as a result, the vdW force plays a major role in the system.

1 Introduction

Nano/micro-electromechanical systems (N/MEMS) are being developed for a wide spectrum of applications in various aspects of life. These devices make the systems faster, more reliable, cheaper and capable of incorporating more complex functions [1,2]. Successful N/MEMS devices rely not only on well-developed fabrication technologies, but also on the knowledge of device behavior [3]. The fact that MEMS devices play key roles in optical systems caused the development of a new class of MEMS called micro-opto-electromechanical systems (MOEMS). MOEMS include a wide variety of devices including optical cross-connects [4], digital micromirror devices (DMD) [5], micro scanning mirrors [6], optical switches [7], etc.

H. Moeenfard (✉)
School of Mechanical Engineering,
Ferdowsi University of Mashhad, Mashhad, Iran
E-mail: moeenfar@umich.edu
Tel.: +98-21-66165503
Fax: +98-21-66000021

A. Darvishian · M. T. Ahmadian
School of Mechanical Engineering, Center of Excellence in Design, Robotics and Automation,
Sharif University of Technology, Azadi Ave, Tehran, Iran
E-mail: alidarvishian@gmail.com

In torsional nano/micro-actuators, the mirror is made to rotate by electrostatic actuation, intermolecular surface forces or capillary force. The rotation of the actuator is opposed by a mechanical restoring torque generated within the supporting torsion beams. So the problem of torsion of beams is a preliminary step in modeling torsional nano/micro-actuators. Timoshenko and Goodier [8] found the torsional rigidity of prismatic bars using a linear elasticity model. Faulkner and Haddow [9] studied the nearly isochoric finite torsion of a compressible isotropic elastic circular cylinder. Torsion of an infinite composite elastic cylindrical shell was investigated by Hemed and Dhaliwal [10]. Chen et al. [11] modeled Saint-Venant torsion problem of compound sections with imperfect interfaces. But up to the authors knowledge, the torsional rigidity of an elastic prismatic bar undergoing finite rotations has not been well presented in the literature yet. It has to be mentioned that here in this paper, it is assumed that the tilting angle of the actuator is very small and a linear elasticity theory can be used for prediction of the elastic torque and elastic force.

Static and dynamic characteristics of torsional nano/micro-actuators have been investigated by many researchers. For example, Degani et al. [12] presented a novel displacement iteration pull-in extraction (DIPIE) scheme for the problem of electrostatic torsion micro-actuators. They [12] showed that their presented method converges 100 times faster than the voltage iteration scheme. Zhang et al. [13] described the static characteristics of an electrostatically actuated torsional micromirror based on parallel plate capacitor model. They extensively studied the snap down phenomenon in micromirrors. Sun et al. [14] analyzed the dynamic behavior of a torsional micromirror. Huang et al. [15] presented a coupled torsion-bending model for electrostatic torsional actuators. Moeenfard and Ahmadian [16] used perturbational-based methods to analytically model the coupled torsion-bending problem in torsional micro-actuators. The pull-in parameters of electrostatic actuators have been investigated by Degani and Nemirovski [17,18]. Darvishian et al. [19] investigated the torsion and bending effects on static stability of torsional micromirrors under capillary force. The dynamic characteristics for a torsional micromirror have been investigated by Zhao and Chen [20]. Khatami and Rezazadeh [21] studied dynamic response of the micromirrors to the effect of mechanical shock.

Intermolecular surface forces which mainly include Casimir and vdW forces play an important role in the stability of nano/micro-actuators [22]. VdW force is a short-range force in nature, but it can lead to long-range effects more than 0.1 μm [23]. This force is the interaction force between neutral atoms, and it differs from covalent and ionic bondings in that it is caused by correlations in the fluctuating polarizations of nearby particle [24]. Casimir force can be simply understood as the long-range analog of the vdW force, resulting from the propagation of retarded electromagnetic waves [25].

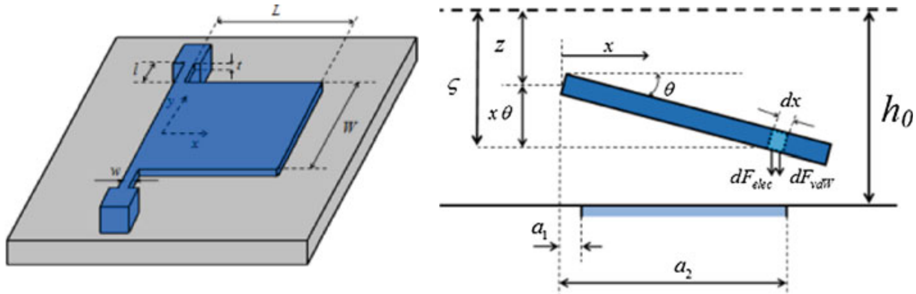
Tahami et al. [26] studied influence of vdW force on the pull-in phenomena of a capacitive nano-beam switch. Effects of vdW force and thermal stresses on pull-in instability of electrostatically actuated microplates have been studied by Batra et al. [27]. Mojahedi et al. [28] modeled nano-switches under the effect of vdW force and electrostatic actuation. Ramezani et al. [29] analyzed effect of vdW force on the pull-in parameters of cantilever type nanoscale electrostatic actuators. Influence of Casimir force on static behavior of micromirrors under the effect of capillary force has been studied by Moeenfard et al. [22]. Guo and Zhao [30] studied dynamic stability of electrostatic torsional actuators with considering vdW effect. They [31] also investigated the effect of vdW force on the pull-in and static behavior of electrostatic torsional actuators and showed that the effect of vdW force in small sized gaps cannot be neglected. Moeenfard and Ahmdian [32] used a pure torsion model to analytically study the static behavior of electrostatically actuated torsional micromirrors considering vdW force, but in their model, the bending effect of the supporting torsion beams was not considered. On the other hand, many researchers [15,16,19–21] showed that torsion models in torsional micromirrors may not accurately predict the stability limits of micromirrors. In the current paper, it will be shown that the same concern exist for the torsional nano/micro-actuators under the effect of electrostatic and vdW force.

2 Problem formulation

2.1 Coupled bending-torsion model

Torsional nano/micro-actuator shown in Fig. 1 is considered. The differential vdW force applied to the differential surface element of the actuator with width W and infinitesimal length dx is [23]:

$$\mathrm{d}F_{\mathrm{vdW}} = \frac{A.W}{6\pi\,(h_0 - z - x\theta)^3}\mathrm{d}x \tag{1}$$



(a) *isometric view* **(b)** *side view*

Fig. 1 Schematic (**a**) isometric and (**b**) side view of a nano/micro-actuator under the combined effect of electrostatic actuation and vdW force

where h_0 is the initial distance between the nano/micro-actuator and the underneath substrate, z is the vertical deflection of the torsion beams, θ is the tilting angle of the nano/micro-actuator and A is the Hamaker constant.

Equation (1) can be expressed in terms of normalized tilting angle Θ and normalized deflection of the mirror Δ, as follows:

$$\mathrm{d}F_{\mathrm{vdW}} = \frac{AW}{6\pi h_0^3\left(1-\Delta-\frac{x}{L}\Theta\right)^3}\mathrm{d}x. \tag{2}$$

In this equation, L is length of the actuator, and Δ and Θ are defined as given in Eqs. (3) and (4), respectively:

$$\Delta = \frac{z}{h_0}, \tag{3}$$

$$\Theta = \frac{\theta}{\theta_{\max}}, \tag{4}$$

where $\theta_{\max}$ is the maximum geometrically possible rotation angle of the nano/micro-actuator and can be calculated as

$$\theta_{\max} = \tan^{-1}\left(\frac{h_0}{L}\right) \approx \frac{h_0}{L}. \tag{5}$$

The vdW force F_{vdW} and vdW torque M_{vdW} applied on the nano/micro-actuator are easily calculated as

$$F_{\mathrm{vdW}} = \int_0^L \mathrm{d}F_{\mathrm{vdW}}, \tag{6}$$

$$M_{\mathrm{vdW}} = \int_0^L x\,\mathrm{d}F_{\mathrm{vdW}}. \tag{7}$$

By substituting Eqs. (2) in (6) and (7), one can easily conclude that

$$F_{\mathrm{vdW}} = \frac{A.W.L}{12\pi h_0^3\Theta}\left(\frac{1}{(\Theta-(1-\Delta))^2}-\frac{1}{(1-\Delta)^2}\right), \tag{8}$$

$$M_{\mathrm{vdW}} = \frac{AWL^2}{6\pi h_0^3\Theta^2}\left(\frac{1}{2\,(1-\Delta)}+\frac{2\Theta-(1-\Delta)}{2\,(\Theta-(1-\Delta))^2}\right). \tag{9}$$

Similarly, the electrostatic force F_{Elec} and electrostatic torque M_{Elec} applied to the actuator plate are obtained as Eqs. (10) and (11), respectively [15]:

$$F_{\mathrm{Elec}} = \frac{\varepsilon_0 V^2.W.L}{2h_0^2\Theta}\left(\frac{1}{1-\Delta-\beta\Theta}-\frac{1}{1-\Delta-\alpha\Theta}\right), \tag{10}$$

$$M_{\mathrm{Elec}} = \frac{\varepsilon_0 V^2 WL^2}{2h_0^2\Theta^2}\left(\frac{1-\Delta}{1-\Delta-\beta\Theta}-\frac{1-\Delta}{1-\Delta-\alpha\Theta}+\ln\left(\frac{1-\Delta-\beta\Theta}{1-\Delta-\alpha\Theta}\right)\right), \tag{11}$$

where V is the applied voltage between the actuator and the underneath electrode, ε_0 is the permittivity of free space, α and β are some normalized geometrical parameters defining the position of the electrodes. Mathematically, α and β are defined as given in Eqs. (12) and (13), respectively:

$$\alpha = \frac{a_1}{L}, \tag{12}$$

$$\beta = \frac{a_2}{L}. \tag{13}$$

In these equations, a_1 and a_2 are some position parameters defining the starting and ending points of the electrodes as illustrated in Fig. 1.

Assuming that the end deflections of the supporting torsion beams, i.e., z and θ are very small, one may assume that the elastic restoring force F_{els} and the elastic restoring torque M_{els} are linearly proportional with z and θ as given in Eqs. (14) and (15), respectively. It has to be noted that in the case z and/or θ are finite, a nonlinear elasticity model which considers the nonlinear von Karman strains should be used for accurate prediction of F_{els} and M_{els}:

$$F_{\mathrm{els}} = -K_0 z = -K_0 h_0 \Delta, \tag{14}$$

$$M_{\mathrm{els}} = -S_0 \theta = -S_0 \frac{h_0}{L} \Theta. \tag{15}$$

In Eqs. (14) and (15), S_0 and K_0 are the overall torsional and vertical stiffnesses of the torsion beams and can be calculated using Eqs. (16) and (17), respectively [15].

$$S_0 = \frac{2 G I_{\mathrm{p}}}{l}, \tag{16}$$

$$K_0 = \frac{24 E I_{\mathrm{b}}}{l^3}, \tag{17}$$

where E and G are the Young's modulus and shear modulus of elasticity of the beams material, respectively, l is the length of each torsion beam, I_{p} is the polar moment of inertia of the beams cross section, and I_{b} is the second moment of inertia of the beams cross section around their neutral axis. For beams with rectangular cross sections shown in Fig. 1, I_{b} and I_{p} would be as [15]:

$$I_{\mathrm{b}} = \frac{1}{12} w t^3, \tag{18}$$

$$I_{\mathrm{p}} = \frac{1}{3} t w^3 - \frac{64}{\pi^5} w^4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \tan\mathrm{h} \frac{(2n-1)\pi t}{2w}, \tag{19}$$

where w and t are the width and length of the beam's cross section, respectively. At equilibrium state, the resultants of the forces and torques applied to the actuator are zero. So, the following equations have to be satisfied:

$$\Xi_{T1} = \sum F = F_{\mathrm{els}} + F_{\mathrm{vdW}} + F_{\mathrm{Elec}} = 0, \tag{20}$$

$$\Xi_{T2} = \sum M = M_{\mathrm{els}} + M_{\mathrm{vdW}} + M_{\mathrm{Elec}} = 0, \tag{21}$$

where $\Xi_{Ti}, 1 \le i \le 2$ is the i'th equilibrium equation. Using some algebraic manipulations, equilibrium equations can be further simplified as Eqs. (22) and (23):

$$\Gamma\Delta - \frac{\xi}{\Theta}\left(\frac{1}{1-\Delta-\beta\Theta} - \frac{1}{1-\Delta-\alpha\Theta}\right) - \frac{\mu}{2\Theta}\left(\frac{1}{(\Theta-(1-\Delta))^2} - \frac{1}{(1-\Delta)^2}\right) = 0, \tag{22}$$

$$\begin{aligned}\Theta &- \frac{\xi}{\Theta^2}\left(\frac{1-\Delta}{1-\Delta-\beta\Theta} - \frac{1-\Delta}{1-\Delta-\alpha\Theta} + \ln\left(\frac{1-\Delta-\beta\Theta}{1-\Delta-\alpha\Theta}\right)\right) \\ &- \frac{\mu}{\Theta^2}\left(\frac{1}{2(1-\Delta)} + \frac{2\Theta-(1-\Delta)}{2(\Theta-(1-\Delta))^2}\right) = 0,\end{aligned} \tag{23}$$

where ξ, μ and Γ are

$$\xi = \frac{\varepsilon V^2 W.L}{2h_0^3 K_\mathrm{t}}, \tag{24}$$

$$\mu = \frac{A.W.L}{6\pi h_0^4 K_\mathrm{t}}, \tag{25}$$

$$\Gamma = \frac{K_\mathrm{b}}{K_\mathrm{t}}. \tag{26}$$

In these equations, K_b and K_t are scaled bending and torsion stiffnesses, respectively, and are defined as follows:

$$K_\mathrm{t} = \frac{S_0}{L^2}, \tag{27}$$

$$K_\mathrm{b} = K_0. \tag{28}$$

By increasing the value of ξ in Eqs. (22) and (23), the values of Δ and Θ are increased. At the pull-in state, ξ has its maximum value. Using the implicit function theorem [18] and Eqs. (20) and (21), it is easily derived that the local maxima for ξ (i.e., pull-in state) is reached when

$$\begin{vmatrix} \frac{\partial \Xi_{T1}}{\partial \Delta} & \frac{\partial \Xi_{T1}}{\partial \Theta} \\ \frac{\partial \Xi_{T2}}{\partial \Delta} & \frac{\partial \Xi_{T2}}{\partial \Theta} \end{vmatrix} = 0. \tag{29}$$

2.2 Torsion model

In the pure torsion model, the equilibrium equation would be obtained by setting the value of Δ equal to zero in Eq. (23):

$$\Theta - \frac{\xi}{\Theta^2}\left(\frac{1}{1-\beta\Theta} - \frac{1}{1-\alpha\Theta} + \ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)\right) - \frac{\mu}{\Theta^2}\left(\frac{1}{2} + \frac{2\Theta-1}{2(\Theta-1)^2}\right) = 0. \tag{30}$$

Additionally, the pull-in condition is simplified as

$$\frac{\partial \xi}{\partial \Theta} = 0. \tag{31}$$

3 Results and discussion

In order to investigate the effect of the electrode's geometrical parameters, i.e., α and β on the pull-in response of the nano/micro-actuator, in Fig. 2, the values of Θ_P and Δ_P (which are the normalized pull-in angle and pull-in deflection of the supporting torsion beams respectively) have been plotted versus β at different values of α for both torsion and coupled models.

Figure 2 shows that the pure torsion model predicts a reduction in Θ_P by increasing the value of β, while the coupled model predicts that with increasing the value of β, Θ_P would be increased and then decreased. Additionally, this figure shows that with increasing the value of α, the torsion model predicts the pull-in angle to be decreased, while the coupled model predicts it to be increased. Figure 2 also shows that by increasing β and α, pull-in deflection is reduced.

In Fig. 3, Θ_P has been plotted versus Γ at various values of μ for both torsion and coupled models. It is observed that regardless of the value of μ, neglecting the bending in modeling a nano/micro-actuator under combined electrostatic actuation and vdW force can lead to considerable overestimation in predicting the stable limits of the nano/micro-actuator. Actually, Γ is a parameter which determines the instability mode of the actuator. When Γ is small, the dominant instability mode of the actuator is the bending mode, and when it is large, the dominant instability mode of the actuator would be the torsion mode.

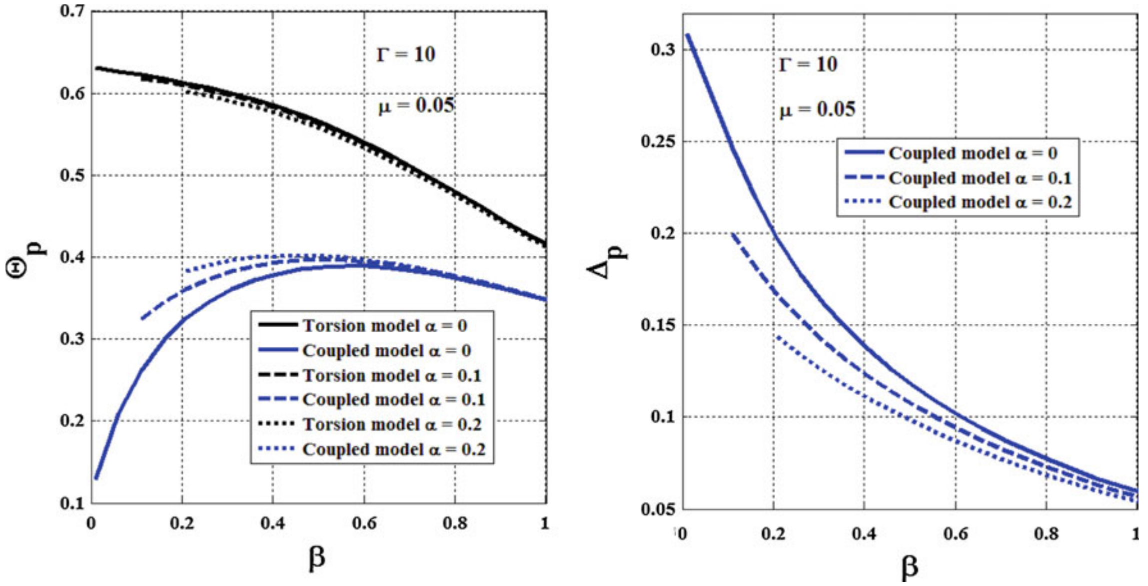


Fig. 2 Θ_P and Δ_P versus β at $\mu = 0.05$ for both torsion and coupled models ($\Gamma = 10$) and various values of α

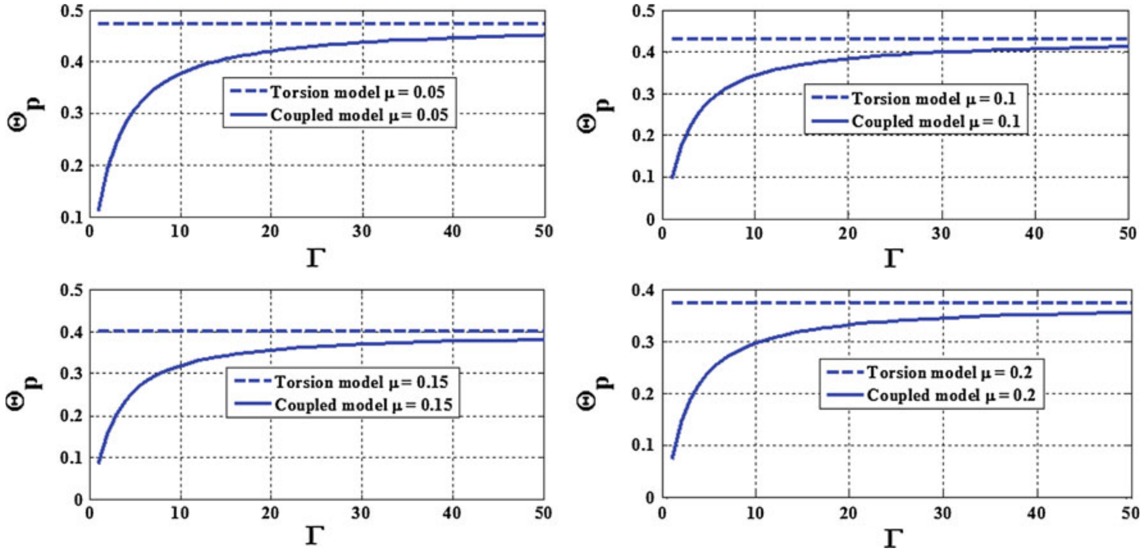


Fig. 3 Normalized pull-in angle of the actuator versus Γ for both torsion and coupled models at $\alpha = 0.2$, $\beta = 0.8$ and different values of μ

In order to investigate the effect of the vdW force on the pull-in response of the nano/micro-actuator, in Fig. 4, the values of Θ_P and Δ_P have been plotted versus Γ at different values of μ. It is observed that by increasing the value of μ, both pull-in angle and pull-in displacement are reduced, and as a result, neglecting the vdW force in modeling nano/micro-actuator can induce significant error in designing a stable actuator. Furthermore, it is observed that by reducing the value of Γ, Θ_P is decreased and Δ_P is increased, which is a sign of changing the instability mode of the actuator from the torsion mode to bending.

In Fig. 5, the values of ξ at the pull-in point, ξ_P is plotted against β for different values of α. It is observed that both coupled and torsion models predict that with increasing the value of β and/or decreasing the value of α, the value of ξ_P would be decreased. But, this figure shows that at a given α and β, the coupled model predicts the value of ξ_P lower than the torsion model prediction which indicates that neglecting the bending effect may lead to several hundred percent of error in predicting ξ_P depending on the value of β.

In Fig. 6, the values of ξ_P have been plotted versus Γ at different values of μ. This figure implicitly shows that regardless the value of μ at low values of Γ, the predicted value of the torsion model for ξ_P is significantly

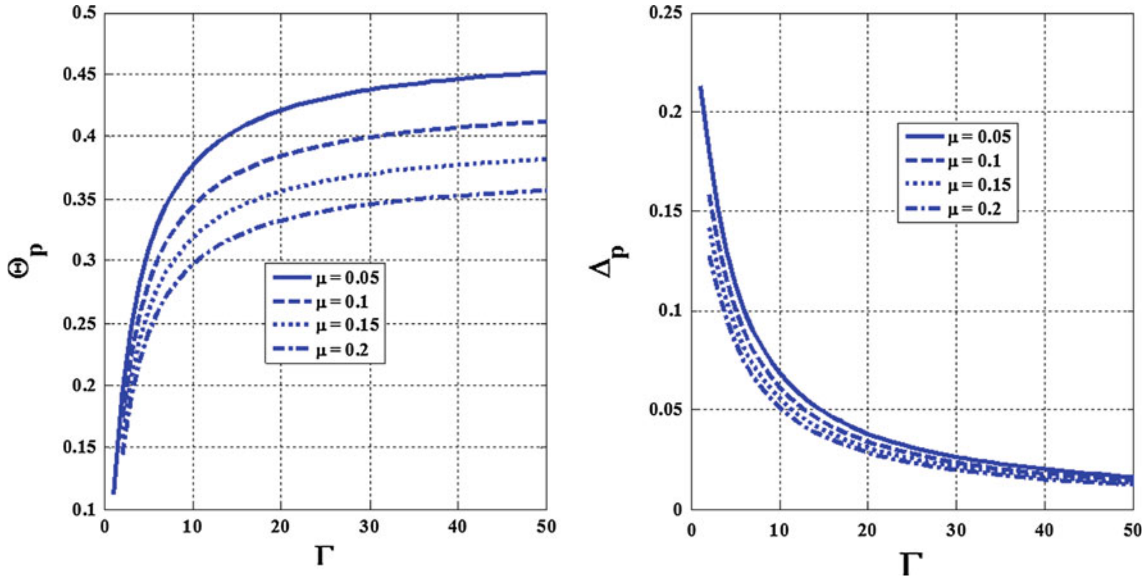


Fig. 4 Normalized pull-in parameters versus Γ at $\alpha = 0.2$, $\beta = 0.8$ and different values of μ

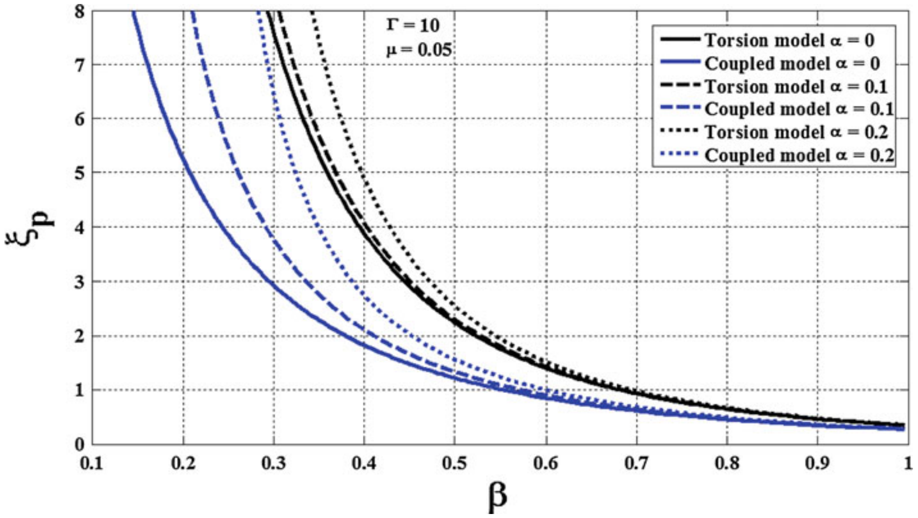


Fig. 5 ξ_P versus β at $\mu = 0.05$ for both torsion and coupled models ($\Gamma = 10$) and various values of α

higher than that of the coupled model, and the torsion model overestimates the instability limit of the actuator, and its related results cannot be trusted for a safe and stable design.

In Fig. 7, the coupled model has been used to investigate the effect of the vdW force on ξ_P. It is observed that with increasing μ, pull-in occurs at lower values of ξ. In fact, this figure shows that the vdW force can significantly reduce the maximum allowable value for ξ, and as a result, the stability limit of the nano/micro-actuator is reduced by the presence of the vdW force. In fact, this figure shows that even without the presence of any electrostatic actuation, the vdW force may lead to the pull-in of the structure.

In order to investigate the nano/micro-actuator's behavior under combined electrostatic actuation and vdW loading, Θ and Δ have been plotted versus ξ in Fig. 8. This figure shows that $\xi - \Theta$ and $\xi - \Delta$ characteristics of the nano/micro-actuator considering bending of the torsion beams considerably differ from that of the torsion model, especially at larger values of ξ.

To study the effect of vdW force on the $\xi - \Theta$ and $\xi - \Delta$ characteristics of the nano/micro-actuator, in Fig. 9, the values of Θ and Δ have been plotted against ξ at various values of μ. As it was expected, it is observed that with increasing the values of μ, Θ and Δ are increased.

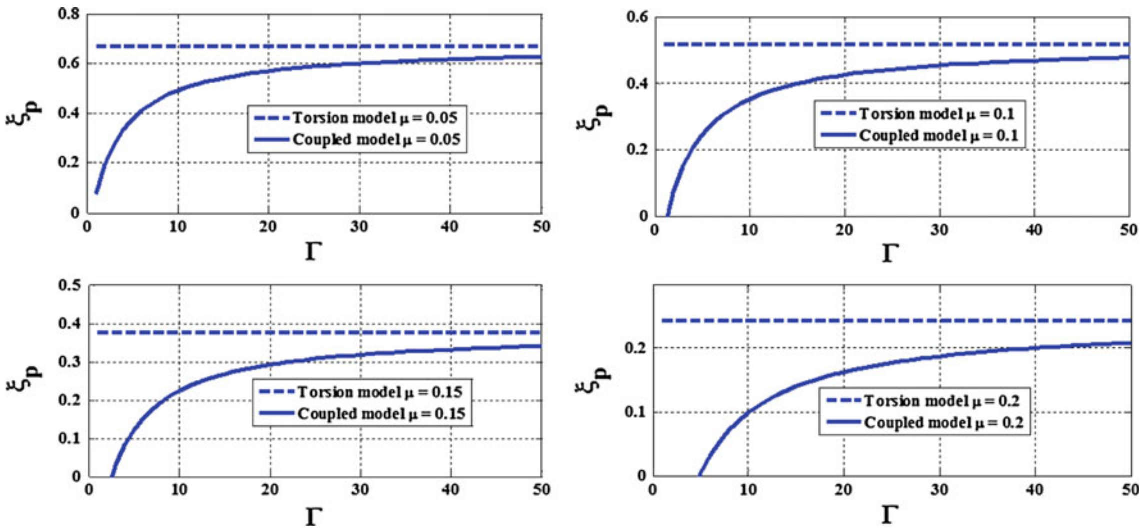


Fig. 6 Values of ξ_P versus Γ at $\alpha = 0.2$, $\beta = 0.8$ and different values of μ for both torsion and coupled models

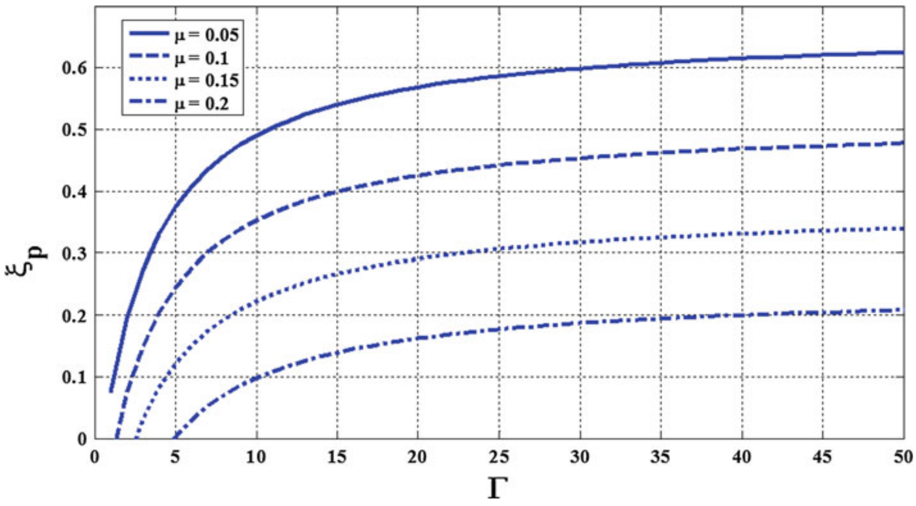


Fig. 7 Values of ξ_P versus Γ at $\alpha = 0.2$, $\beta = 0.8$ and different values of μ

4 Conclusion

A coupled bending-torsion model for electrostatically actuated nano/micro-actuators under the effect of vdW force was presented in this paper. The equilibrium equations governing the system behavior were obtained using Newton's second law. Furthermore, the equations governing the pull-in state of the nano/micro-actuator were found using the implicit function theorem. It was observed that the pure torsion model can lead to considerable overestimation of the pull-in parameters of the actuator. It was also observed that the presence of vdW force can significantly reduce the pull-in angle, pull-in deflection and pull-in voltage of the actuator depending on the instability mode of the system. The equilibrium behavior of the nano/micro-actuator was studied, and it was shown that vdW force as well as the bending ability of the supporting torsion beams can

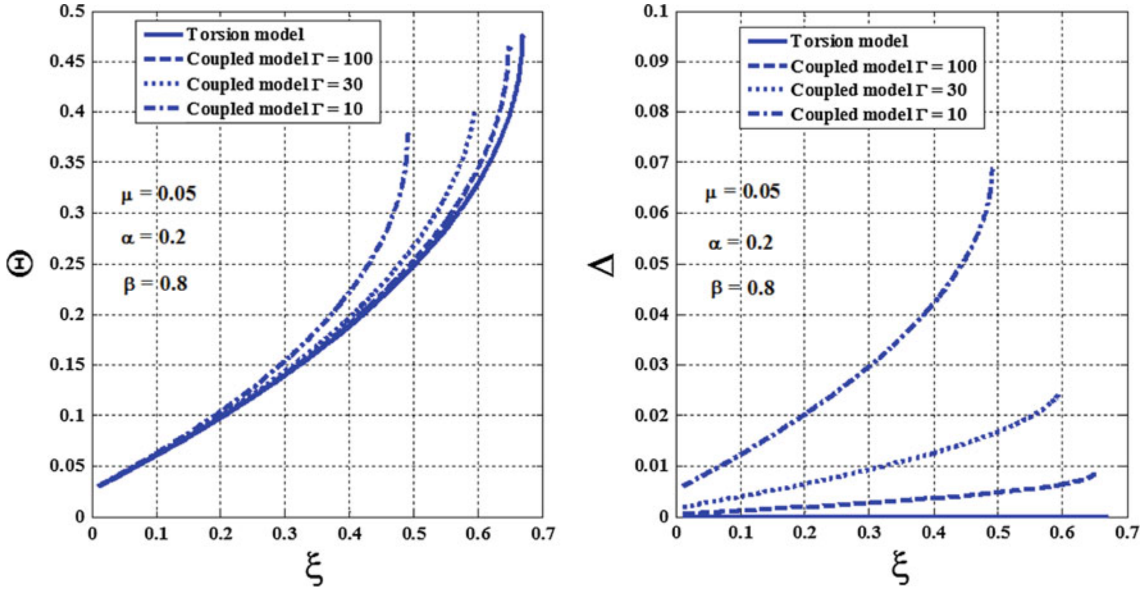


Fig. 8 Θ and Δ versus ξ at $\alpha = 0.2$, $\beta = 0.8$ and for $\mu = 0.05$ for both torsion and coupled model

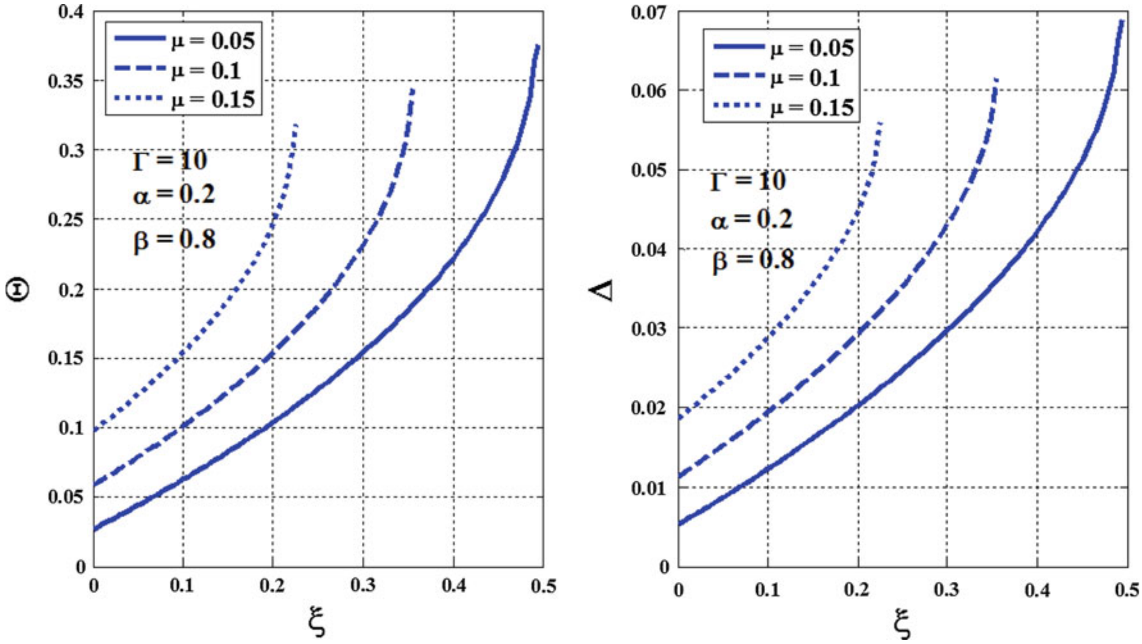


Fig. 9 Θ and Δ versus ξ for $\Gamma = 10$ at $\alpha = 0.2$ and $\beta = 0.8$ and different values of μ

significantly reduce the stability limits of the actuator, and their effects have to be carefully considered in design and modeling of nano/micro-actuators.

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