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ANALYTICAL MODELING OF THE EFFECTS OF ELECTROSTATIC ACTUATION AND CASIMIR FORCE ON THE PULL-IN INSTABILITY AND STATIC BEHAVIOR OF TORSIONAL NANO/MICRO ACTUATORS

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This paper studies the effect of Casimir force on the pull-in instability of electrostatically actuated torsional nano/micro actuators. Dependence of the actuator's pull-in angle and pull-in voltage on several design parameters are investigated and it is found that Casimir force can considerably reduce the stability limits of the torsional actuators. Nonlinear equilibrium equation is solved numerically and analytically using straight forward perturbation expansion method. It is observed that a fourth-order perturbation approximation can precisely model the behavior of a torsional actuator. The results of this paper can be used for safe and stable design of torsional nano/micro actuators.

Keywords: Torsional nano/micro actuators; electrostatic actuation; Casimir force; pull-in instability; straight forward perturbation expansion.

1. Introduction

Electronic devices in micro and nano scales have experienced lots of progress recently. Their low manufacturing cost, durability, light weight, small size, batch production, low energy consumption and compatibility with integrated circuits, makes them attractive.[1,2] Successful nano/micro electro-mechanical system (N/MEMS) devices rely on well developed fabrication technologies as well as the knowledge of device behavior.[3]

A review on modeling electrostatic MEMS can be found in Ref. 4. The important roles of N/MEMS in optical systems, initiated the development of a new class of systems called micro-opto-electro-mechanical systems (MOEMS). These systems include a wide variety of devices such as micro scanning mirrors,[5] digital micromirror devices (DMD),[6] optical cross connects,[7,8] etc. Torsional nano/micro actuators play key roles in MOEMS. Many researches have been done on these devices.[9] For example, Degani *et al.*[10] presented a novel displacement iteration pull-in extraction (DIPIE) scheme for the problem of electrostatic torsion micro actuators. Zhang *et al.*[11] explained the characteristics of an electrostatically actuated micromirror based on parallel plate capacitor model. They investigated snap down phenomenon in these devices. Switching response of the micro actuators have been studied by Bhaskar *et al.*[12] Khatami and Rezazadeh[13] investigated dynamic response of a torsional micromirror to electrostatic force and mechanical shock.

Intermolecular surface forces, which mainly include Casimir and van der Waals (vdW) forces, play key roles in MOEMS design.[14] vdW force is the interaction force between neutral atoms and its difference from covalent and ionic bondings is that it is caused by correlations in the fluctuating polarizations of nearby particles.[15] Casimir force is understood as the long range analog version of the vdW force, resulting from the propagation of retarded electromagnetic waves and its effect has to be considered in distance ranges from a few nano-meters up to a few micrometers.[16] As a result, investigating the effect of Casimir force on nano/micro actuators can be extremely important in their design.

Tahami *et al.*[17] discussed pull-in phenomena and dynamic response of a capacitive nano beam switches under effect of electrostatic and Casimir forces. Casimir effect on the pull-in parameters of nano-meter switches has been studied by Lin and Zhao.[18] They[19] also studied nonlinear behavior of nano scale electrostatic actuators with Casimir force. Ramezani *et al.*[20,21] investigated the two point boundary value problem of the deflection of a nano cantilever subjected to Casimir and electrostatic forces using analytical and numerical methods to obtain the instability point of the nano beam. Modeling and simulation of electrostatically actuated nano switches under the effect of Casimir forces have been investigated by Mojahedi *et al.*[22] Sirvent *et al.*[23] theoretically studied pull-in control in capacitive micro switches actuated by Casimir forces using external magnetic fields. Effect of the Casimir force on the static deflection and stiction of membrane strips in MEMS have been studied by Serry *et al.*[24] Gusso and Delben[16] analyzed the influence of surface roughness and temperature on the pull-in parameters of silicon based actuators by considering Casimir force. Guo and Zhao[25] discussed the effect of Casimir force on the pull-in of electrostatic torsional actuators. In their problem a small angle perturbation could lead to pull-in, but when the actuator is single sided, there would be two equilibrium points which one of them is stable and the other one is unstable. In this paper, the behavior of the stable equilibrium point would be investigated analytically using straight forward perturbation expansion method.

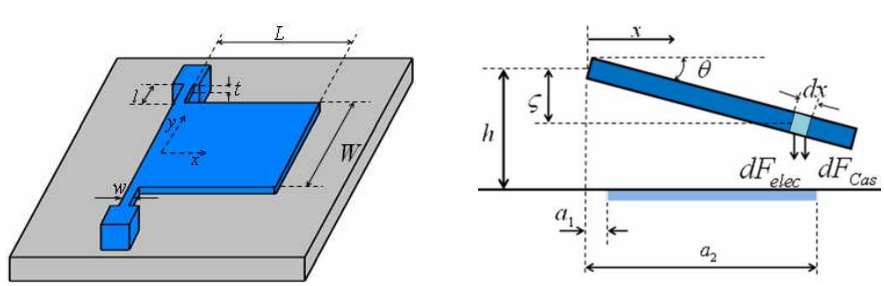
Perturbation based methods have been widely used to analytically solve the nonlinear problems in N/MEMS. For example, Moeenfard *et al.*[26] investigated the behavior of nano/micromirrors under effect of capillary and vdW forces by using homotopy perturbation method (HPM). Abdel-Rahman and Nayfeh[27] used the multiple-scale perturbation method to model secondary resonances in electrically actuated micro beams. Younis and Nayfeh[28] used the same method to study the response of a resonant micro beam to an electric actuation. Moeenfard *et al.*[29] used HPM for modeling the nonlinear behavior of Timoshenko micro beams. Mojahedi *et al.*[30] applied the same method to simulate the static response of nano switches to electrostatic actuation and intermolecular surface forces. But so far no analytic solution has been presented to model the behavior of torsional nano/micro actuators under the effects of electrostatic actuation and Casimir force.

In this paper, the equations governing the static behavior of torsional nano/micro actuators are obtained using the minimum total potential energy principle. Then energy method is used to investigate the stability of torsional nano/micro actuators equilibrium points and the effect of different design parameters on the stable equilibrium point is investigated. At the end, tilting angle of a torsional nano/micro actuator under electrostatic and Casimir forces is calculated analytically using straight forward perturbation expansion method.

2. Problem Formulation

Here the total potential energy principle is utilized for finding equilibrium equation of torsional nano/micro actuators under the combined effect of electrostatic and Casimir force. The total potential energy of the torsional nano/micro actuators shown in Fig. 1 can be divided into two parts: the potential strain energy of the torsion beams and the potential energy of applied loads which is equal to the minus of work done by external forces.[31,32]

$$\Pi = U + \Psi = U - W_e \,, \tag{1}$$



(a) (b)

Fig. 1. Schematic (a) 3D and (b) 2D view of an electrostatically actuated torsional nano/micro actuator under the effect of Casimir force.

where Π is the total potential energy of the system, U is the total mechanical and electrical potential energy of the system, Ψ is the potential energy of applied loads and W_e is the work done by external forces (i.e., Casimir force).

When the distance between the nano/micro actuator and the underneath substrate is within some applicable ranges (approximately between 100 nm and 1000 nm), the Casimir force plays important role in the actuator behavior.[16] For two parallel plates slightly held apart from each other, Casimir force can be modeled using the following equation.[33]

$$\hat{F}_{\text{Cas}} = \frac{\pi^2 \hbar c}{240 D^4}\,, \tag{2}$$

where $\hat{F}_{\text{Cas}}$ is the Casimir force per unit area, c is the speed of light, $\hbar$ is the Planck's constant divided by 2π and D is the distance between the plates.

Usually the rotation angle of the nano/micro actuators are small (less than 2° (Ref. 11)), So the actuator plate and the underneath substrate will remain approximately parallel and Eq. (2) can still be used for modeling the Casimir effect on the torsional nano/micro actuators. Furthermore, when the rotation angle of the torsional actuator is small, one can say that $\tan\theta \approx \theta$ where θ is the rotation angle of the actuator. So the differential Casimir force exerted to the differential surface element of the actuator shown in Fig. 1 would be[33]:

$$dF_{\text{Cas}} = \frac{\pi^2 \hbar c}{240(h - x\theta)^4} W \cdot dx\,, \tag{3}$$

where h is the initial distance between the actuator and the substrate and W is the width of the actuator.

For convenient purpose, the normalized rotation angle Θ is introduced as follows.

$$\Theta = \theta/\theta_{\max}\,, \tag{4}$$

where $\theta_{\max}$ is the maximum physically possible rotation angle of the actuator and is calculated as

$$\theta_{\max} = \sin^{-1}\left(\frac{h}{L}\right) \approx \frac{h}{L}\,. \tag{5}$$

In Eq. (5), L is the length of the actuator.

Casimir force can be expressed in terms of normalized variable Θ in the following manner.

$$dF_{\text{Cas}} = \frac{\pi^2 \hbar c}{240 h^4 \left(1 - \frac{x}{L}\Theta\right)^4} W \cdot dx\,. \tag{6}$$

The external work done on the actuator by Casimir force is:

$$W_e^{\text{Cas}} = \iint dF_{\text{Cas}} \cdot d\varsigma\,, \tag{7}$$

where ς is a position parameter as shown in Fig. 1. Obviously for small angles, ς can be calculated using the following equation.

$$\varsigma = x\theta \tag{8}$$

substituting Eqs. (6) and (8) into Eq. (7), one can easily conclude that:

$$W_e^{\mathrm{Cas}} = \frac{\pi^2 \hbar c W}{240 h^3 L} \int_0^L \int_0^\Theta x \frac{d\Theta'}{\left(1 - \frac{x}{L}\Theta'\right)^4} dx \,. \tag{9}$$

When there exist a potential difference between the actuator plate and the underneath electrode, the system acts like a capacitor. The overall capacitance of this system can be computed by integrating the capacitance of differential capacitors each of them having a capacity of[34]:

$$dC = \frac{\varepsilon_0 W \cdot dx}{h - x\theta} \,, \tag{10}$$

where ε_0 is permittivity of free space. By integrating Eq. (10), the total capacitance of the torsional nano/micro actuators, C would be obtained as follows.

$$C = \int_0^L dC = \frac{\varepsilon_0 W}{h} \int_{a_1}^{a_2} \frac{1}{1 - \frac{x}{L}\Theta} dx \,, \tag{11}$$

where a_1 and a_2 are some position parameters defining the start and end point of the electrode as illustrated in Fig. 1. The potential energy of electrostatic force can be calculated as[34]:

$$U_E = -\frac{1}{2} C V^2 \,, \tag{12}$$

where V is the applied voltage between the actuator and the electrode. The potential strain energy stored in the torsion beams is:

$$U_M = \frac{1}{2} S_0 \theta^2 = \frac{h^2}{2L^2} S_0 \Theta^2 \,. \tag{13}$$

In this equation, U_M is the potential strain energy of the beams and S_0 is the overall torsional stiffness of the two torsion beams and can be calculated using Eq. (14).

$$S_0 = \frac{2GI_p}{l} \,, \tag{14}$$

where l is the length of each beam as shown in Fig. 1, G is the shear modulus of elasticity of beams material and I_p is the polar moment of inertia of the beams cross-section. For a rectangular cross-section beam, I_p is as below[9]:

$$I_p = \frac{1}{3} t w^3 - \frac{64}{\pi^5} w^4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \tanh \frac{(2n-1)\pi t}{2w} \,, \tag{15}$$

where w and t are the width and length of the beam's cross-section respectively.

The overall potential energy of the system, U is simply obtained by adding the mechanical and electrical strain energies as follows.

$$U = U_M + U_E \,. \tag{16}$$

Using equations (1), (12), (13) and (16), the total potential energy of the system, Π is calculated as follows.

$$\Pi = \frac{h^2}{2L^2} S_0 \Theta^2 - \frac{1}{2} C V^2 - W_e^{\mathrm{Cas}} \,. \tag{17}$$

At equilibrium state, Π has no variation,[31,32] so the following equation has to be satisfied.

$$\frac{\partial \Pi}{\partial \Theta} = \frac{h^2}{L^2} S_0 \Theta - \frac{1}{2} V^2 \frac{\partial C}{\partial \Theta} - \frac{\partial W_e^{\mathrm{Cas}}}{\partial \Theta} = 0\,. \tag{18}$$

Using some algebraic manipulations, equilibrium equation, can be more simplified as equation (17).

$$\begin{aligned}\Xi(\Theta, \bar{V}, \lambda) = \Theta &- \frac{\bar{V}^2}{\Theta^2}\left(\frac{1}{1-\beta\Theta} - \frac{1}{1-\alpha\Theta} + \ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)\right) \\ &- \frac{\lambda}{\Theta^2}\left(\frac{1}{6} - \frac{3\Theta - 1}{6(\Theta-1)^3}\right) = 0\,.\end{aligned} \tag{19}$$

In this equation $\Xi(\Theta, \bar{V}, \lambda)$ is the equilibrium equation, $\bar{V}$ is defined as the normalized voltage and λ, α and β are calculated as follows.

$$\bar{V} = V\sqrt{\frac{\varepsilon_0 W L^3}{2h^3 S_0}}\,, \tag{20}$$

$$\lambda = \frac{\pi^2 \hbar c W L^3}{240 h^5 S_0} \tag{21}$$

$$\alpha = a_1/L\,, \tag{22}$$

$$\beta = a_2/L\,. \tag{23}$$

By performing the second variation operator on Eq. (17) and using equilibrium equation, it is easily concluded that:

$$\begin{aligned}\delta^2\Pi = \frac{(\delta\Theta)^2 h^2 S_0}{L^2}\Bigg[1 &+ \frac{\bar{V}^2}{\Theta^3}\left(\frac{2+\beta\Theta}{1-\beta\Theta} - \frac{2+\alpha\Theta}{1-\alpha\Theta} + 2\ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)\right. \\ &\left. - \frac{\beta\Theta}{(1-\beta\Theta)^2} + \frac{\alpha\Theta}{(1-\alpha\Theta)^2}\right) + \frac{\lambda}{\Theta^3}\left(\frac{1}{3} - \frac{3\Theta-2}{6(\Theta-1)^3} - \frac{\Theta(3\Theta-1)}{2(\Theta-1)^4}\right)\Bigg] > 0\,.\end{aligned} \tag{24}$$

According to minimum total potential energy principle an equilibrium point is stable when $\delta^2\Pi > 0$ and is unstable when $\delta^2\Pi < 0$.[32] So the stability condition is reduced to:

$$\begin{aligned}&I(\bar{V}, \lambda, \Theta, \alpha, \beta) \\ &\quad = 1 + \frac{\bar{V}^2}{\Theta^3}\left(\frac{2+\beta\Theta}{1-\beta\Theta} - \frac{2+\alpha\Theta}{1-\alpha\Theta} + 2\ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right) - \frac{\beta\Theta}{(1-\beta\Theta)^2} + \frac{\alpha\Theta}{(1-\alpha\Theta)^2}\right) \\ &\qquad + \frac{\lambda}{\Theta^3}\left(\frac{1}{3} - \frac{3\Theta-2}{6(\Theta-1)^3} - \frac{\Theta(3\Theta-1)}{2(\Theta-1)^4}\right) > 0\,.\end{aligned} \tag{25}$$

Finding $\bar{V}$ from Eq. (19) and substituting it into Eq. (25) leads to:

$$I(\lambda,\Theta,\alpha,\beta) = 1 + \frac{1 - \dfrac{\lambda}{\Theta^4}\left(\dfrac{1}{6} - \dfrac{3\Theta - 1}{6(\Theta-1)^3}\right)}{\dfrac{1}{1-\beta\Theta} - \dfrac{1}{1-\alpha\Theta} + \ln\left(\dfrac{1-\beta\Theta}{1-\alpha\Theta}\right)}$$
$$\times\left(\frac{2+\beta\Theta}{1-\beta\Theta} - \frac{2+\alpha\Theta}{1-\alpha\Theta} + 2\ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right) - \frac{\beta\Theta}{(1-\beta\Theta)^2} + \frac{\alpha\Theta}{(1-\alpha\Theta)^2}\right)$$
$$+\frac{\lambda}{\Theta^3}\left(\frac{1}{3} - \frac{3\Theta-2}{6(\Theta-1)^3} - \frac{\Theta(3\Theta-1)}{2(\Theta-1)^4}\right) \tag{26}$$

Figure 2 shows the function $I(\lambda,\Theta,\alpha,\beta)$ versus Θ at some values of λ.

An equilibrium point is stable if $I(\lambda,\Theta,\alpha,\beta) > 0$ and unstable if $I(\lambda,\Theta,\alpha,\beta) < 0$. It is observed that at certain value of Θ, which is called Θp, $I(\lambda,\Theta,\alpha,\beta)$ becomes zero. When $\Theta < \Theta P$, $I(\lambda,\Theta,\alpha,\beta)$ is positive and the resulting equilibrium point would be stable and when $\Theta > \Theta p$ $I(\lambda,\Theta,\alpha,\beta)$ is negative and the resulting equilibrium point would be unstable.

At the pull-in state the following equation is satisfied.

$$I(\lambda,\Theta,\alpha,\beta) = 0\,. \tag{27}$$

By using equations (19), (26) and (27), Θ_P and $\bar{V}_P$ where $\bar{V}_P$ is the value of $\bar{V}$ at pull-in state, can be plotted versus β at various value of α and λ as depicted in Figs. 3 and 4, respectively. Figure 3 shows that by increasing the values of β, Θ_P and $\bar{V}_P$ would be decreased. From Fig. 3 it is observed that by increasing α, normalized pull-in angle is reduced and pull-in voltage is increased. Figure 4 shows that the values of Θ_P and $\bar{V}_P$ are decreased when λ is increased.

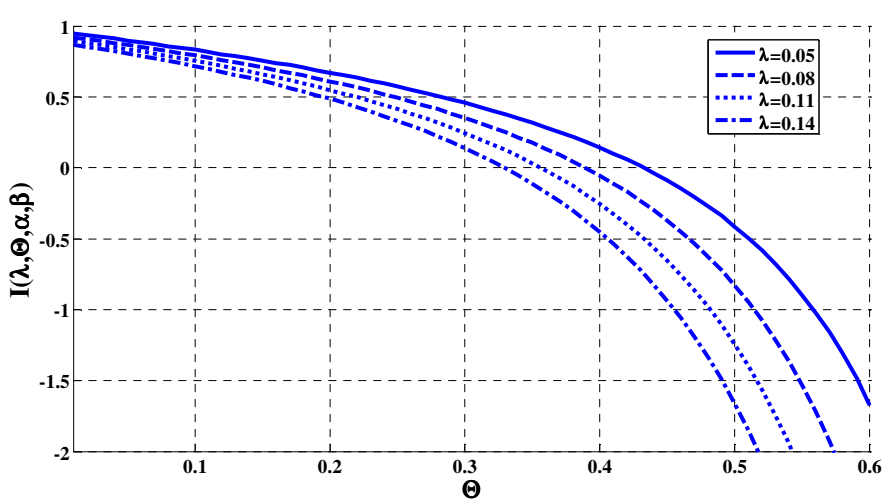


Fig. 2. Function $I(\lambda,\Theta,\alpha,\beta)$ versus Θ at $\alpha = 0.2$ and $\beta = 0.7$.

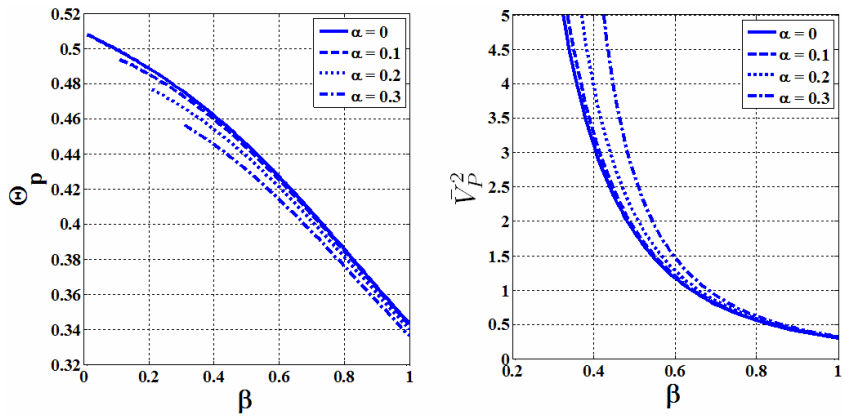


Fig. 3. Θ_P and $\bar{V}_P^2$ versus β at $\lambda = 0.05$ and various values of α.

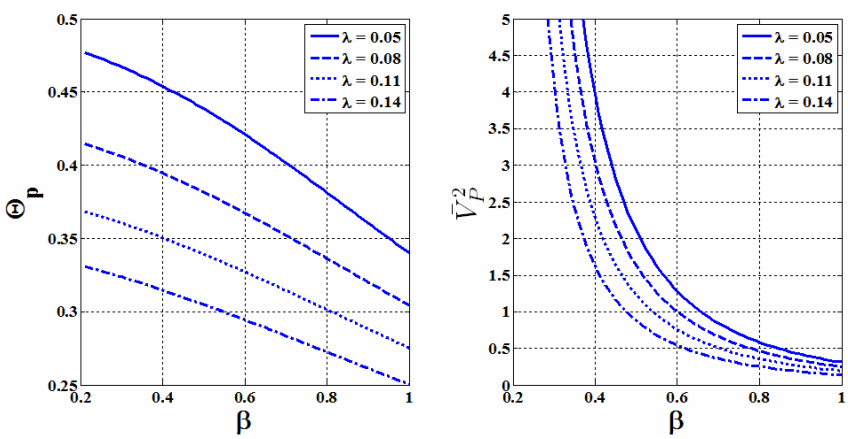


Fig. 4. Θ_P and $\bar{V}_P^2$ versus β at $\alpha = 0.2$ and various values of λ.

In Fig. 5, $\bar{V}_P^2$ has been plotted against λ. It is observed that with increasing λ pull-in occurs at lower values of $\bar{V}_P^2$. In fact this figure shows that Casimir force can significantly reduce the maximum allowable value for $\bar{V}_P^2$ and as a result, it would reduce the stability limits of the torsional nano/micro actuators. In addition it can be concluded that even in the absence of electrostatic force (i.e., $\bar{V}_P = 0$), Casimir force can lead to the occurrence of pull-in.

In order to avoid pull-in in electrostatically actuated nano/micro actuators under the effect of Casimir force, the maximum applied voltage should be less than a certain value and the inequality given in Eq. (26) has to be satisfied.

$$V < \sqrt{\frac{2\bar{V}_P^2 h^3 S_0}{\varepsilon_0 W L^3}}\,. \tag{28}$$

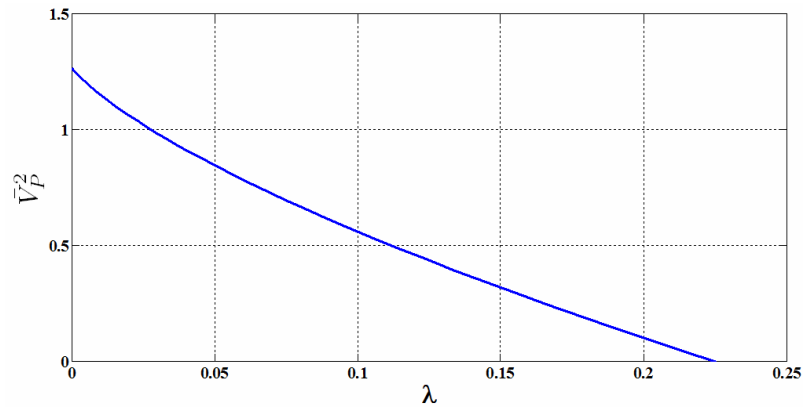


Fig. 5. $\bar{V}_P^2$ versus λ when $\alpha = 0.2$ and $\beta = 0.7$.

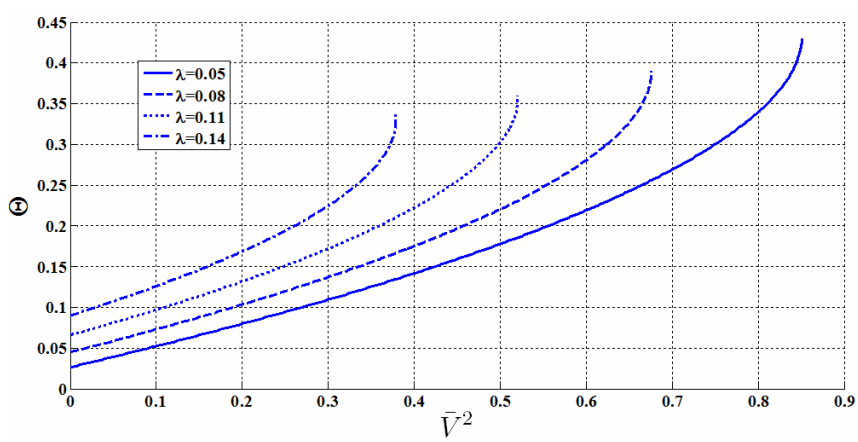


Fig. 6. Stable equilibrium angle versus $\bar{V}^2$.

In order to investigate the actuator's behavior under combined electrostatic and Casimir loading, Θ has been plotted versus $\bar{V}^2$ in Fig. 6.

It is observed that by increasing the value of $\bar{V}$ the rotation angle of the torsional nano/micro actuators is increased, but the maximum value of $\bar{V}$ at pull-in, highly depends on the value of λ and it is verified that by increasing λ, the maximum allowable value for $\bar{V}$ is reduced. Furthermore, it is concluded that at a constant $\bar{V}$, increasing the values of λ would lead to larger values for stable equilibrium angle.

3. Analytical Solution of Equilibrium Equation

In this part, it is tried to obtain the value of the rotation angle of the torsional nano/micro actuators analytically in terms of $\bar{V}$ and λ To this goal, the straightforward perturbation expansion method is utilized as follows.

By using Taylor series expansion, the linear part of the equilibrium equation can be found as follows.

$$L(\Theta, \bar{V}, \lambda) = L_1 + L_2\Theta\,, \tag{29}$$

where $L(\Theta, \bar{V}, \lambda)$ is the linear part of the equilibrium equation and L_1 and L_2 are defined as,

$$L_1(\bar{V}, \lambda) = -\frac{\lambda + \bar{V}^2(\beta^2 - \alpha^2)}{2}\,, \tag{30}$$

$$L_2(\bar{V}, \lambda) = 1 - \frac{4}{3}\lambda - \frac{2}{3}\bar{V}^2(\beta^3 - \alpha^3)\,. \tag{31}$$

The nonlinear part of equilibrium equation is obtained by subtracting Eq. (29) from the equilibrium equation

$$N(\Theta, \bar{V}, \lambda) = \bar{V}^2 N_1(\Theta) + \lambda N_2(\Theta)\,, \tag{32}$$

where

$$N_1(\Theta) = \frac{(\beta^2 - \alpha^2)}{2} + \frac{2}{3}(\beta^3 - \alpha^3)\Theta - \frac{1}{\Theta^2}\left(\frac{1}{1-\beta\Theta} - \frac{1}{1-\alpha\Theta} + \ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)\right), \tag{33}$$

$$N_2(\Theta) = \frac{1}{2} + \frac{4}{3}\Theta - \frac{1}{\Theta^2}\left(\frac{1}{6} + \frac{3\Theta - 1}{6(\Theta - 1)^3}\right). \tag{34}$$

The normalized voltage $\bar{V}$ and the normalized Casimir parameter can be scaled as:

$$\bar{V}^2 = \vartheta\hat{V}^2\,, \tag{35}$$

$$\lambda = \vartheta\hat{\lambda}\,, \tag{36}$$

where ϑ is a small book-keeping parameter and will be used as perturbation parameter. Since $N(\Theta, \bar{V}, \lambda)$ is a homogenous linear combination of $\bar{V}^2$ and λ, one can conclude that

$$N(\Theta, \bar{V}, \lambda) = \vartheta N(\Theta, \hat{V}, \hat{\lambda})\,. \tag{37}$$

The equilibrium equation can be reconstructed using Eqs. (29) and (32) as follows.

$$\Xi(\Theta, \bar{V}, \lambda) = L(\Theta, \bar{V}, \lambda) + N(\Theta, \bar{V}, \lambda)\,. \tag{38}$$

Using Eqs. (29) and (37), Eq. (38) can be restated as,

$$\Xi(\Theta, \bar{V}, \lambda) = L_1 + L_2\Theta + \vartheta N(\Theta, \hat{V}, \hat{\lambda})\,. \tag{39}$$

In the next step, Θ is expanded in terms of ϑ as follows.

$$\Theta = \Theta_0 + \vartheta\Theta_1 + \vartheta^2\Theta_2 + O(\upsilon^3) \tag{40}$$

by substituting Θ from Eq. (40) into Eq. (39) and computing the Taylor series expansion of the resulting equation, one would get

$$\Xi(\Theta, \bar{V}, \lambda) = L_1 + L_2\Theta_0 + (L_2\Theta_1 + N(\Theta_0, \hat{V}, \hat{\lambda}))\vartheta + \left(L_2\Theta_2 + \Theta_1 \frac{\partial N(\Theta_0, \hat{V}, \hat{\lambda})}{\partial \Theta_0}\right)\vartheta^2 + O(\vartheta^3)\,. \tag{41}$$

By equating the coefficients of all powers of ϑ with zero, the following equations are obtained.

$$\vartheta^0 : L_1 + L_2\Theta_0 = 0\,, \tag{42}$$

$$\vartheta^1 : L_2\Theta_1 + N(\Theta_0, \hat{V}, \hat{\lambda}) = 0\,, \tag{43}$$

$$\vartheta^2 : L_2\Theta_2 + \Theta_1 \frac{\partial N(\Theta_0, \hat{V}, \hat{\lambda})}{\partial \Theta_0} = 0\,. \tag{44}$$

These equations can be solved consecutively and the result would be as,

$$\Theta_0 = -L_1/L_2\,, \tag{45}$$

$$\Theta_1 = -N(\Theta_0, \hat{V}, \hat{\lambda})/L_2\,, \tag{46}$$

$$\Theta_2 = -\frac{\Theta_1}{L_2}\frac{\partial N(\Theta_0, \hat{V}, \hat{\lambda})}{\partial \Theta_0} = \frac{N(\Theta_0, \hat{V}, \hat{\lambda})}{L_2^2}\frac{\partial N(\Theta_0, \hat{V}, \hat{\lambda})}{\partial \Theta_0}\,. \tag{47}$$

By substituting the values of Θ_0, Θ_1 and Θ_2 into Eq. (40), the final solution for Θ is simply obtained as,

$$\Theta = -\frac{L_1}{L_2} - \vartheta N(-L_1/L_2, \hat{V}, \hat{\lambda})/L_2 + \vartheta^2 \frac{N(-L_1/L_2, \hat{V}, \hat{\lambda})}{L_2^2}\frac{\partial N(\Theta_0, \hat{V}, \hat{\lambda})}{\partial \Theta_0}\bigg|_{\Theta_0 = -L_1/L_2}\,. \tag{48}$$

Using Eq. (37), Eq. (48) can be more simplified as follows.

$$\Theta = -\frac{L_1}{L_2} - N(-L_1/L_2, \bar{V}, \lambda)/L_2 + \frac{N(-L_1/L_2, \bar{V}, \lambda)}{L_2^2}\frac{\partial N(\Theta_0, \bar{V}, \lambda)}{\partial \Theta_0}\bigg|_{\Theta_0 = -L_1/L_2}\,. \tag{49}$$

It is observed that as it was expected, the final solution for Θ, is independent of the value of the book-keeping parameter ϑ.

In Fig. 7 the results of the numerical simulations are compared with those of analytical perturbation results at different values of λ. It is observed that straight forward perturbation expansion method can closely approximate the rotation angle of the torsional nano/micro actuators. By increasing the order of perturbation approximation, the obtained results become more precise, but increasing the order of the perturbation approximation more than four will not improve the accuracy of the obtained response appreciably. Therefore, a fourth-order perturbation approximation can precisely predict the torsional nano/micro actuator behavior under the combined effects of electrostatic actuation and Casimir force.

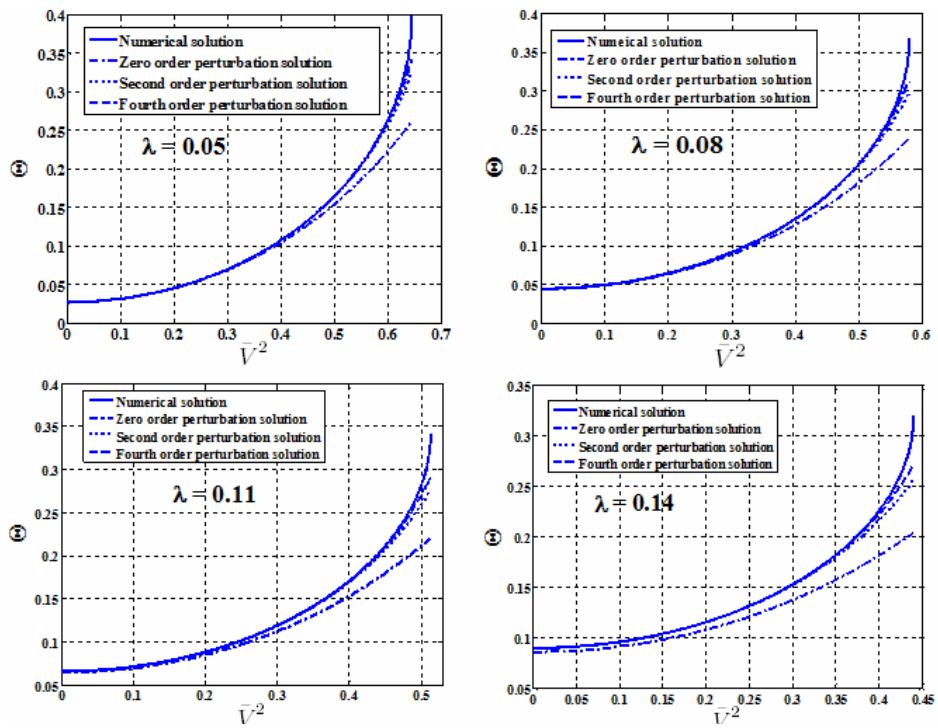


Fig. 7. Estimation of nano/micro actuator's rotation angle using straight forward perturbation expansion for a torsional actuator with $\alpha = 0.1$ and $\beta = 0.9$ at different values of λ.

4. Conclusion

Effect of Casimir force on the pull-in behavior of electrostatically actuated torsional nano/micro actuators was investigated in this paper. First static equilibrium equation was found using the minimum total potential energy principle and stability of equilibrium points were analyzed. Then a parametric study was performed to study the dependence of pull-in angle and pull-in voltage of the actuator to its design parameters. It was observed that the effect of Casimir force may lead to significant reduction in the stability limits of the torsional nano/micro actuators. Finally, straight forward perturbation expansion method was used to analytically model the behavior of the stable equilibrium angle of the actuator. Comparison between presented analytical solutions and numerical results show that straight forward perturbation expansion method can effectively model the torsional nano/micro actuators behavior under combined electrostatic actuation and Casimir force. The analytical model presented in this paper can be used in design optimization and determination of the stable operative range of electrostatic torsional nano/micro actuators where the gap between the actuator and the underneath electrodes is small enough for the Casimir force to play a major role in the system.

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