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RESEARCH PAPER

Analytical modeling of static behavior of electrostatically actuated nano/micromirrors considering van der Waals forces

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Abstract In this paper, the effect of van der Waals (vdW) force on the pull-in behavior of electrostatically actuated nano/micromirrors is investigated. First, the minimum potential energy principle is utilized to find the equation governing the static behavior of nano/micromirror under electrostatic and vdW forces. Then, the stability of static equilibrium points is analyzed using the energy method. It is found that when there exist two equilibrium points, the smaller one is stable and the larger one is unstable. The effects of different design parameters on the mirror's pull-in angle and pull-in voltage are studied and it is found that vdW force can considerably reduce the stability limit of the mirror. At the end, the nonlinear equilibrium equation is solved numerically and analytically using homotopy perturbation method (HPM). It is observed that a sixth order perturbation approximation can precisely model the mirror's behavior. The results of this paper can be used for stable operation design and safe fabrication of torsional nano/micro actuators.

Keywords Nano/micromirror · Electrostatic actuation · vdW force · Pull-in · Homotopy perturbation method (HPM)

1 Introduction

Technology of N/MEMS has recently witnessed lots of progress in testing and fabricating new devices. Their advantages of low manufacturing cost, batch production, light weight, small size, durability, low energy consumption and compatibility with integrated circuits, make them even more attractive [1, 2].

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Successful MEMS devices rely not only on well developed fabrication technologies, but also on the knowledge of device behavior, based on which a favorable structure of the device can be forged [3].

The fact that MEMS devices play important roles in optical systems, leads to the development of a new class of systems called micro-opto-electro-mechanical systems (MOEMS). MOEMS include a wide variety of devices including digital micromirror devices (DMD) [4], optical switches [5], micro scanning mirrors [6], optical cross connects [7, 8], etc. Torsional actuators play an important role in MOEMS systems [9]. Many researches have been done on torsional micro actuators using novel numerical schemes. For example, Degani et al. [10] presented a novel displacement iteration pull-in extraction (DIPIE) scheme for the problem of electrostatic torsion micro-actuators, and showed that their method converges 100 times faster than the voltage iteration scheme. Zhang et al. [11] described the static characteristics of an electrostatically actuated torsional micromirror based on parallel plate capacitor model, and studied extensively the snap down phenomenon in micromirrors. Dynamic response of a torsional micromirror to electrostatic force and mechanical shock has been investigated by Khatami and rezazadeh [12]. Bhaskar et al. [13] studied Switching response of torsional micromirrors.

Intermolecular surface forces which mainly include Casimir and vdW force, play an important role in the stability of micromirrors. vdW force is a short range force in nature, but it can lead to long range effects more than 0.1 μm [14]. This force is the interaction force between neutral atoms, and it differs from covalent and ionic bonds in that it is caused by correlations in the fluctuating polarizations of nearby particles [15]. Casimir force can be simply understood as the long range analog of the vdW force, resulting from the propagation of retarded electromagnetic wave [16].

Influence of vdW and Casimir forces in pull-in phenomena and dynamic response of a capacitive nano-beam switch has been studied by Tahami et al. [17]. Batra et al. [18] studied the effects of vdW force and thermal stresses on pull-in instability of electrostatically actuated microplates. Modelling and simulation of nano switches under the effects of vdW and electrostatic forces have been studied by Mojahedi et al. [19]. Ramezani et al. [20] investigated influence of vdW force on the pull-in parameters of cantilever type nanoscale electrostatic actuators. Guo and Zhao [21] studied dynamic stability of electrostatic torsional actuators with vdW effects. They [22] also discussed the pull-in of torsional actuators under the effects of vdW and electrostatic forces. In their problem, a small angle perturbation could lead to pull-in, but when the micromirror is single sided there would exist two equilibrium points, one of which is stable and the other is unstable. In this paper, the behavior of the stable equilibrium point is analytically investigated by HPM.

Perturbation methods have been used to analytically solve the nonlinear problems in MEMS. Younis and Nayfeh [23] investigated the response of a resonant microbeam to an electric actuation using the multiple-scale perturbation method. Abdel-Rahman and Nayfeh [24] used the same method to model secondary resonances in electrically actuated microbeams. Since perturbation methods are based upon the assumption that there is a small parameter in the equations, they have some limitations for problems without small parameters involved. In order to overcome this limitation a new perturbational based method, namely HPM was developed by He et al. [25]. His new method takes full advantages of the traditional perturbation methods and homotopy techniques. HPM has also been used for solving the nonlinear problems encountered in N/MEMS. For example, Moeenfard et al. [26] used HPM to model the nonlinear vibrational behavior of Timoshenko micobeams. Mojahedi et al. [27] adopted the HPM method to simulate the static response of nano-switches to electrostatic actuation and intermolecular surface forces.

The common approach for finding the equilibrium angle of a mirror under different kinds of excitations is the voltage-iteration technique, but this technique is time consuming and requires many iterations to converge to the solution with the desired accuracy. So, to overcome this limitation, there have been several analytical/numerical attempts to find the equilibrium angle of a mirror in an analytic closed form [10, 28]. But, all the methods presented so far for finding the equilibrium angle of the mirror are still based on numerical simulations. In the current paper, for the first time, an entirely analytical closed form solution is presented for finding the equilibrium angle of the nano/micromirror under the combined effect of vdW and electrostatic force.

In the current paper, the equation governing the static behavior of electrostatically actuated nano/micromirrors considering vdW forces is obtained using the minimum potential energy principle. Then energy method is used to investigate the stability of nano/micromirrors equilibrium points, and the effect of different design parameters on the stable equilibrium point is investigated. At the end, tilting angle of an electrostatically actuated nano/micromirror is calculated both numerically and analytically using HPM with vdW force taken into consideration.

2 Problem formulation

Here, the minimum total potential energy principal is utilized to obtain the equilibrium equations of an electrostatically actuated nano/micromirror under the effect of vdW force. According to this principal, at the equilibrium state, the variation of the total potential energy of the system with respect to all degrees of freedom of the system is zero. So, the first step in using this principal to find the equilibrium equation is to calculate the total potential energy of the system under investigation. On the other hand, the total potential energy of the nano/micromirror shown in Fig. 1 can be divided into two parts: the potential strain energy of the torsion beams and the potential energy of applied loads which is equal to the minus of work done by external forces [29, 30] on the mirror

$$\Pi = U + \Upsilon = U - W_{\mathrm{e}}, \tag{1}$$

where Π is the total potential energy of the system, U is the potential strain energy of the torsion beams, Υ is the potential energy of applied loads and W_{e} is the work done by external forces (i.e. electrostatic and vdW force).

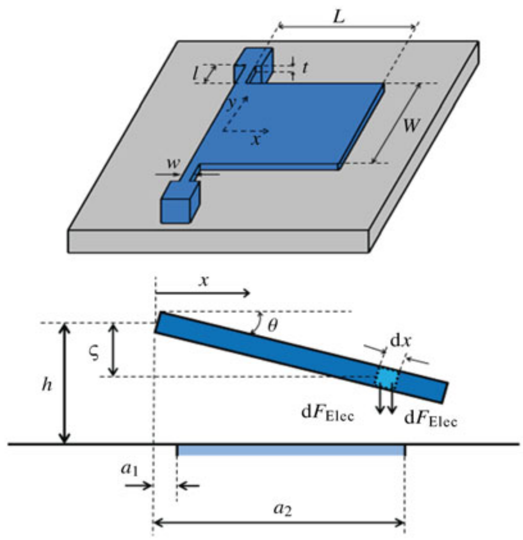

Fig. 1 Schematic view of an electrostatically actuated nano/micromirror under the effect of vdW force

When the rotation angle of mirror is small, the differential vdW force applied to the differential surface element of the mirror shown in Fig. 1 [14]

$$\mathrm{d}F_{\mathrm{vdW}} = \frac{AW}{6\pi (h - x\theta)^3}\mathrm{d}x, \tag{2}$$

where A is the Hamaker constant, W is the width of the mirror, h is the initial distance between the mirror and the underneath electrode and θ is the rotation angle of the mirror. vdW force can be simplified to the following manner

$$\mathrm{d}F_{\mathrm{vdW}} = \frac{AW}{6\pi h^3 (1 - x\Theta/L)^3}\mathrm{d}x. \quad (3)$$

In Eq. (3), L is the length of the mirror and Θ is dimensionless rotation angle of the mirror and is defined as

$$\Theta = \frac{\theta}{\theta_0}, \quad (4)$$

where θ_0 is the maximum physically possible rotation angle of the mirror and is calculated using Eq. (5)

$$\theta_0 = \frac{h}{L}. \quad (5)$$

The external work done on the mirror by the vdW force is

$$W_{\mathrm{e}}^{\mathrm{vdW}} = \int \mathrm{d}F_{\mathrm{vdW}}\mathrm{d}\varsigma, \quad (6)$$

where ς is a position parameter shown in Fig. 1. Obviously, for small angles, ς is

$$\varsigma = x\theta. \quad (7)$$

Substituting Eqs. (3) and (7) into Eq. (6), one can easily conclude that

$$W_{\mathrm{e}}^{\mathrm{vdW}} = \frac{1}{2}\mathfrak{R}, \quad (8)$$

where

$$\mathfrak{R} = \frac{AW}{3\pi h^2}\int_0^L \int_0^\Theta \frac{x}{L(1 - x\Theta'/L)^3}\mathrm{d}\Theta'\mathrm{d}x. \quad (9)$$

When there exist a potential difference between the mirror plate and the underneath electrode, the system acts like a capacitor. The capacitance of this system can be computed by integrating the capacitance of differential capacitors each of them having a capacity of [31]

$$\mathrm{d}C = \frac{\varepsilon_0 W \mathrm{d}x}{h - x\theta}, \quad (10)$$

where ε_0 is permittivity of free space. By integrating Eq. (10), the total capacitance of the nano/micromirror, C, would be as follows

$$C = \int_{a_1}^{a_2} \mathrm{d}C = \frac{\varepsilon_0 W}{h}\int_{a_1}^{a_2} \frac{1}{1 - x\Theta/L}\mathrm{d}x, \quad (11)$$

where a_1 and a_2 are some position parameters defining the start and end points of the electrode as illustrated in Fig. 1. The potential energy of electrostatic force can be calculated as [31]

$$U_E = -\frac{1}{2}CV^2, \quad (12)$$

where V is the applied voltage between the mirror and the electrode. The potential energy stored in the torsion beams is

$$U = \frac{1}{2}S_0\theta^2. \quad (13)$$

In this equation, S_0 is the total torsional stiffness of the two torsion beams and can be calculated using Eq. (14)

$$S_0 = \frac{2GI_{\mathrm{p}}}{l}, \quad (14)$$

where l is the length of each torsion beam, G is the shear modulus of elasticity of the beam's material and I_{p} is the polar moment of inertia of the beams cross section. For beams with rectangular cross section, I_{p} would be [9]

$$I_{\mathrm{p}} = \frac{1}{3}tw^3 - \frac{64w^4}{\pi^5}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}\tanh\frac{(2n-1)\pi t}{2w}, \quad (15)$$

where t and w are the length and width of the beams cross section, respectively. Using Eqs. (8), (12) and (13), the total potential energy of the system Π would be

$$\Pi = \frac{S_0 h^2 \Theta^2}{2L^2} - \frac{1}{2}CV^2 - \frac{1}{2}\mathfrak{R}. \quad (16)$$

At equilibrium state, Π has no variation [29, 30], so the following equation has to be satisfied

$$\Xi = \frac{\partial \Pi}{\partial \Theta} = \frac{\partial U}{\partial \Theta} - \frac{V^2 \partial C}{2\partial \Theta} - \frac{1}{2}\frac{\partial \mathfrak{R}}{\partial \Theta} = 0, \quad (17)$$

where Ξ is the equilibrium equation. For convenience, it is usually desirable to present the equilibrium equations in a dimensionless form [32, 33]. So, after some algebraic manipulations, Eq. (17) can be further simplified to

$$\Theta - \frac{\xi}{\Theta^2}\left[\frac{1}{1-\beta\Theta} - \frac{1}{1-\alpha\Theta} + \ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)\right] - \frac{\mu}{\Theta^2}\left[\frac{1}{2} + \frac{2\Theta - 1}{2(\Theta-1)^2}\right] = 0, \quad (18)$$

where ξ, μ, α and β are defined as

$$\xi = \frac{\varepsilon_0 W L^3 V^2}{2h^3 S_0}, \quad (19)$$

$$\mu = \frac{AWL^3}{6\pi h^4 S_0}, \quad (20)$$

$$\alpha = \frac{a_1}{L}, \quad (21)$$

$$\beta = \frac{a_2}{L}, \quad (22)$$

where ξ and μ represent the magnitude of the ratio of the electrostatic and vdW forces to the restoring force, respectively, the same method as used by Lin and Zhao [32, 33]. Performing the second variation operator on Eq. (16) and using equilibrium equation yields

$$\begin{aligned}\delta^2\Pi &= \frac{(\delta\Theta)^2\, h^2 S_0}{L^2}\left\{1+\frac{\xi}{\Theta^3}\left[\frac{2+\beta\Theta}{1-\beta\Theta}-\frac{2+\alpha\Theta}{1-\alpha\Theta}\right.\right.\\ &\quad\left.+2\ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)-\frac{\beta\Theta}{(1-\beta\Theta)^2}+\frac{\alpha\Theta}{(1-\alpha\Theta)^2}\right]\\ &\quad\left.+\frac{\mu}{\Theta^3}\left[1-\frac{1}{(1-\Theta)^3}+\frac{3}{(1-\Theta)^2}-\frac{3}{(1-\Theta)}\right]\right\}. \qquad (23)\end{aligned}$$

According to minimum total potential energy principle [29, 30], an equilibrium point is stable when $\delta^2\Pi > 0$ and is unstable when $\delta^2\Pi < 0$. So, the stability condition is reduced to

$$\begin{aligned}I(\xi,\mu,\Theta,\alpha,\beta) &= 1+\frac{\xi}{\Theta^3}\left[\frac{2+\beta\Theta}{1-\beta\Theta}-\frac{2+\alpha\Theta}{1-\alpha\Theta}\right.\\ &\quad+2\ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)-\frac{\beta\Theta}{(1-\beta\Theta)^2}\\ &\quad\left.+\frac{\alpha\Theta}{(1-\alpha\Theta)^2}\right]+\frac{\mu}{\Theta^3}\left[1-\frac{3}{1-\Theta}\right.\\ &\quad\left.+\frac{3}{(1-\Theta)^2}-\frac{1}{(1-\Theta)^3}\right]>0. \qquad (24)\end{aligned}$$

Finding ξ from Eq. (18) and substituting it into Eq. (24) leads to

$$\begin{aligned}&I(\mu,\Theta,\alpha,\beta)\\ &= 1+\frac{1-(\mu/\Theta^3)\left\{1/2+(2\Theta-1)/[2(\Theta-1)^2]\right\}}{1/(1-\beta\Theta)-1/(1-\alpha\Theta)+\ln[(1-\beta\Theta)/(1-\alpha\Theta)]}\\ &\quad\times\left[\frac{2+\beta\Theta}{1-\beta\Theta}-\frac{2+\alpha\Theta}{1-\alpha\Theta}+2\ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)-\frac{\beta\Theta}{(1-\beta\Theta)^2}\right.\\ &\quad\left.+\frac{\alpha\Theta}{(1-\alpha\Theta)^2}\right]+\frac{\mu}{\Theta^3}\left[1-\frac{1}{(1-\Theta)^3}\right.\\ &\quad\left.+\frac{3}{(1-\Theta)^2}-\frac{3}{(1-\Theta)}\right]. \qquad (25)\end{aligned}$$

Figure 2 shows the function $I(\mu,\Theta,\alpha,\beta)$ versus Θ for some values of μ.

An equilibrium point is stable if $I(\mu,\Theta,\alpha,\beta) > 0$ and unstable if $I(\mu,\Theta,\alpha,\beta) < 0$. It is observed that for certain value of Θ, which is called Θ_P, $I(\mu,\Theta,\alpha,\beta)$ becomes zero. When $\Theta < \Theta_P$, $I(\mu,\Theta,\alpha,\beta)$ would be positive and the resulting equilibrium point is stable and when $\Theta > \Theta_P$, $I(\mu,\Theta,\alpha,\beta)$ would be negative and the resulting equilibrium point is unstable. At the pull-in state the following equation is satisfied

$$I(\mu,\Theta,\alpha,\beta) = 0. \qquad (26)$$

Using Eqs. (18), (25) and (26), Θ_P can be plotted versus ξ_P, where ξ_P is the value of ξ at pull-in state. The related results are depicted in Fig. 3.

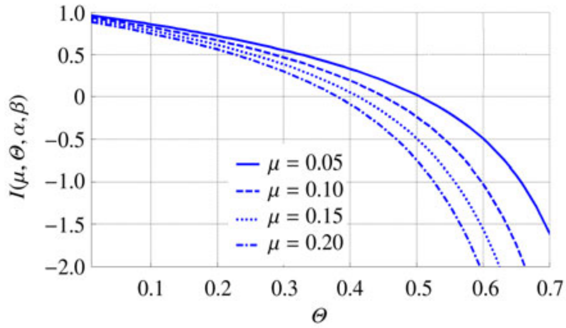

Fig. 2 Function $I(\mu,\Theta,\alpha,\beta)$ versus Θ for $\alpha = 0.2$ and $\beta = 0.7$

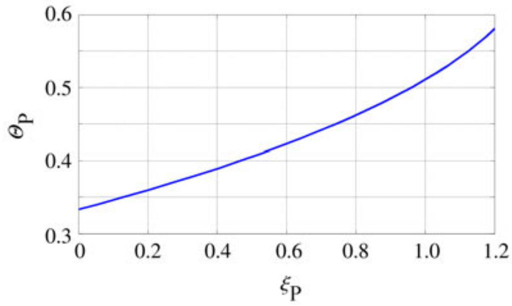

Fig. 3 Pull-in angle of mirror versus ξ_P for $\alpha = 0.2$ and $\beta = 0.7$

By using Eqs. (25) and (26), Θ_P and ξ_P can be plotted versus β for various value of α and μ, as depicted in Figs. 4 and 5. These figures show that with increasing β, Θ_P and ξ_P are reduced. It is also observed that with increasing α, Θ_P is decreased and ξ_P is increased. Figure 5 shows that the values of Θ_P and ξ_P are decreased when μ is increased.

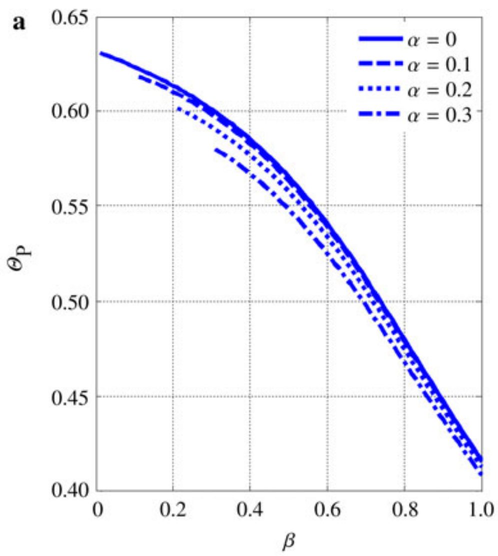

Fig. 4 Θ_P and ξ_P versus β for $\mu = 0.05$ and various values of α

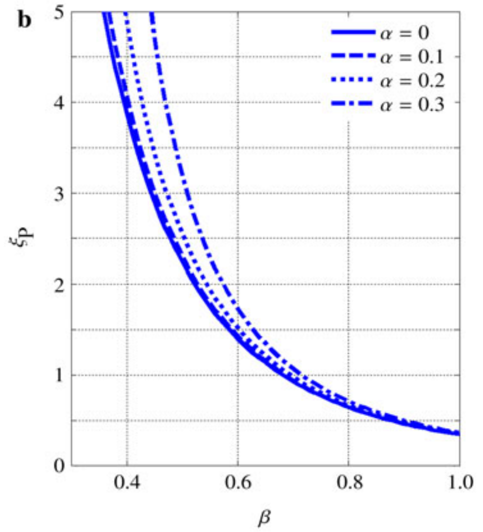

Fig. 4 Θ_P and ξ_P versus β for $\mu = 0.05$ and various values of α (contiuned)

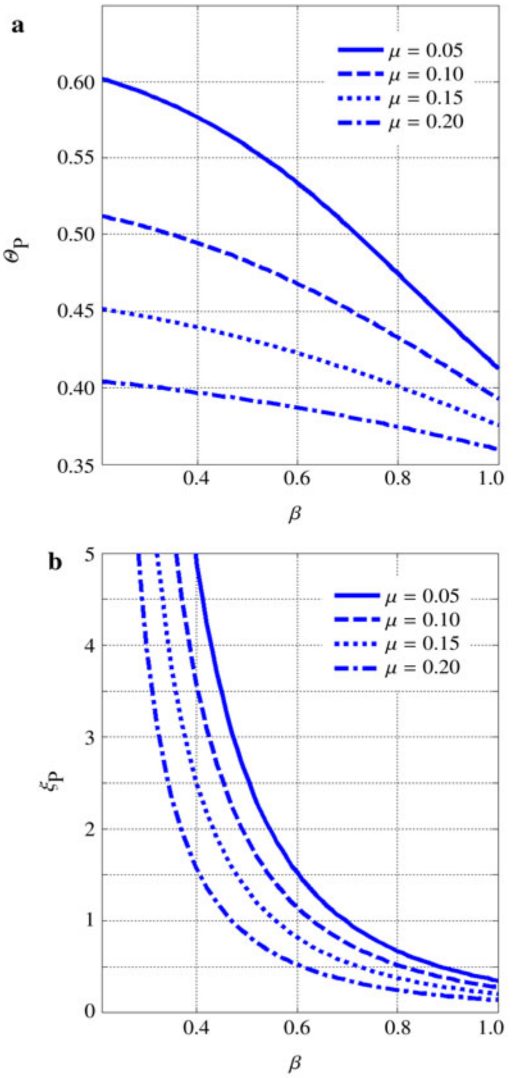

Fig. 5 Θ_P and ξ_P versus β for $\alpha = 0.2$ and various values of μ

In Fig. 6, the values of ξ_P has been plotted versus μ. It is observed that with increasing μ, pull-in occurs at lower values of ξ_P. In fact, this figure shows that vdW force can significantly reduce the maximum allowable value for ξ and as a result, the stability limit of the nano/micromirror is reduced in the presence of vdW force.

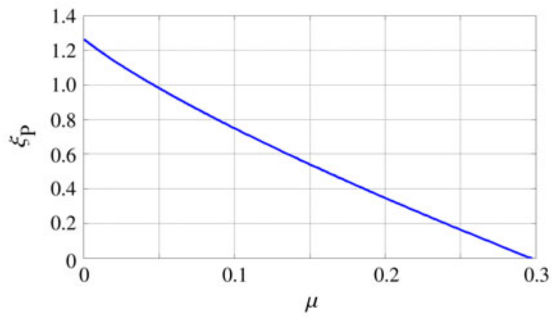

Fig. 6 ξ_P versus μ for $\alpha = 0.2$ and $\beta = 0.7$

From Fig. 6 it can be concluded that even in the absence of electrostatic force ($\xi = 0$), vdW force can lead to the occurrence of pull-in. So, in order to avoid pull-in in electrostatically actuated nano/micromirrors for a given μ, the inequality given in Eq. (27) has to be satisfied

$$V < \left(\frac{2\xi_P h^3 S_0}{\varepsilon_0 W L^3}\right)^{1/2}. \tag{27}$$

In order to investigate the mirror's behavior under combined electrostatic and vdW loading, Θ has been plotted versus ξ in Fig. 7.

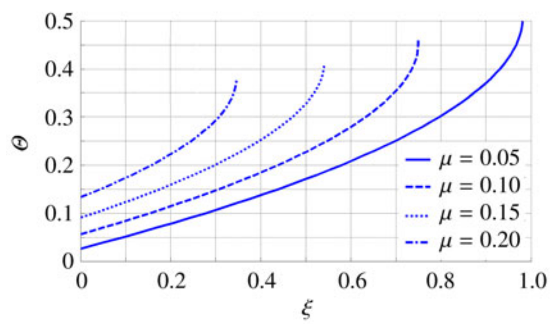

Fig. 7 Stable equilibrium angle versus ξ

It is observed that with increasing ξ, the rotation angle of the nano/micromirror is increased, but the maximum value of ξ at pull-in is highly dependent on the value of μ, and it is verified that with increasing μ, the maximum allowable value for ξ is reduced. Furthermore it is concluded that for a constant ξ, larger values of μ would lead to larger values of stable equilibrium angle.

3 Analytical solutions of the equilibrium equation

In this section, it is tried to find the value of the rotation angle of the nano/micromirror analytically in terms of ξ and μ. For this purpose, the analytical tool, HPM is utilized.

The linear part of Eq. (18) can be found by using Taylor series expansion of the equilibrium equation as follows

$$L(\Theta,\xi,\mu) = -\frac{\mu+\xi\left(\beta^2-\alpha^2\right)}{2} + \left[1-\mu-\frac{2}{3}\xi\left(\beta^3-\alpha^3\right)\right]\Theta, \tag{28}$$

where $L(\Theta,\xi,\mu)$ is the linear part of Eq. (18). Obviously the nonlinear part of equilibrium equation is obtained by subtracting $L(\Theta,\xi,\mu)$ from Eq. (18)

$$N(\Theta,\eta,\mu) = \frac{\mu+\xi\left(\beta^2-\alpha^2\right)}{2} + \left[\mu+\frac{2}{3}\xi\left(\beta^3-\alpha^3\right)\right]\Theta - \frac{\xi}{\Theta^2}\left[\frac{1}{1-\beta\Theta}-\frac{1}{1-\alpha\Theta}+\ln\left(\frac{1-\beta\Theta}{1-\alpha\Theta}\right)\right] - \frac{\mu}{\Theta^2}\left[\frac{1}{2}+\frac{2\Theta-1}{2(\Theta-1)^2}\right]. \tag{29}$$

Now, the homotopy form is constructed as follows

$$\Im(\Theta,\xi,\mu,P) = L(\Theta,\xi,\mu) + P.N(\Theta,\xi,\mu) = 0. \tag{30}$$

In Eq. (30), $\Im(\Theta,\xi,\mu,P)$ is the homotopy form and P is an embedding parameter which serves as perturbation parameter. When $P = 1$, the homotopy form would be the same as the equilibrium equation and when $P = 0$, homotopy form would be the linear part of equilibrium equation. The value of the dimensionless rotation angle Θ can also be expanded in terms of the embedded parameter P

$$\Theta = \Theta_0 + P\Theta_1 + P^2\Theta_2 + P^3\Theta_3 + \dots. \tag{31}$$

Substituting Eq. (31) into homotopy form yields

$$\Im(\Theta,\xi,\mu,P) = L\left(\Theta_0+P\Theta_1+P^2\Theta_2+\dots,\ \xi,\ \mu\right) + P.N\left(\Theta_0+P\Theta_1+P^2\Theta_2+\dots,\ \xi,\ \mu\right) = 0. \tag{32}$$

The Taylor series expansion of right hand side of Eq. (32) in terms of P would be

$$\begin{aligned}\Im(\Theta,\xi,\mu,P) =\ & L(\Theta_0,\xi,\mu) + \left[\Theta_1\frac{\partial L(\Theta_0,\xi,\mu)}{\partial\Theta_0} + N(\Theta_0,\xi,\mu)\right]P \\ & + \left[\Theta_2\frac{\partial L(\Theta_0,\xi,\mu)}{\partial\Theta_0} + \Theta_1\frac{\partial N(\Theta_0,\xi,\mu)}{\partial\Theta_0}\right]P^2 \\ & + \left[\Theta_3\frac{\partial L(\Theta_0,\xi,\mu)}{\partial\Theta_0} + \Theta_2\frac{\partial N(\Theta_0,\xi,\mu)}{\partial\Theta_0} + \frac{\Theta_1^2\partial^2 N(\Theta_0,\xi,\mu)}{2\partial\Theta_0^2}\right]P^3 \\ & + \dots \\ =\ & 0.\end{aligned} \tag{33}$$

Since the homotopy form must be unified with zero, the coefficients of all powers of P must be zero. This, leads to the following equations

$$L(\Theta_0,\xi,\mu) = 0, \tag{34}$$

$$\Theta_1\frac{\partial L(\Theta_0,\xi,\mu)}{\partial\Theta_0} + N(\Theta_0,\xi,\mu) = 0, \tag{35}$$

$$\Theta_2\frac{\partial L(\Theta_0,\xi,\mu)}{\partial\Theta_0} + \Theta_1\frac{\partial N(\Theta_0,\xi,\mu)}{\partial\Theta_0} = 0, \tag{36}$$

$$\Theta_3\frac{\partial L(\Theta_0,\xi,\mu)}{\partial\Theta_0} + \Theta_2\frac{\partial N(\Theta_0,\xi,\mu)}{\partial\Theta_0}, + \frac{\Theta_1^2\partial^2 N(\Theta_0,\xi,\mu)}{2\partial\Theta_0^2} = 0. \tag{37}$$

In solving Eqs. (34) to (37), the parameters Θ_i $(0 \leqslant i \leqslant 3)$ are found as follows

$$\Theta_0 = \frac{\mu/2+\xi\left(\beta^2-\alpha^2\right)/2}{1-\mu-2\xi\left(\beta^3-\alpha^3\right)/3}, \tag{38}$$

$$\Theta_1 = -N(\Theta_0,\xi,\mu)\Big/\left[\frac{\partial L(\Theta_0,\xi,\mu)}{\partial\Theta_0}\right], \tag{39}$$

$$\Theta_2 = -\Theta_1\left[\frac{\partial N(\Theta_0,\xi,\mu)}{\partial\Theta_0}\right]\Big/\left[\frac{\partial L(\Theta_0,\xi,\mu)}{\partial\Theta_0}\right], \tag{40}$$

$$\Theta_3 = -\left[\Theta_2\frac{\partial N(\Theta_0,\xi,\mu)}{\partial\Theta_0} + \frac{\Theta_1^2\partial^2 N(\Theta_0,\xi,\mu)}{2\partial\Theta_0^2}\right]\Big/\left[\frac{\partial L(\Theta_0,\xi,\mu)}{\partial\Theta_0}\right]. \tag{41}$$

The value of Θ is found by substituting Θ_i $(0 \leqslant i \leqslant 3)$ and $P = 1$ into Eq. (31). In Fig. 8, the results of the numerical simulations are compared with those of analytical HPM. It is observed that HPM closely approximates the rotation angle of the mirror. Obviously, increasing the order of perturbation approximation would lead to more precise results, but increasing the order of perturbation approximation to more than 6 will not significantly improve the accuracy of the obtained response. As a result, a sixth order perturbation approximation used in HPM can precisely predict the nano/micromirror behavior under the combined electrostatic and vdW loading. But as it can be seen from this figure, the accuracy of all results are reduced by approaching the pull-in state. This reduction in the accuracy is due to the fact that at the pull-in state, the denominator of the electrostatic force and the vdW force is reduced and as a result the nonlinearity reaches its maximum level.

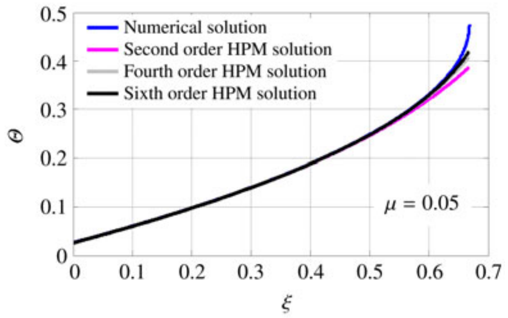

Fig. 8 Estimation of nano/micromirror's rotation angle using HPM

4 Conclusion

The effect of vdW force on the pull-in behavior of electrostatically actuated nano/micromirrors was investigated in this paper. First, static equilibrium equation was found using the minimum potential energy principle and stability of equilibrium points was analyzed. Then, a parametric study was performed to study the dependence of pull-in angle and pull-in voltage of the mirror to design parameters. It was observed that the effect of vdW force may lead to significant reduction in the stability limits of the nano/micromirror. Finally, HPM was used to analytically model the behavior of the stable equilibrium angle of the mirror. Comparison between the presented analytical solutions and numerical results show that HPM can effectively model the nano/micromirros behavior under combined electrostatic actuation and vdW force. The analytical models presented in this paper can be used in design to optimize and determine the stable operative range of electrostatic torsional nano/micro actuators where the gap between the mirror and the underneath electrodes is so small that vdW force still plays a major role in the system.

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