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A homotopy perturbation analysis of nonlinear free vibration of Timoshenko microbeams †

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Abstract

This paper uses He's Homotopy Perturbation Method (HPM) to analyze the nonlinear free vibrational behavior of clamped-clamped and clamped-free microbeams considering the effects of rotary inertia and shear deformation. Galerkin's projection method is used to reduce the governing nonlinear partial differential equation. to a nonlinear ordinary differential equation. HPM is used to find analytic expressions for nonlinear natural frequencies of the pre-stretched microbeam. A parametric study investigated the effects of design parameters such as applied axial loads and slenderness ratio. The effect of rotary inertia and shear deformation on the nonlinear natural frequency was investigated. For verification, a numerical approach was implemented to solve the nonlinear equation. of vibration. A comparison between analytical and numerical results shows that HPM can predict system nonlinear vibrational behavior significantly more accurately than previously used methods in the literature.

Keywords: Homotopy perturbation method; Modified lindstedt-poincare method; Shear deformation; Rotary inertia; Microbeam; Nonlinear free vibration

1. Introduction

Beams are the most important building blocks of most engineering structures. They are applied in various structures, from micro/nano dimensions such as micro/nano resonators, resonant sensors, and capacitive switches, to macro dimensions such as airplane wings, flexible satellites, and long-span bridges.

In these structures, large vibration amplitudes of beam- or plate-like structures often occur [1], inducing a dynamic behavior different from that predicted by linear structural dynamics theories. The sources of nonlinearities may be geometric, inertial, or material. The geometric nonlinearity may be caused by nonlinear stretching or large curvatures. Nonlinear inertial effects are caused by the presence of concentrated or distributed masses. Material nonlinearity occurs when the stresses are nonlinear functions of strains [2].

In the Euler-Bernoulli beam theory, the assumption that cross sections normal to the neutral axis continue to remain so after deformation and do not undergo any strain in their planes [3] is utilized to present a simple theory for the statical, dynamical, and vibrational behavior of beams.

Pirbodaghi et al. [4] used the homotopy analysis method (HAM) to investigate the nonlinear vibrational behavior of Euler-Bernoulli beams subjected to axial loads and provided analytical expressions for geometrically nonlinear vibrations of beams. Pillai and Rao [5] examined the problem of large amplitude free vibrations of simply supported uniform beams and found the frequency response of the system through methods such as the elliptic function method, the harmonic balance method, and the method in which simple harmonic oscillations is assumed.

Effects of rotary inertia and shear deformation are not negligible for thick beams or even thin beams vibrating at high frequencies such as micro/nano scale resonators, which vibrate at extremely high frequencies [6]. Micro/Nano mechanical resonators tend to behave nonlinearly at very small amplitudes [7, 8]. Therefore, micro/nano electromechanical resonators vibrate nonlinearly, which includes the effect of rotary inertia and shear deformation.

Effects of shear deformation and rotary inertia in the vibration of beams have been considered by only a few researchers. Zhong and Liao [9] studied higher-order nonlinear vibrations of Timoshenko beams with immovable ends. They considered the nonlinear effects of axial deformation, bending curvatures, and transverse shear strains and solved nonlinear differential equations using a spline-based differential quadrature method. Chen [10] developed a differential quadrature element method

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on the out-of-plane vibration analysis of curved nonprismatic beam structures considering the effect of shear deformation. Liao and Zhong [11] analyzed the nonlinear flexural vibration of tapered beams considering the effect of nonlinear transverse deformation, nonlinear curvature, and nonlinear axial deformation. Foda [12] used the multiple scales method to analyze the nonlinear vibrations of a beam with pinned ends considering the effects of shear deformation and rotary inertia. Ramezani et al. [13] used the same method for the same problem with doubly clamped boundary conditions. They concluded that when the theory of beams is used to study micro/nano electromechanical structures, shear deformation and rotary inertia effects should be considered for an accurate dynamic analysis.

Finding an exact solution to the nonlinear vibration of Timoshenko beams is difficult. Consequently, approximate analytical approach or numerical techniques can be used for this purpose. Besides the advantages of numerical methods, due to convenience for parametric studies and accounting for the physics of the problem, an analytical solution appears more appealing than the numerical one. Analytical solutions give a reference frame to verify and validate the numerical approaches [4].

This paper uses He's Homotopy Perturbation Method (HPM) to analyze nonlinear free vibration of microbeams. He [14] presented a new perturbation technique that does not depend on the assumption of small parameters. He used the well-known Duffing equation as an example and found that even using a first-order approximation, the maximal relative error of the period of vibration is less than 7% even when parameter ε approaches infinity. He [15] proposed this new perturbation method, which does not require a small parameter in an equation. His new method takes full advantage of traditional perturbation methods and homotopy techniques. Belendez et al. [16] solved the nonlinear differential equation, which governs the nonlinear oscillations of a simple pendulum, and showed that even only one iteration leads to a relative error of less than 2% for the approximated period of vibration even for amplitudes as high as 130°. Belendez et al. [17] found improved approximate solutions to conservative truly nonlinear oscillators using He's HPM. They found that for the second-order approximation, the relative error in the analytical approximate frequency is approximately 0.03% for any parameter values involved.

In HPM literature, this method overcomes the limitations of classical perturbation methods and accurately predicts the behavior of the nonlinear systems. Thus, this method was used in conjunction with the modified Lindstedt-Poincare method to solve the problem of nonlinear free vibrations of microbeams considering the effects of shear deformation and rotary inertia.

2. Problem formulation

Utilizing the Hamilton's principle, Ramezani et al. [13] showed that when the effects of rotary inertia and shear deformation are not negligible, the nonlinear free vibrations of microbeams considering midplane stretching is governed by the following nonlinear partial differential equation.

$$\begin{aligned} &EI\frac{\partial^4\hat{w}}{\partial\hat{x}^4}+m\frac{\partial^2\hat{w}}{\partial\hat{t}^2}-\left(mr^2+\frac{mEI}{kAG}\right)\frac{\partial^4\hat{w}}{\partial\hat{x}^2\partial\hat{t}^2}+\frac{m^2r^2}{kAG}\frac{\partial^4\hat{w}}{\partial\hat{t}^4}\\ &-N\left[\frac{\partial^2\hat{w}}{\partial\hat{x}^2}-\frac{EI}{kAG}\frac{\partial^4\hat{w}}{\partial\hat{x}^4}+\frac{mr^2}{kAG}\frac{\partial^4\hat{w}}{\partial\hat{x}^2\partial\hat{t}^2}\right]=0 \end{aligned} \tag{1}$$

In this equation, E is the Young's modulus of elasticity of the beam material, I is the second moment of area of the cross section with respect to the bending axis, $\hat{w}$ is the beam deflection, m is the longitudinal density, $\hat{t}$ is the time, A is the cross-sectional area of the beam, G is the shear modulus, and k is the shear correction factor that depends only on the geometric properties of the cross section of the beam. The parameters r and N are defined as follows:

$$N=N_0+\frac{EA}{2L}\int_0^L\left(\frac{\partial\hat{w}}{\partial\hat{x}}\right)^2d\hat{x} \tag{2}$$

$$r^2=\frac{I}{A} \tag{3}$$

where N_0 is the pretension of the beam and L is the length of the beam. Assuming the beam is vibrating at its n 'th natural frequency, and introducing the non-dimensionalized variables t, x, w, which are defined in Eqs. (4)-(7), Eq. (1) is non-dimensionalized as Eq. (8):

$$t=\frac{\hat{t}}{T_n} \tag{4}$$

$$x=\frac{\hat{x}}{L} \tag{5}$$

$$w=\frac{\hat{w}}{L} \tag{6}$$

where

$$T_n=\frac{1}{\beta_n{}^2}\sqrt{\frac{mL^4}{EI}} \tag{7}$$

$$\begin{aligned} &\frac{\partial^4 w}{\partial x^4}+\beta_n{}^4\frac{\partial^2 w}{\partial t^2}-\left(1+\frac{E}{kG}\right)\left(\frac{r}{L}\right)^2\beta_n{}^4\frac{\partial^4 w}{\partial x^2\partial t^2}+\frac{E\beta_n{}^8}{kG}\left(\frac{r}{L}\right)^4\frac{\partial^4 w}{\partial t^4}\\ &-\frac{N_0}{EA}\left(\frac{L}{r}\right)^2\left[\frac{\partial^2 w}{\partial x^2}-\frac{E}{kG}\left(\frac{r}{L}\right)^2\frac{\partial^4 w}{\partial x^4}+\frac{E\beta_n{}^4}{kG}\left(\frac{r}{L}\right)^4\frac{\partial^4 w}{\partial x^2\partial t^2}\right]=0. \end{aligned} \tag{8}$$

For clamped-clamped and clamped-free boundary conditions, β_n is the n th positive root of Eqs. (9) and (10), respectively [2].

$$\cosh\beta_n\cos\beta_n-1=0 \tag{9}$$

$$\cosh\beta_n\cos\beta_n+1=0 \tag{10}$$

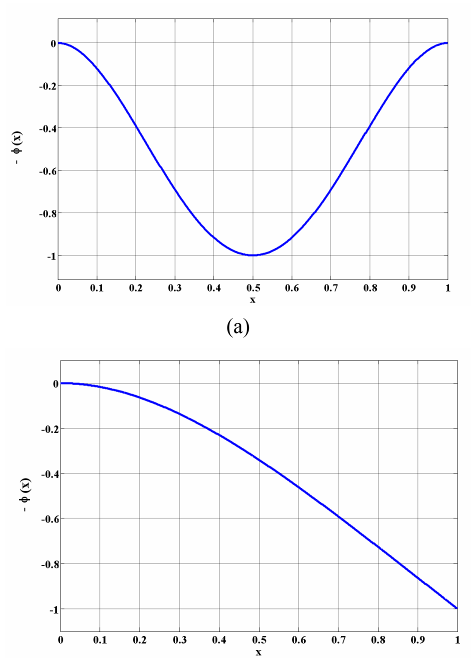


Fig. 1. First linear mode shape of (a) clamped-clamped beam; (b) clamped-free beam.

Considering the first mode of vibration, which is usually the dominant vibrational mode, the solution of Eq. (8) can be assumed as:

$$w(x,t) = \phi(x) q(t) \tag{11}$$

where $\phi(x)$ is the first linear undamped vibrational mode of the beam. For clamped-clamped and clamped-free boundary conditions, the normalized form of $\phi(x)$ can be stated as Eqs. (12) and (13), respectively,

$$\phi(x) = \frac{Q_1(x)}{Q_1(0.5)} \tag{12}$$

$$\phi(x) = \frac{\mathcal{Q}_1(x)}{\mathcal{Q}_1(1)} \tag{13}$$

where $Q_n(x)$ and $\mathcal{Q}_n(x)$ are defined in Eqs. (14) and (15), respectively.

$$Q_n(x) = \cosh\beta_n x - \cos\beta_n x - \frac{\cosh\beta_n - \cos\beta_n}{\sinh\beta_n - \sin\beta_n} (\sinh\beta_n x - \sin\beta_n x) \tag{14}$$

$$\mathcal{Q}_n(x) = \cosh\beta_n x - \cos\beta_n x - \frac{\cosh\beta_n + \cos\beta_n}{\sinh\beta_n + \sin\beta_n} (\sinh\beta_n x - \sin\beta_n x) \tag{15}$$

Fig. 1 represents the first linear mode shape of clamped-clamped and clamped-free beams.

Note that $\phi(x)$ satisfies the geometric and forcing boundary conditions of the beam, which can be expressed as Eqs. (16) to (19) and (20) to (23) for doubly clamped and clamped-free beams, respectively.

$$w(0,t) = 0 \tag{16}$$

$$\left.\frac{\partial w(x,t)}{\partial x}\right|_{(x,t)=(0,t)} = 0 \tag{17}$$

$$w(1,t) = 0 \tag{18}$$

$$\left.\frac{\partial w(x,t)}{\partial x}\right|_{(x,t)=(1,t)} = 0 \tag{19}$$

$$w(0,t) = 0 \tag{20}$$

$$\left.\frac{\partial w(x,t)}{\partial x}\right|_{(x,t)=(0,t)} = 0 \tag{21}$$

$$\left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{(x,t)=(1,t)} = 0 \tag{22}$$

$$\left.\frac{\partial^3 w(x,t)}{\partial x^3}\right|_{(x,t)=(1,t)} = 0 \tag{23}$$

The initial conditions are assumed as:

$$w(x,0) = \frac{W_{\max}}{L}\phi(x) \tag{24}$$

$$\left.\frac{\partial w(x,t)}{\partial t}\right|_{(x,t)=(x,0)} = \left.\frac{\partial^2 w(x,t)}{\partial t^2}\right|_{(x,t)=(x,0)} = \left.\frac{\partial^3 w(x,t)}{\partial t^3}\right|_{(x,t)=(x,0)} = 0. \tag{25}$$

Using Galerkin's procedure, by substituting $\phi(x)$ into Eq. (8) and integrating the residual by weight $\phi(x)$ over the problem domain, the nonlinear ordinary differential Eq. (26) for the first vibrational mode can be derived,

$$\dddot{q} + (\alpha_1 + \alpha_2 q_1^2)\ddot{q} + \alpha_3 q + \alpha_4 q^3 = 0 \tag{26}$$

where

$$\alpha_1 = \frac{1}{\beta^4 F_1}\left(\frac{L}{r}\right)^4 \left[\frac{kG}{E}F_1 + \left(1 + \frac{kG}{E}\right)\left(\frac{r}{L}\right)^2 F_2 + \frac{N_0}{EA}\left(\frac{r}{L}\right)^2 F_2\right] \tag{27}$$

$$\alpha_2 = \frac{1}{2\beta^4 F_1}\left(\frac{L}{r}\right)^2 F_2^2 \tag{28}$$

$$\alpha_3 = \frac{1}{\beta^8 F_1}\left(\frac{L}{r}\right)^4 \left[\frac{kG}{E}F_3 + \frac{kG}{E}\frac{N_0}{EA}\left(\frac{L}{r}\right)^2 F_2 + \frac{N_0}{EA}F_3\right] \tag{29}$$

$$\alpha_4 = \frac{1}{2\beta^8 F_1}\left(\frac{L}{r}\right)^4 \left[\frac{kG}{E}\left(\frac{L}{r}\right)^2 F_2^2 + F_2 F_3\right]. \tag{30}$$

In Eqs. (27) to (30), F_i's, $1 \le i \le 3$ are defined as follows.

$$F_1 = \int_0^1 \phi(x)^2\, dx \tag{31}$$

$$F_2 = \int_0^1 \phi'(x)^2 \, dx \tag{32}$$

$$F_3 = \int_0^1 \phi''(x)^2 \, dx \tag{33}$$

Homotopy perturbation is applied to solve Eq. (26). The initial conditions (24) and (25) can be converted to the following initial condition for $q(t)$.

$$q(0) = \frac{W_{\max}}{L} \tag{34}$$

$$\dot{q}(0) = 0 \tag{35}$$

$$\ddot{q}(0) = 0 \tag{36}$$

$$\dddot{q}(0) = 0 \tag{37}$$

The homotopy form is constructed as follows:

$$(1-P)\left[\ddddot{q} + \alpha_1\ddot{q} + \alpha_3 q\right] + P\left[\ddddot{q} + \alpha_1\ddot{q} + \alpha_3 q + \alpha_2 q^2\ddot{q} + \alpha_4 q^3\right] = 0 \tag{38}$$

Eq. (38) can be simplified as Eq. (39).

$$\ddddot{q} + \alpha_1\ddot{q} + \alpha_3 q + P\left[\alpha_2 q^2\ddot{q} + \alpha_4 q^3\right] = 0 \tag{39}$$

From Eq. (39), linear frequencies of the Timoshenko beam are calculated as Eqs. (40) and (41).

$$\omega_{10}^2 = \frac{\alpha_1}{2} - \sqrt{\frac{\alpha_1^2}{4} - \alpha_3} \tag{40}$$

$$\omega_{20}^2 = \frac{\alpha_1}{2} + \sqrt{\frac{\alpha_1^2}{4} - \alpha_3} \tag{41}$$

The existence of two natural frequencies in the Timoshenko beam is because the kinetic energy is composed of translational and rotary parts. The strain potential energy is composed of bending and shear parts. Adding rotary kinetic and shear strain energies to the kinetic and potential energies that appear in Euler-Bernoulli beams would lead to two frequencies, namely ω_{10} and ω_{20}. The ω_{10} and ω_{20} are known as bending natural frequency and rotary natural frequency, respectively. Eqs. (40) and (41) can be solved for α_1 and α_3, respectively. The results are as follows.

$$\alpha_1 = \omega_{10}^2 + \omega_{20}^2 \tag{42}$$

$$\alpha_3 = \omega_{10}^2\omega_{20}^2 \tag{43}$$

Using the modified Lindstedt-Poincare method [18, 19], $q(t)$, ω_{10}^2, and ω_{20}^2 are perturbed using homotopy parameter P.

$$q(t) = q_0(t) + Pq_1(t) + O(P^2) \tag{44}$$

$$\omega_{10}^2 = \omega_1^2 + P\omega_{11} + O(P^2) \tag{45}$$

$$\omega_{20}^2 = \omega_2^2 + P\omega_{21} + O(P^2) \tag{46}$$

Eqs. (45) and (46) are substituted into Eqs. (42) and (43) to find the first-order perturbation expansion of coefficients α_1 and α_3,

$$\alpha_1 = C_1 + C_2 P + O(P^2) \tag{47}$$

$$\alpha_3 = C_3 + C_4 P + O(P^2) \tag{48}$$

where

$$C_1 = \omega_1^2 + \omega_2^2 \tag{49}$$

$$C_2 = \omega_{11} + \omega_{21} \tag{50}$$

$$C_3 = \omega_1^2\omega_2^2 \tag{51}$$

$$C_4 = \omega_1^2\omega_{21} + \omega_2^2\omega_{11} . \tag{52}$$

Substituting Eqs. (44), (47), and (48) into Eq. (39) and setting the coefficients of each power of P to zero lead to the following sets of equations:

$$\ddddot{q}_0 + C_1\ddot{q}_0 + C_3 q_0 = 0 \tag{53}$$

$$\ddddot{q}_1 + C_1\ddot{q}_1 + C_3 q_1 + C_2\ddot{q}_0 + C_4 q_0 + \alpha_2 q_0^2\ddot{q}_0 + \alpha_4 q_0^{\,3} = 0 \tag{54}$$

Initial conditions (34) to (37) are translated as initial conditions (55) to (58) for Eq. (53) and initial conditions (59) to (62) for Eq. (54).

$$q_0(0) = \frac{W_{\max}}{L} \tag{55}$$

$$\dot{q}_0(0) = 0 \tag{56}$$

$$\ddot{q}_0(0) = 0 \tag{57}$$

$$\dddot{q}_0(0) = 0 \tag{58}$$

$$q_1(0) = 0 \tag{59}$$

$$\dot{q}_1(0) = 0 \tag{60}$$

$$\ddot{q}_1(0) = 0 \tag{61}$$

$$\dddot{q}_1(0) = 0 \tag{62}$$

Solving Eq. (53) yields Eq. (63) for q_0,

$$q_0(t) = A\cos\omega_1 t + B\cos\omega_2 t \tag{63}$$

where

$$A = \frac{W_{\max}}{L}\frac{\omega_2^2}{\omega_2^2 - \omega_1^2} \tag{64}$$

$$B = \frac{W_{\max}}{L}\frac{\omega_1^2}{\omega_1^2 - \omega_2^2} . \tag{65}$$

Substituting q_0 from Eq. (63) to Eq. (54) can conclude Eq. (66).

$$\begin{aligned} &\ddddot{q}_1 + C_1\ddot{q}_1 + C_3 q_1 - C_2\left(A\omega_1^2\cos\omega_1 t + B\omega_2^2\cos\omega_2 t\right) \\ &+C_4\left(A\cos\omega_1 t + B\cos\omega_2 t\right) \\ &-\alpha_2\left(A\cos\omega_1 t + B\cos\omega_2 t\right)^2\left(A\omega_1^2\cos\omega_1 t + B\omega_2^2\cos\omega_2 t\right) \\ &+\alpha_4\left(A\cos\omega_1 t + B\cos\omega_2 t\right)^3 = 0 \end{aligned} \tag{66}$$

Table 1. Microbeam properties [13].

Property name	E	G	ρ	A	k
Property value	$169Gpa$	$66Gpa$	$2330\,kg/m^3$	$15\times 6\mu m^2$	$5/6$

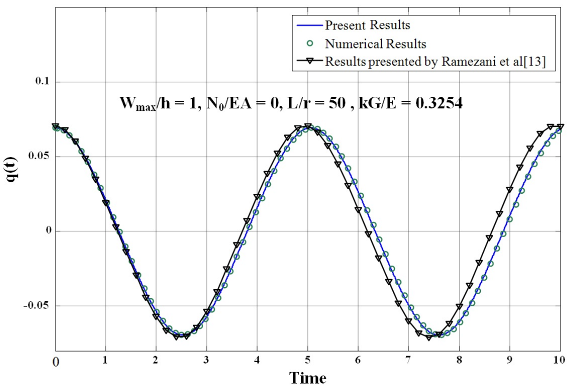


Fig. 2. Time domain response of a clamped-clamped microbeam.

Eliminating secular terms in Eq. (66) yields Eqs. (67) and (68).

$$C_4A - C_2A\omega_1^2 - \frac{3}{4}\alpha_2A^3\omega_1^2 - \alpha_2AB^2\omega_2^2 - \frac{1}{2}\alpha_2AB^2\omega_1^2 + \frac{3}{2}\alpha_4AB^2 + \frac{3}{4}\alpha_4A^3 = 0 \quad (67)$$

$$C_4B - C_2B\omega_2^2 - \frac{3}{4}\alpha_2B^3\omega_2^2 - \alpha_2A^2B\omega_1^2 - \frac{1}{2}\alpha_2A^2B\omega_2^2 + \frac{3}{2}\alpha_4A^2B + \frac{3}{4}\alpha_4B^3 = 0 \quad (68)$$

Letting $P=1$ in Eqs. (47) and (48) and noting that $C_1=\omega_1^2+\omega_2^2$ and $C_3=\omega_1^2\omega_2^2$, C_2 and C_4 are determined as follows.

$$C_2 = \alpha_1 - \left(\omega_1^2+\omega_2^2\right) \quad (69)$$

$$C_4 = \alpha_3 - \omega_1^2\omega_2^2 \quad (70)$$

By substituting Eqs. (69) and (70) into Eqs. (67) and (68), respectively, two coupled fourth-order polynomial algebraic equations in terms of ω_1 and ω_2 are obtained.

$$A\omega_1^4 - \left(\alpha_1A + \frac{3}{4}\alpha_2A^3 + \frac{1}{2}\alpha_2AB^2\right)\omega_1^2 - \left(\alpha_2AB^2\right)\omega_2^2 + \alpha_3A + \frac{3}{2}\alpha_4AB^2 + \frac{3}{4}\alpha_4A^3 = 0 \quad (71)$$

$$B\omega_2^4 - \left(\alpha_1B + \frac{3}{4}\alpha_2B^3 + \frac{1}{2}\alpha_2A^2B\right)\omega_2^2 - \left(\alpha_2A^2B\right)\omega_1^2 + \alpha_3B + \frac{3}{2}\alpha_4A^2B + \frac{3}{4}\alpha_4B^3 = 0 \quad (72)$$

Eqs. (71) and (72) can be simplified as Eqs. (73) and (74),

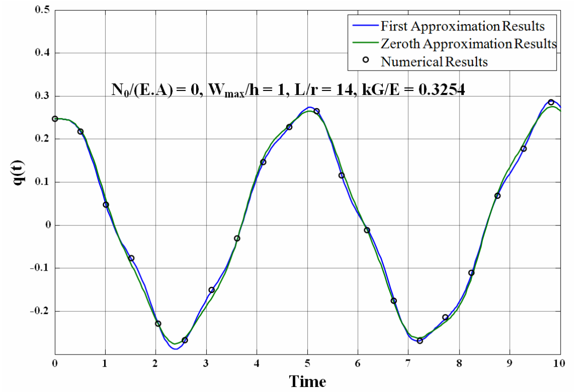


Fig. 3. Comparison of zeroth and first-order response of the system obtained by the presented method with numerical results.

respectively.

$$\omega_1^8 - \left(2\omega_2^2 + \alpha_1 + \frac{1}{2}\alpha_2a^2\right)\omega_1^6 + \left(\omega_2^4 + \left(2\alpha_1 - \alpha_2a^2\right)\omega_2^2 + \alpha_3 + \frac{3}{2}\alpha_4a^2\right)\omega_1^4 - \left(\left(\alpha_1 + \frac{3}{4}\alpha_2a^2\right)\omega_2^4 + 2\alpha_3\omega_2^2\right)\omega_1^2 + \frac{3}{4}\alpha_4a^2\omega_2^4 + \alpha_3\omega_2^4 = 0 \quad (73)$$

$$\omega_2^8 - \left(2\omega_1^2 + \alpha_1 + \frac{1}{2}\alpha_2a^2\right)\omega_2^6 + \left(\omega_1^4 + \left(2\alpha_1 - \alpha_2a^2\right)\omega_1^2 + \alpha_3 + \frac{3}{2}\alpha_4a^2\right)\omega_2^4 - \left(\left(\alpha_1 + \frac{3}{4}\alpha_2a^2\right)\omega_1^4 + 2\alpha_3\omega_1^2\right)\omega_2^2 + \frac{3}{4}\alpha_4a^2\omega_1^4 + \alpha_3\omega_1^4 = 0 \quad (74)$$

where

$$a = \frac{W_{\max}}{L}. \quad (75)$$

Eqs. (73) and (74) are solved to find the nonlinear natural frequencies ω_1 and ω_2.

3. Vibrational behavior of microbeam

A silicon microbeam with properties given in Table 1 is considered.

In Fig. 2, the presented time domain response of the system was compared with previously published and numerical results. This Fig. shows that HPM gives better approximation in such strongly nonlinear systems.

Fig. 3 shows the convergence of the method used. In this figure, the zeroth and first-order time domain responses of the system are plotted and compared with numerical results. There is no appreciable difference between zeroth and first-order results and between first-order and numerical results. Since numerical results resemble the exact solution, Fig. (3) implies that including any more terms in the perturbation expansion of the parameters would not change the results appreciably, which is a sign of convergence of the presented proce-

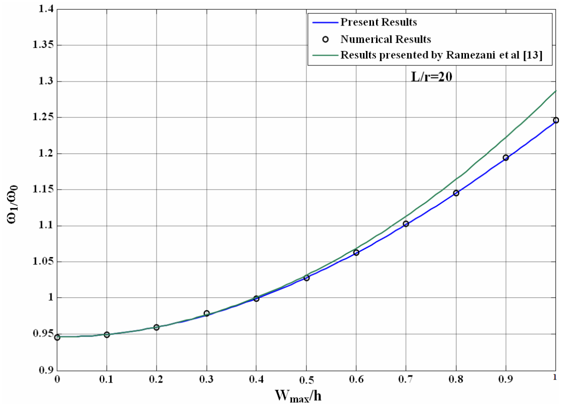


Fig. 4. Nonlinear non-dimensionalized frequency of vibration of clamped-clamped microbeam versus $W_{\max}/h$ for $L/r = 20$.

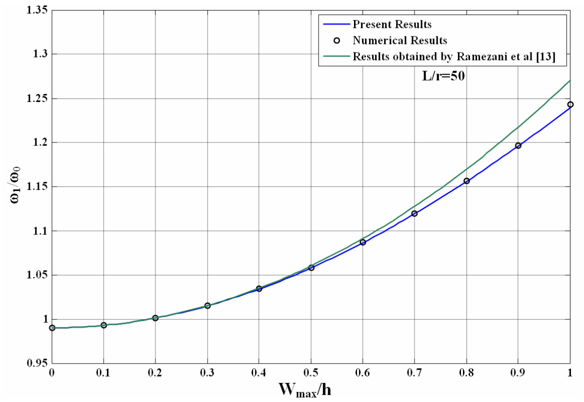


Fig. 5. Nonlinear non-dimensionalized frequency of vibration of clamped-clamped microbeam versus $W_{\max}/h$ for $L/r = 50$.

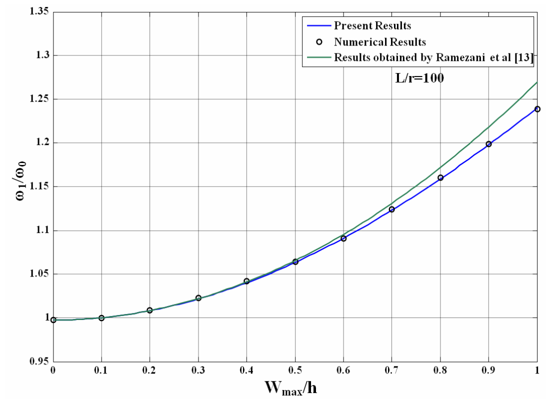


Fig. 6. Nonlinear non-dimensionalized frequency of vibration of clamped-clamped microbeam versus $W_{\max}/h$ for $L/r = 100$.

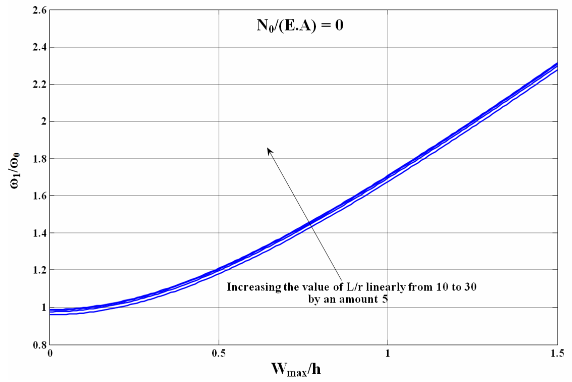


Fig. 7. Variation of nonlinear non-dimensionalized natural frequency with $W_{\max}/h$ for the case of clamped-free microbeams at different values of L/r .

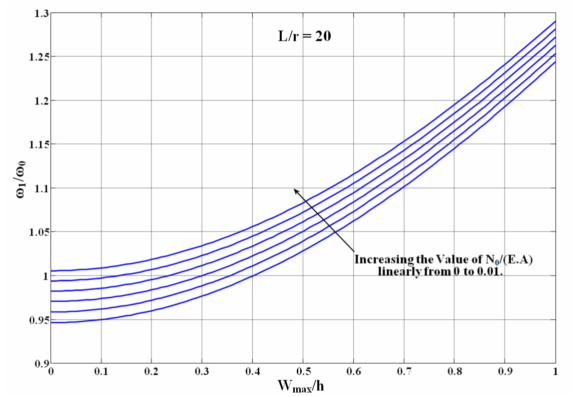


Fig. 8. Effect of the parameter $N_0/(E.A)$ on the nonlinear frequency of vibration of clamped-clamped microbeam for $L/r = 20$.

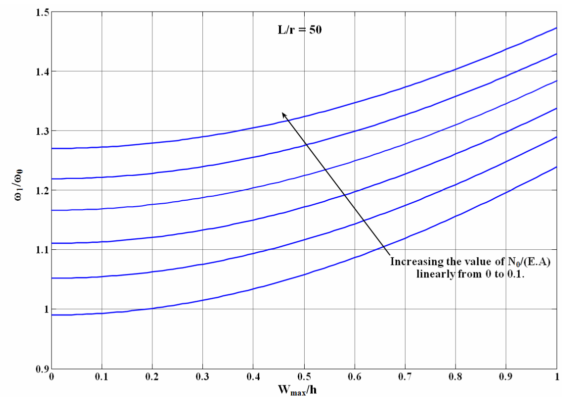


Fig. 9. Effect of the parameter $N_0/(E.A)$ on the nonlinear frequency of vibration of clamped-clamped microbeam for $L/r = 50$.

dure.

The ratio of the nonlinear natural frequency of vibration ω_1 to the linear natural frequency ω_0 of the Euler-Bernoulli beam theory is plotted in Figs. 4, 5, and 6 against $W_{\max}/h$ for different slenderness ratios, where h is the thickness of the beam. These figures compare the results of HPM, of Ramezani et al. [13] and numerical results for clamped-clamped microbeams, respectively. In Fig. 6, the expression suggested by Ramezani et al. [13] was used to generate corresponding results. HPM better predicts nonlinear frequency than the multiple time scales perturbation method used by Ramezani et al. [13].

Fig. 7 shows the variation of nonlinear non-dimensionalized natural frequency with $W_{\max}/h$ for the case of clamped-free

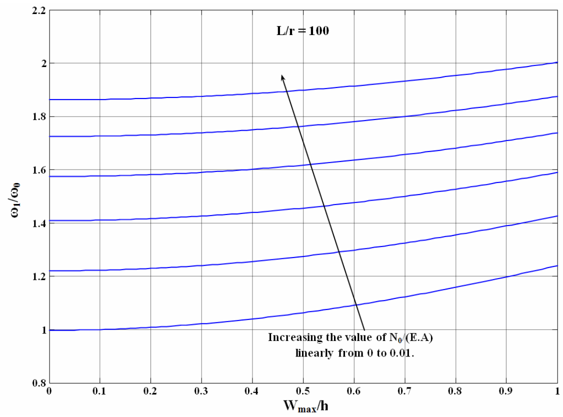


Fig. 10. Effect of the parameter $N_0/(E.A)$ on the nonlinear frequency of vibration of clamped-clamped microbeam for $L/r = 100$.

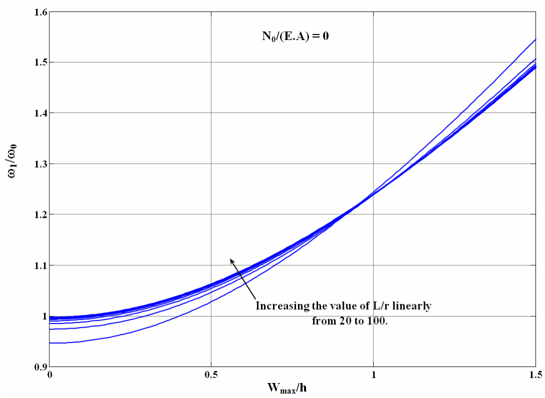


Fig. 11. Effect of the slenderness ratio on the nonlinear natural frequency of clamped-clamped microbeam when $N_0/(E.A) = 0$.

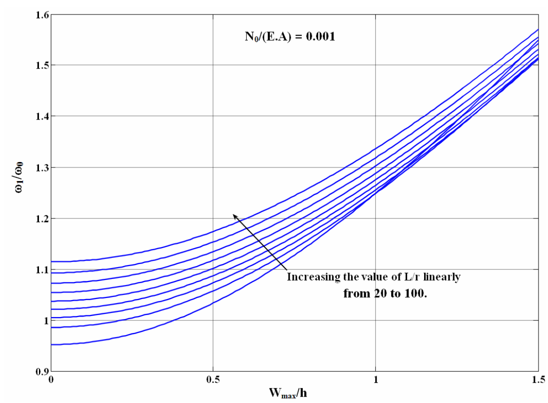


Fig. 12. Effect of the slenderness ratio on the nonlinear natural frequency of clamped-clamped microbeam when $N_0/(E.A) = 0.001$.

microbeams at different values of L/r . Just like the clamped-clamped case, the value of nonlinear natural frequency increases with increasing initial deflection of the beam.

Figs. 8, 9 and 10 show the effect of the parameter $N_0/(E.A)$ on the nonlinear natural frequency of vibration for clamped-clamped microbeams. It may be concluded that applying pre-tensile loads will increase the nonlinear natural

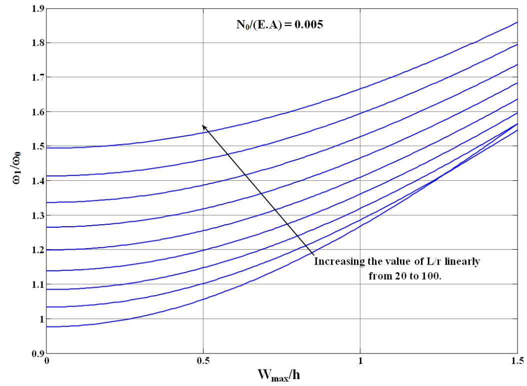


Fig. 13. Effect of the slenderness ratio on the nonlinear natural frequency of clamped-clamped microbeam when $N_0/(E.A) = 0.005$.

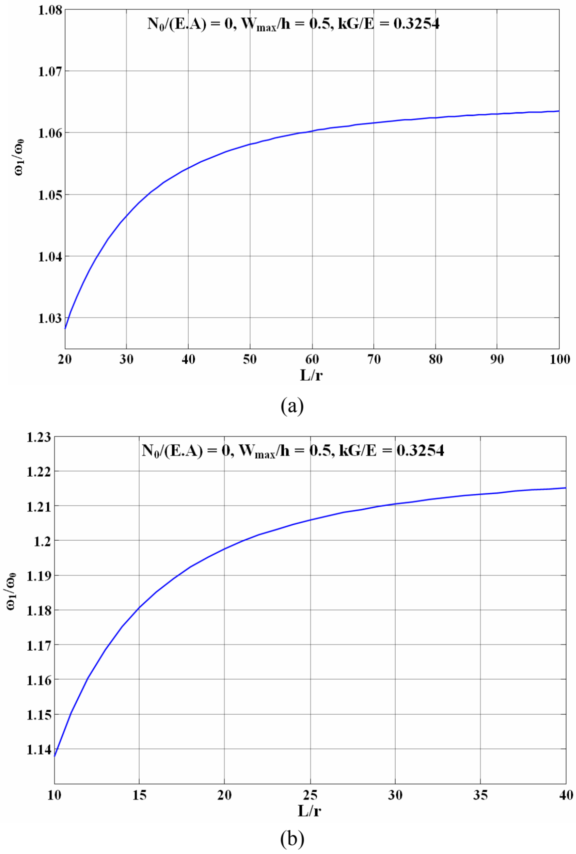
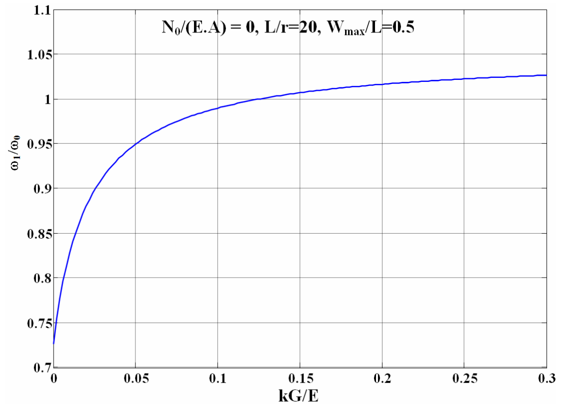


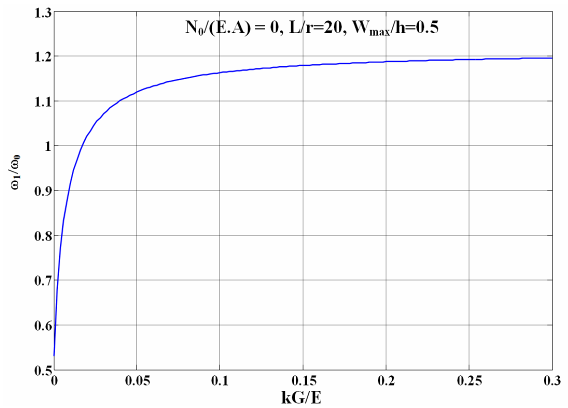
Fig. 14. Variation of non-dimensionalized natural frequency of vibration with slenderness ratio: (a) clamped-clamped and (b) clamped-free boundary conditions.

frequency of the system.

Figs. 11, 12 and 13 were predicted to investigate the effect of the slenderness ratio to the nonlinear natural frequency of the doubly clamped microbeam. For small values of W_{max}/h with increasing the slenderness ratio, the nonlinear natural frequency would increase regardless of the value of the pre-tensile axial load applied to the beam. For large values of W_{max}/h , the system may behave otherwise, depending on the value of $N_0/(E.A)$.



(a)



(b)

Fig. 15. Variation of non-dimensionalized natural frequency of vibration with kG/E : (a) clamped-clamped and (b) clamped-free boundary conditions.

To investigate the effects of rotary inertia and shear deformation on the nonlinear natural frequency, the ratio of the nonlinear natural frequency of vibration ω_1 to the linear natural frequency ω_0 of Euler-Bernoulli beam theory was plotted against L/r and kG/E in Figs. 14 and 15, respectively. These figures reveal that with increasing values of L/r and kG/E , the nonlinear frequency would converge increasingly and asymptotically to a definite value.

4. Conclusion

In this study, the combined methods of homotopy perturbation and modified Lindstedt-Poincare were used to study the nonlinear free vibrational behavior of microbeams considering the effects of shear deformation and rotary inertia. The results of HPM are significantly more accurate than previously reported analytical results. A parametric study was applied to characterize the behavior of the beam due to changes in applied pre-tensile loads and changes in the slenderness ratio. Applying tensile loads would increase the nonlinear natural frequency, while increasing the value of the slenderness ratio may increase or decrease the nonlinear natural frequency depending on the value of the initial deflection of the beam and the applied axial load. Effects of rotary inertia and shear deformation were investigated, and by increasing the effect of rotary inertia and shear deformation, the nonlinear natural frequency would increasingly converge to a definite value. This study also opens up a new feature for solving strongly nonlinear fourth-order initial value problems.

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